Analysis of GOCE Omission Error and its Contribution to Vertical Datum Offsets in Greece and its Islands

Martin Willberg[1], Thomas Gruber[2], Georgios S. Vergos[3]

[1]*Technical University of Munich, Institute of Astronomical and Physical Geodesy*
+49 89 289 23187
martin.willberg@tum.de

[2]*Technical University of Munich, Institute of Astronomical and Physical Geodesy*
+49 89 289 23192
thomas.gruber@tum.de

[3]*GravLab, Department of Geodesy and Surveying, Aristotle University of Thessaloniki,*
+30 231 0994366
vergos@topo.auth.gr

Abstract – In this paper we evaluate three different geoid models (a pure and an extended satellite-only model and a local geoid solution) for the mainland of Greece and fourteen of its biggest islands in terms of signal content and applicability for height system unification. By comparing local geoid heights from GPS and spirit levelling with the three geoid models it is possible to make statements about the Earth's gravity signal that is omitted in these models (omission error). In a further step we try to quantify the contribution of the omission error to the height system unification between the investigated islands. It becomes obvious that a satellite-only gravity field model (GOCO05S) until degree and order 200 is not sufficient for the mountainous islands of Greece due to an omission error of up to 2m. The same model with high frequency corrections from EGM08 as well as topography is able to reduce the omission error drastically and shows similar results as for the local geoid model. As an outcome, we can see homogenous omission errors for the smaller islands and in general a high correlation between the size of the island and the amplitudes of the omission error.

Keywords – Omission error, GOCE, height systems, local vertical datum

1 Introduction and Problem Definition

The connection and unification of height systems has been identified as one of the most important tasks in physical geodesy. The International Association of Geodesy (IAG) accommodated this and issued a resolution about the establishment of an International Height Reference System (IHRS) (Drewes et al. 2016). The basis of height system unification is a globally consistent Earth gravity field as it is observed by the Gravity field and steady-state Ocean Circulation Explorer mission (GOCE) satellite (Drinkwater et al. 2003) in combination with other satellite information as it is provided by the Gravity Recovery And Climate Experiment mission (GRACE) (Tapley et al. 2004). It has been proven that GRACE/GOCE based Earth gravity field models deliver the static part of the geoid with an accuracy of 1-2cm at spatial scales of 100km and larger (Brockmann et al. 2014). Still missing are geoid signals with smaller spatial resolution, which cannot be observed by satellites due to their distance from the Earth surface. This is the so-called omission error, which plays an important role in height system unification and is the major topic of this paper.

We investigate the possibility to account for the omission error (or in other words the omitted signal) in a satellite-only gravity field model depending on local characteristics at the evaluation points (e.g. availability of gravity observations, terrain roughness, land-ocean transition). We regard three possible approaches: (1) The omission error is neglected at all, assuming that no surface gravity data is available. (2) The omitted signal is estimated from a global high resolution gravity field model incorporating surface and altimetry-derived gravity data, e.g. the EGM2008 model (Pavlis et al. 2012, Pavlis et al. 2013), and topography-induced gravity field information (confer Hirt et al. 2010). (3) A regional geoid model (Grigoriadis 2009) based on a satellite model and terrestrial/altimetry gravity data is used, assuming that this model contains the full gravity signal. The results obtained from these three approaches are finally compared to independent geoid heights as they are derived from GPS and spirit levelling. This will allow us to gain accuracy estimates about the incorporated data sets and/or the estimation of the omission error at each individual point investigated. Finally, the impact of the omission error on offsets between different height systems can be quantified as well.

With its hundreds of islands Greece is an ideal test area for such analyses. The mainland of Greece and the islands have numerous different (orthometric) height systems, also known as locally realised vertical datums, which have never been connected through hydrostatic levelling. Most of the islands show large topographic effects and the omission error frequently lies far above the global average of about three decimetres as it is determined from standard degree variance models (Gruber et al. 2011; Gruber et al. 2014). Furthermore, the islands in the Aegean and Ionian Sea have already been subject to several studies about the geopotential value W_0 and the height offsets (Kotsakis et al. 2012; Grigoriadis et al. 2014). These can be complemented with the different aspects of this analysis. The origin of the Hellenic Vertical Datum is defined by the tide gauge station in Piraeus harbour near Athens, but only the mainland of Greece is connected to this official vertical datum. All islands have their own vertical datum installed by the Hellenic Military Geographic Service between 1963 and 1986 according to the local mean sea level at one point respectively (Grigoriadis et al. 2014).

