Local vertical datum validation through the incorporation of GOCE variance and covariance information

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1. Introduction

After the completion of the GOCE mission, the information on height system validation and unification has been enriched, especially in the low to medium frequency of the gravity field spectrum. GOCE information is used for estimating height and/or geopotential offsets with respect to a conventional reference geopotential value, employing available GNSS/Leveling observations on trigonometric BMs and a GOCE-based geoid model. The scope of this work is to investigate the influence of GOCE errors in the determination of the Hellenic LVD. This is facilitated through a least-squares (LS) based adjustment of collocated GNSS/Leveling and GOCE geoid heights over a network of 1542 BMs in mainland Greece. The latest TIM-R5 and GOCO05s GOCE and GOCE/GRACE global geopotential models are used to represent the contribution of GOCE and GRACE to the Earth's gravity field. Four different weighting scenarios are used including standard a-priori error for the BMs heights, the GNSS heights and the geoid undulations, cumulative errors for the geoid heights and variance/covariance information from GOCE geoid models. The local geopotential offset W_o^{LVD} is also calculated utilizing the weighting scenarios. Finally, variance component estimation is performed to evaluate the height (*h, H, N*) variance/covariance matrices using the GPS/leveling errors from the above weighting schemes.

2. Data availability

The available GPS/leveling data refer to stations belonging to the Hellenic Triangulation Network (see Figure 1). The leveling data were measured by the Hellenic Military Geographic Service using spirit and trigonometric leveling. Although each orthometric height is accompanied by an accuracy value, the experience of the project team has shown that these values may not be reliable. Additionally, there is no scientific documentation available for the vertical datum of Greece and inconsistencies are known to exist between the mainland and the islands. On the other hand, the GPS data (geodetic coordinates - φ, λ, h) originate from measurements carried out using Geodetic GPS receivers in the frame of the HEPOS project (Gianniou 2008). The GPS data from the HEPOS project were measured at 2430 trigonometric benchmarks and refer to ITRF00 - epoch 2007.236. Their horizontal accuracy is estimated to be between 1 and 4 cm, while their estimated vertical accuracy ranges from 2 to 5 cm. These data cover the mainland and some of the islands of Greece as presented in Figure 1.

