GOCE-based evaluation of the Hellenic Vertical Datum within the GOCE+++ project

Ilias N. Tziavos, Georgios S. Vergos, Vassilios N. Grigoriadis, Dimitrios A. Natsiopoulos and Eleni A. Tzanou
Laboratory of Gravity Field Research and Applications, Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Greece

Introduction and Problems

With GOCE having completed its mission at the end of October 2013, there still exists a wide range of applications that GOCE-derived products can have a significant contribution to.

The Height System Unification (HSU) over Greece is an issue of major importance since neither the mainland nor the islands use a common zero level geopotential value and/or common referend. HSU can be performed with GOCE so that a unified vertical reference datum can be generated for Greece.

The GOCE+++ project follows as a natural continuation of the successful GOCESeaComp project, with the main goal being to use GOCE data, either in the form of SGG observations and GGMs, to achieve the unification of the Hellenic vertical system .

Data used and methodology

The Hellenic Vertical Datum (HVD) was established by the Hellenic Geographic Military Service within the period 1963-1986. The physical heights in the HVD were modeled as Helmert orthometric heights in the mean-tide system and they refer to the TG station at the Piraeus harbor.

Over the Greek islands, the corresponding vertical datums were established by the Hellenic Military Geographic Service through the fixed MSL at a single tide-gauge station in each island. In essence, each island has its own LVD, which is not connected to the mainland.

The GOCE+++ project tackles the use of GOCE observations for the generation of end-products that will be useful for the wider scientific community and users.

The methodology for HSU over mainland Greece and the Greek islands will be based on an estimation of W_o^{LVD} using surface gravity and geoid heights computed from a GGM and GPS/Levelling data.

$$W_i^{LVD} = W_0^{CVD} - \Delta C_i^{CVD/LVD}$$

$$W_o^{LVD} = \frac{\sum_1^m W_i^{LVD}}{m} = W_0^{CVD} - \frac{\sum_1^m \Delta C_i^{CVD/LVD}}{m}$$

$$\Delta C_i^{CVD/LVD} = \left(h_i - H_i^{helm} - N_i - N_o\right)\bar{g}_i$$

$$\Delta C_i^{CVD/LVD} = \left(h_i - H_i^{helm} - N_{GOCE}\Big|_1^{n_1} - N_{EGM2008}\Big|_{n_1+1}^{2160} - N_o\right)\bar{g}_i$$

For mainland and islandic Greece, a total number of 2430 GPS/Leveling BMs is available and refer to stations belonging to the Hellenic Triangulation Network. The GPS data from the HEPOS project were measured at 2430 trigonometric benchmarks and refer to ITRF00 - epoch 2007.236.

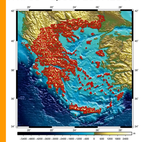


Figure 1:The 1542 GPS/leveling BMs over continental Greece and 797 BMs over the Greek islands.

All available data were first scanned for missing or invalid values. Six records were removed because they did not contain an orthometric height. The next step was to remove outliers with the aid of the EGM.

$$\Delta N = h_{TF}^{GPS} - H_{TF}^{lev} - N_{TF}^{EGM08}$$

A total of 41 values were rejected and removed mainly found on islands or in coastal areas.

Table 1: Statistics of a) the differences between geoid undulations from GPS/leveling data and EGM08, b) orthometric heights, c) geometric heights and d) geoid undulations from GPS/leveling. Unit: [m].

ΔN	max	min	mean	std
Before outlier test	4.10	-3.97	0.20	±0.25
After outlier test	0.75	-0.78	0.21	±0.17
H_{TF}^{lev}				
Before outlier test	2518.87	0.08	418.47	±401.25
After outlier test	2518.87	0.08	422.63	±401.91
h_{TF}^{GPS}				
Before outlier test	2563.85	24.43	453.87	±401.81
After outlier test	2563.85	24.43	458.07	±402.45
N				
Before outlier test	45.00	15.73	35.40	±6.92
After outlier test	45.00	15.73	35.43	±6.90

W_o determination over mainland Greece

The W_o^{LVD} over mainland Greece was determined employing the available local data and GOCE-derived GGMs without the use of any weighting scheme. For the GOCE GGMs two scenarios have been studied.

In the first one, the GOCE models are used to their maximum degree and order of expansion (300 for DIR-R5 and 280 for TIM-R5 and GOCO05s), while in the second, the GOCE derived geoid heights are synthesized as:

$$\Delta N = N^{GPS/Lev} - N^{GOCE}\Big|_2^{n_1} - N^{EGM2008}\Big|_{n_1+1}^{2160} - N_o$$

The evaluation is carried out with the GOCE/GRACE GGMs to maximum degree of expansion (n_1), while the rest of the geoid signal is represented by EGM2008 from degree n_1+1 to degree 2160. The optimum degree for this combination is 175.

Table 2: Estimated $\hat{W}_o^{LVD}$ for the Greek mainland from EGM2008 and the GOCE-based models, all evaluated to their maximum d/o of expansion. Unit: [m^2/s^2].

MODEL	$\hat{W}_o^{LVD}$
EGM2008	62636859.9129±0.0317
TIM-R5	62636859.8530±0.1094
TIM-R5 Comb	62636859.9647±0.0314
DIR-R5	62636859.7890±0.1134
DIR-R5 Comb	62636859.9216±0.319
GOCO05s	62636859.7953±0.1168
GOCO05s Comb	62636859.8902±0.0312

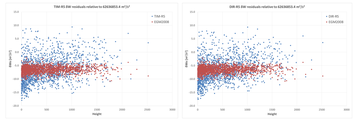


Figure 2: Potential residuals of the estimated at the GPS/Leveling BMs based on TIM-R5 and EGM2008 (top-left), DIR-R5 and EGM2008 (top-right), GOCO05s and EGM2008 (bottom).

The GOCE-based models exhibit a significant correlation with height, while those based on EGM2008 do not show such a correlation. The correlation coefficient with height is at the 7.46% for EGM2008 and reaches 35.24% for TIM-R5, 35.86% for DIR-R5 and 35.23% for GOCO05s.

Table 3: Statistics of the geoid height residuals after the estimation for the GOCE and enhanced GOCE GGMs. Unit: [m].

	max	min	mean	std
Mainland Greece				
EGM2008	0.437	-0.456	0.000	±0.127
TIM-R5	1.173	-1.617	0.000	±0.469
TIM-R5 Comb	0.478	-0.391	0.000	±0.126
DIR-R5	1.140	-1.606	0.000	±0.455
DIR-R5 Comb	0.481	-0.410	0.000	±0.127
GOCO05s	1.177	-1.616	0.000	±0.468
GOCO05s Comb	0.480	-0.388	0.000	±0.125

A significant reduction of both the range, by ~1.9 m, and std, by 34 cm, is observed, which is due to the improvement modeling of the medium-to-high and high frequencies by the EGM2008 Stokes coefficients. For all combined GGMs the statistics of the height residuals are better than those of EGM2008, with lower range and std. For TIM-R5 Comb the range is smaller by 2.4 cm and the std by 0.1 cm, for DIR-R5 Comb the range is smaller by 0.2 cm and the std is the same as that of EGM2008, and for GOCO05s the range is smaller by 2.5 cm and the std by 0.2 cm.

In each observation a spectral weight is assigned. Two scenarios are investigated, one with $p_i=1/H_i$, where H_i is the orthometric height of each BM and another with $p_i=1/L_i$, where L_i is the spherical distance between each BM and the Piraeus TG station.

	max	min	mean	std
δH_i	0.480	-0.388	0.000	±0.125
δH_i	0.487	-0.403	0.000	±0.113
$a_i^T x$	0.089	-0.117	0.000	±0.053

Table 5: Statistics of the parametric surface and the geoid height residuals before and after the basic tilt fit. Unit: [m].

Table 4: Estimated for the Greek mainland from the GOCE-based models, various weights and a height dependent parameter (left) and from the spectrally enhanced GOCE-based models (right). Unit: [m^2/s^2].