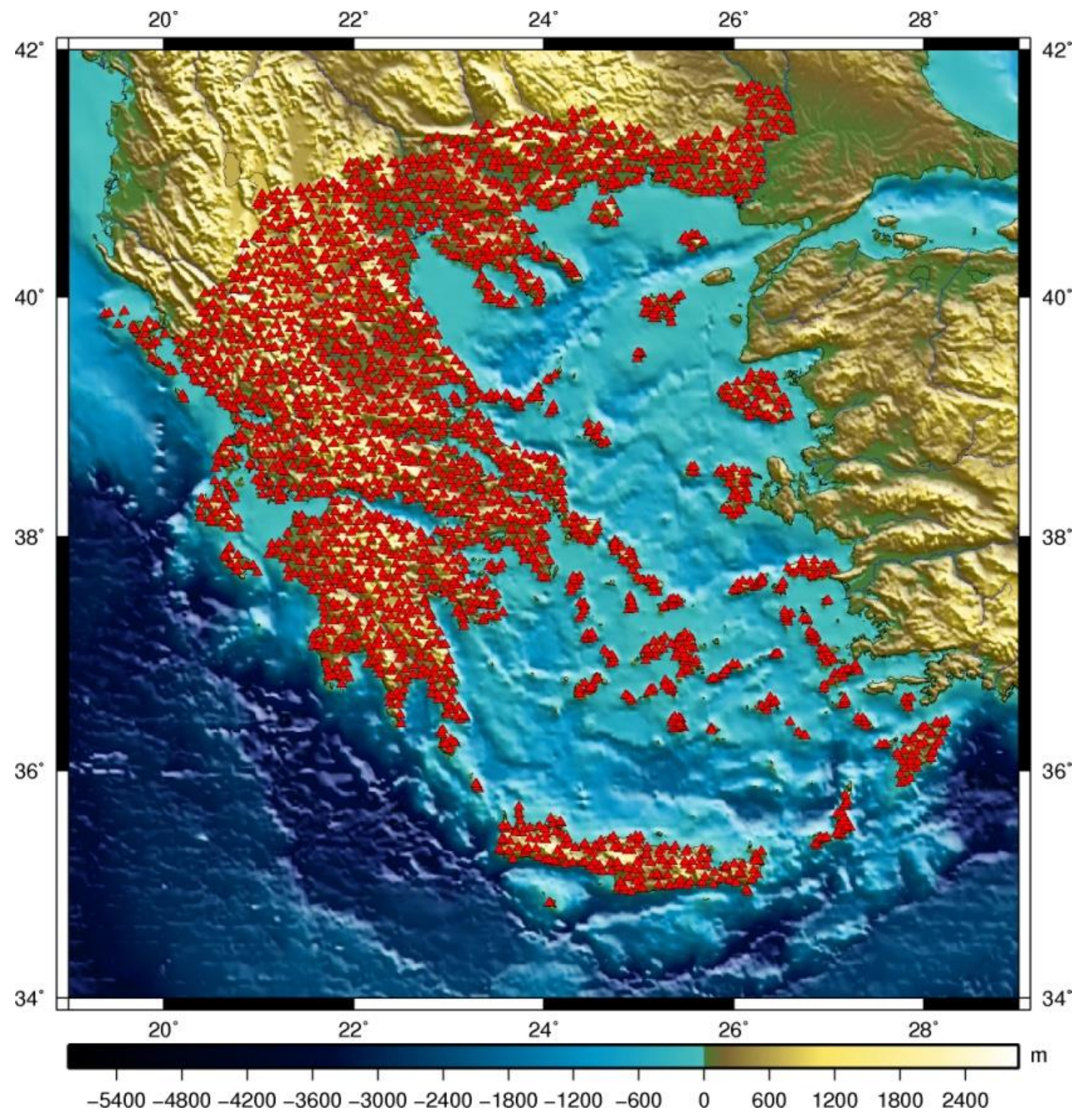


Figure 1: Spatial distribution of the GPS/Levelling points.

3. Adjustment Weighting Scenarios

Scenario 1: Equally weighted heights

$$E\{\mathbf{v}_h\mathbf{v}_h^T\}=\sigma_h^2\mathbf{Q}_h,\quad E\{\mathbf{v}_H\mathbf{v}_H^T\}=\sigma_H^2\mathbf{Q}_H,\quad E\{\mathbf{v}_N\mathbf{v}_N^T\}=\sigma_N^2\mathbf{Q}_N$$

$$\mathbf{Q}_h=\mathbf{Q}_H=\mathbf{Q}_N=\mathbf{I}$$

Scenario 2: Geoid heights weights based on geoid model cumulative errors

$$\varepsilon_{cml}^2=R^2\sum_{m=0}^{n}(\sigma_{\bar{c}_{nm}}^2+\sigma_{\bar{s}_{nm}}^2)$$

$$\mathbf{Q}_N=\varepsilon_{cml}^2\cdot\mathbf{I}=(\varepsilon_{N_{GGM}\,to\,nmax}^2+\varepsilon_{N_{08}\,nmax\,to\,2190}^2)\cdot\mathbf{I}$$

Scenario 3: Geoid height weights based on propagated error variances

$$\mathbf{Q}_N=\sigma_{prop}^2\cdot\mathbf{I}=(\sigma_{prop_{N_{GGM}\,to\,nmax}}^2+\varepsilon_{N_{08}\,nmax\,to\,2190}^2)\cdot\mathbf{I}$$

Scenario 4: Geoid height weights using full geoid model variance-covariance matrix

$$\mathbf{Q}_N=\mathbf{C}_{prop_{N_{GGM}\,to\,nmax}}^{full}+(\varepsilon_{N_{08}\,nmax\,to\,2190}^2)\cdot\mathbf{I}$$

4. Wo estimation methodology

It is possible to estimate the zero-height geopotential value by means of a LS adjustment introducing as observation equation:

$$\hat{W}_o^{LVD}=W_o-\frac{\sum_{i=1}^{m}(h_i-H_i-N_i)g_i}{m}$$

where W_o is set to 62 636 853.4 m²s⁻² (IAG Resolution No 1/2015), h_i is the geometric height of each BM derived by GPS measurements, H_i is the known orthometric height of each BM, N_i is the geoid height derived from the GMs used, g_i is the gravity at each BM computed from GMs, and m the total number of the available stations.