	$\hat{W}_o^{LVD}$		$\hat{W}_o^{LVD}$
EGM2008	62636859.9129±0.0317	EGM2008	62636859.9129±0.0317
$(p_i = 1/H_i)$	62636860.2078±0.0048	$(p_i = 1/H_i)$	62636860.2078±0.0048
$(p_i = 1/L_i)$	62636859.7933±0.0023	$(p_i = 1/L_i)$	62636859.7933±0.0023
(λ only) λ=-0.000209921 m/s^2	62636860.0200±0.0316	*(λ only)* λ=-0.000209921 m/s^2	62636860.0200±0.0312
$(\lambda\ \&p_i = 1/H_i)$ λ=-0.000621924 m/s^2	62636860.230±0.0048	$(\lambda\ \&p_i = 1/H_i)$ λ=-0.000621924 m/s^2	62636860.230±0.0048
		TIM-R5 Comb	62636859.9647±0.0314
TIM-R5 $(p_i = 1/H_i)$	62636863.3736±0.0913	$(p_i = 1/H_i)$	62636860.0874±0.0456
TIM-R5 $(p_i = 1/L_i)$	62636859.5032±0.0087	$(p_i = 1/L_i)$	62636859.7072±0.0022
(λ only) λ=-0.003660826 m/s^2	62636861.7202±0.1094	*(λ only)* λ=-7.03404x10^{-5} m/s^2	62636860.0006±0.0312
$(\lambda\ \&p_i = 1/H_i)$ λ=-0.007422668 m/s^2	62636863.6390±0.0176	$(\lambda\ \&p_i = 1/H_i)$ λ=-2.58664x10^{-4} m/s^2	62636860.0967±0.0046
DIR-R5 $(p_i = 1/H_i)$	62636863.0419±0.0168	DIR-R5 Comb	62636859.9216±0.319
DIR-R5 $(p_i = 1/L_i)$	62636859.3879±0.0084	$(p_i = 1/H_i)$	62636860.0814±0.0454
		$(p_i = 1/L_i)$	62636859.6978±0.0023
(λ only) λ=-0.003660814 m/s^2	62636861.6317±0.1059	*(λ only)* λ=-8.79239x10^{-5} m/s^2	62636859.9664±0.0312
$(\lambda\ \&p_i = 1/H_i)$ λ=-0.007422679 m/s^2	62636863.2879±0.0163	$(\lambda\ \&p_i = 1/H_i)$ λ=-3.37023x10^{-4} m/s^2	62636860.0935±0.0045
GOCO05s $(p_i = 1/H_i)$	62636863.2679±0.0180	GOCO05s Comb	62636859.8902±0.0312
GOCO05s $(p_i = 1/L_i)$	62636859.4429±0.0087	$(p_i = 1/H_i)$	62636860.0237±0.0045
		$(p_i = 1/L_i)$	62636859.6342±0.0022
(λ only) λ=-0.003660826 m/s^2	62636861.6610±0.1093	*(λ only)* λ=-5.41992x10^{-5} m/s^2	62636859.9178±0.0312
$(\lambda\ \&p_i = 1/H_i)$ λ=-0.007422668 m/s^2	62636863.5305±0.0176	$(\lambda\ \&p_i = 1/H_i)$ λ=-2.81476x10^{-5} m/s^2	62636860.0338±0.0045

To evaluate the existence of a tilt in the Hellenic LVD, a simple plane was fitted to the enhanced GOCO05s residuals, as:

$$\delta H_i = H_i - \frac{\hat{W}_o^{LVD} - W_o^{CVD}}{\bar{g}_i^{Helmert}}$$

$$\delta H_i = \delta H_i - \chi_o - \chi_1 \sin\varphi_i + v_i$$

$$a_i^T x = \chi_o + \chi_1 \sin\varphi_i$$

To construct a normal-gravity equivalent to the height residuals, a new variable showing the variations of the normal gravity for each point relative to the mean value in the area was determined, as:

$$\delta\gamma_i = \gamma_i - \bar{\gamma}_i$$

Finally, to investigate the spatial pattern of both the residual heights and the normal gravity variations, the ratio of the normalized height residuals and the normalized residual normal gravity , from Somigliana's formula, for each point has been estimated .

$$\delta\gamma_i = \frac{\delta\gamma_i - \bar{\delta\gamma}_i}{\sigma_{\delta\gamma_i}}$$

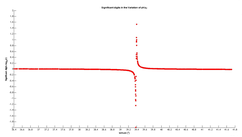


Figure 7: Significant digits in the variation of $\frac{\delta H_i}{\delta\gamma_i}$

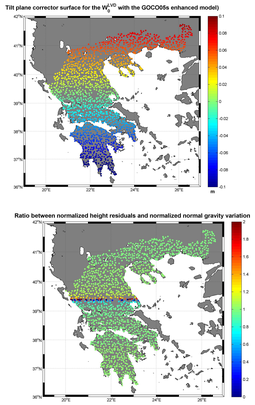
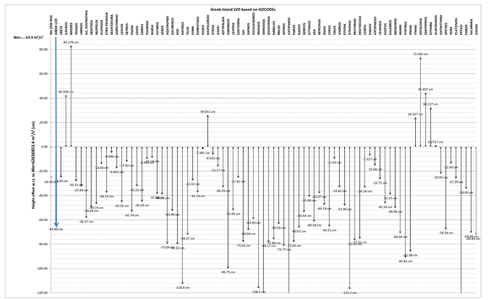


Figure 3: The corrector surface for the Hellenic LVD based on a simple plane fit to the unweighted enhanced GOCO05s height residuals (top left). Spatial distribution of normal gravity variations w.r.t. the mean normal gravity of the area (top right). Spatial distribution of ratio between height residuals and residual normal gravity

W_o determination over Greek islands



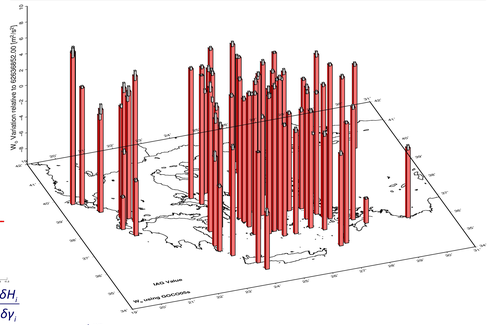
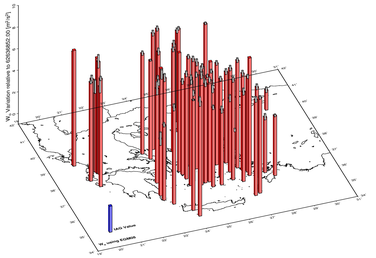
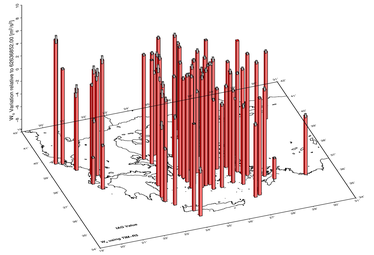


Figure 4: $\hat{W}_o^{LVD}$ estimation for the Greek islands using GOCO05s (top) and distribution (bottom)





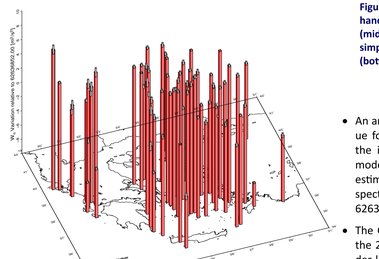


Figure 5: Distribution of $\hat{W}_o^{LVD}$ for the Greek islands using EGM2008 (top), DIR-R5 (middle), TIM-R5(bottom)

Table 6: Statistics of the parametric surface and the geoid height residuals before and after the basic tilt fit over Crete. Unit: [m].

	max	min	mean	std
δH_i	0.504	-0.434	0.000	±0.180
δH_i	0.360	-0.358	0.000	±0.153
$a_i^T \hat{x}$	0.096	-0.260	0.000	±0.177

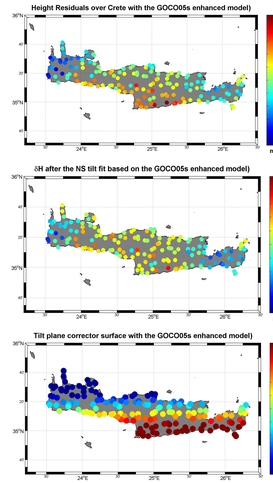


Figure 6: Distribution of height residuals in Crete from the enhanced GOCO05s $\hat{W}_o^{LVD}$ estimation (top), after the tilt correction (middle) and the corrector surface for the Crete LVD based on a simple plane fit to the enhanced GOCO05s height residuals (bottom).

Conclusions

- An analytical determination of the local zero-level geopotential value for the Hellenic Vertical Reference System (VRS) and those of the islands has been presented. The latest GOCE, GOCE/GRACE modes and EGM2008 have been employed, with the most rigorous estimate for the of the Hellenic vertical datum being that from the spectrally enhanced GOCO05s model, found equal to 62636859.8902±0.0312 m^2/s^2.
- The Greek LVD is 64.90 cm above that of the IHRS defined during the 2015 IUGG Assembly by IAG to 62636853.4 m2/s2 In the border line significant biases exist, as for instance between Chios and Izmir, being at the 67 cm level.
- The results for the islands present a high variability, due both to the small number of BMs on some of them (2-3 BMs) and the inherent problems of the Hellenic LVD.
- Future work will employ the determination of the improved DOT model(s) for the Mediterranean Sea an investigation of its influence on the geopotential differences.

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