The weighting of the abovementioned equation observation is based on a-priori information on GPS measurements and BM orthometric height accuracy. The geoid height weights followed the four scenarios according to the GM used in each case.

4. Results

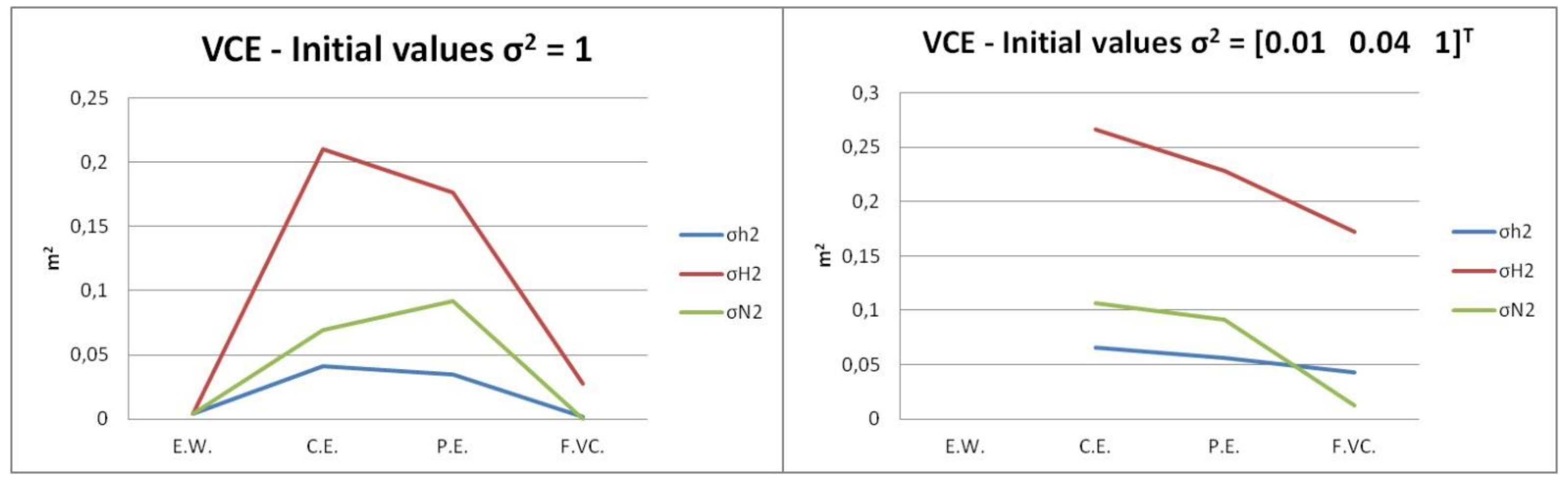
The validation is performed at first by adjusting various parametric models to the geoid height difference $(h_i-H_i-N_i)$ using parametric models, as described in Andritsanos et al. (2017).

The weighting effect on the adjustment results was studied using the four abovementioned scenarios. The statistics after the parametric adjustment remain exactly the same as in Scenario 1 showing minimal effect of the weighting in the final results. The major differences using Scenario 2 – 4 weighting procedure can be seen in the estimation of the parameters of each corrector surface as well as in the accuracy of this estimation and the a-posteriori variance of the adjustment (see Table 1). In Table 1 the degradation due to the higher coefficients of the GOCE models is also identified in the a-posteriori sd estimation of the parametric adjustment. A 6 cm a-posteriori std is computed when equally weighting heights are used. Nevertheless, this is not the case in real applications. The introduction of more realistic information of the height error led to worse results. It is of great importance that with the incorporation of more realistic errors for the geoid heights (cumulative errors, propagated errors and full variance / covariance matrix) the statistical results are improved in the case of the a-posteriori std

Table 1: Effect of the various weighting scenarios in the a-posteriori std of the parametric adjustment – parametric MODEL C. E.W.: Equally Weighted heights, C.E.: Cumulative Errors weighting scenario, P.E.: Propagated Errors weighting scenario, F.V.C.: Full Variance / Covariance weighting scenario. [m]

Geoid model	E.W.	C.E.	P.E.	F.V.C.
DIR – R5 (175)	0.0652	0.5030	0.4786	-
DIR – R5 (300)	0.1412	1.0669	0.7985	-
TIM – R5 (175)	0.0644	0.4948	0.4335	-
TIM – R5 (280)	0.1213	0.8147	0.5851	-
GOCO05S (175)	0.0643	0.4953	0.4967	0.4938
GOCO05S (280)	0.1216	0.8086	0.7651	0.6100
GOCO05C (175)	0.0648	0.5001	0.5009	0.5002
GOCO05C (720)	0.0663	0.4466	0.5127	0.5074

Figure 2: Variance Component Estimation results for the different weighting schemes. Parametric MODEL C and geopotential model GOCO05C (720 expansion degree). E.W.: Equally weighting heights, C.E.: Cumulative errors based heights. [m²]



The variance component estimation of the various heights used in the adjustment was performed using the MINQUE method. Two different cases of initial values were chosen. The variance component estimation results presented in Figure 2 confirm the statement that with the introduction of a more realistic weighting scenario, the estimations of height variance components are smaller, signalling the importance of introducing real error information in such height adjustment schemes.

Table 2: Wo for the Greek mainland derived from orthometric heights, geometric heights and GMs by a) averaging and b) by least squares adjustment with weights equal to the inverse orthometric height of each station. [m²/s²]

GM	Max Degree	Average Value (no LS adjustment)		LS adjustment with $p_i = 1/H$	
		Wo	σ	Wo	σ
DIR-R5	300	62636859.789	0.113	62636863.042	0.101
DIR-R5/EGM08	175/2160	62636859.814	0.034	62636859.950	0.028
EGM08	2160	62636859.664	0.035	62636860.239	0.029
GOCO05c	200	62636859.342	0.142	62636863.632	0.128
GOCO05c	720	62636859.647	0.058	62636860.375	0.036
GOCO05c/EGM08	175/2160	62636859.809	0.035	62636860.019	0.028
GOCO05s	200	62636859.366	0.141	62636863.533	0.127
GOCO05s	280	62636859.526	0.124	62636863.355	0.108
GOCO05s/EGM08	175/2160	62636859.843	0.034	62636859.991	0.028
TIM-R5	280	62636859.525	0.124	62636863.410	0.109
TIM-R5/EGM08	175/2160	62636859.859	0.034	62636859.987	0.029

Table 3: Wo for the Greek mainland derived from orthometric heights, geometric heights and GMs using least squares adjustment with weighting schemes based on a) the cumulative geoid error of the GMs and b) the propagated variances from the diagonal error covariance matrix of the GOCO models. [m²/s²]

GM	Max Degree	LS adjustment with $p_i=(\sigma_h^2+\sigma_H^2+\sigma_N^2)^{-1}$ and cumulative geoid error for σ_N^2		LS adjustment with $p_i=(\sigma_h^2+\sigma_H^2+\sigma_N^2)^{-1}$ and propagated variances for σ_N^2	
		Wo	σ	Wo	σ
DIR-R5	300	62636859.789	0.113	n/a	n/a
DIR-R5/EGM08	175/2160	62636859.814	0.034	n/a	n/a
EGM08	2160	62636859.664	0.035	n/a	n/a
GOCO05c	200	62636859.342	0.142	62636859.333	0.142
GOCO05c	720	62636859.647	0.058	62636859.596	0.057
GOCO05c/EGM08	175/2160	62636859.809	0.035	62636859.809	0.035
GOCO05s	200	62636859.366	0.141	62636859.379	0.141
GOCO05s	280	62636859.526	0.124	62636859.557	0.123
GOCO05s/EGM08	175/2160	62636859.843	0.034	62636859.844	0.034
TIM-R5	280	62636859.525	0.124	n/a	n/a
TIM-R5/EGM08	175/2160	62636859.859	0.034	n/a	n/a

5. Conclusions

Both the degree of expansion of the used GMs as well as the adopted weighting scheme affect the finally computed value of Wo. Moreover, different GMs lead to different values although their difference corresponds to less than 10 cm in terms of height. Therefore, the selection of the best value to be adopted for the Greek mainland is currently not possible due to the accuracy of the source data used as well as to inhomogeneities present in the Greek vertical datum (see also Andritsanos et al., 2017).

References

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