The situation between various islands is exemplified by two different vertical height systems and an ocean in between (Figure 1). The following description is a summary of Gruber et al. (2012), but adapted to the situation in Greece. As orthometric heights are chosen as height coordinates in Greece we stick to these in the following analyses, but all conclusions are applicable to normal heights as well. For more details about height systems, geoid determination from spherical harmonics, or regional approaches we refer to Heiskanen and Moritz (1967).

Local height systems are defined by the local equipotential surface through the origin of the vertical system, which in most cases is set to the observed mean sea level at one point at the coast (e.g. tide gauge) (brown solid line). Orthometric heights (brown dotted lines) can then be transferred from the origin to every other point on the Earth surface by spirit levelling and gravimetry.

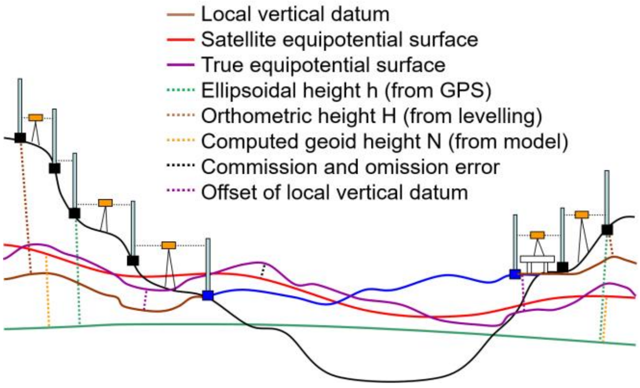

Fig. 1 Overview of different heights and reference surfaces as used in this paper (figure adapted from Gruber et al. 2012)

With the combination of ellipsoidal heights determined from GPS (green dotted lines) and in case of error-free orthometric heights one can compute the height of the local equipotential surface above the reference ellipsoid, which is named local geoid throughout the paper. But neither the local geoid height nor the orthometric height can be compared between different height systems, because of different origins in the vertical datum (Figure 1). Satellite based global gravity field models are able to deliver a globally consistent equipotential surface (red solid line), but as explained above this still differs from the true equipotential surface (purple solid line) due to the commission and omission error (black dotted line).

The paper is structured as follows: Chapter 2 describes the different data sets used in this study. After that, we present the procedures and their results for three different geoid models with a special emphasis on the accuracies of each model in chapter 3 and the influence of these accuracies on height systems in chapter 4. The final chapter 5 summarizes the results and provides some conclusions.

2 Data Sets used for the Study

For the evaluation we need geoid information from a Global Gravity Model (GGM) based on GOCE, a local geoid calculation based on terrestrial gravimetry information and GPS/levelling data, which we use to check our three approaches at selected stations. Of all the Greek islands we select only those with twelve or more GPS/levelling stations available for our study. In addition, we include data from the Greek mainland. The procedure itself is not limited to the chosen islands, but a minimum number of GPS/levelling stations helps to derive conclusive results. A map with the 14 selected islands (Andros, Chios, Corfu, Crete, Eyvoia, Karpathos, Kefalonia, Kos, Lesvos, Limnos, Naxos, Rhodes, Samos and Zakynthos) can be found in chapter 4 (Figure 3).

Geoid Solutions from Global Model

With GOCO05S we use a state-of-the-art satellite-only GGM based on all data from the GRACE and GOCE missions (Mayer-Gürr et al. 2015). Comparisons to other combined GGMs (EGM2008, GOCO05C) show that it has full signal content approximately up to degree 200 to 220. Therefore, using this model up to degree and order 200, assuming that no terrestrial data is available, is a good starting point for our initial analysis (confer case 1 in the introduction). For case 2 the omitted signal is approximated in two steps: first, by adding the EGM2008 model geoid from degree 201 to degree 2190, and second, by adding the geoid impact computed from a Residual Terrain Model (RTM) above the resolution of

EGM2008. A more detailed description of the general approach of using a GGM in order to estimate the omission error can be found in Gruber et al. (2011).

Local Hellenic Geoid Model

The Hellenic Geoid Model 2009 (HGM2009) was derived from a thoroughly validated gravity database, which contains terrestrial data for land and sea areas as well as satellite altimetry derived gravity anomalies. The HGM2009 was estimated by employing the spherical Stokes kernel and the 1D spherical FFT approach (Haagmans et al. 1993). Regarding the necessary reductions, the EGM96 (Lemoine et al. 1998) was chosen as the geopotential reference model, while a Digital Terrain and Bathymetry Model, obtained from the combination of SRTM3 (Farr et al. 2007) and SRTM30-Plus (Becker et al. 2009), was used for computing the terrain corrections.

GPS-Levelling Data

The GPS measurements used in this paper originate from a nation-wide campaign carried out in 2007 and their resulting height accuracy is given as 2-5cm (Vergos et al. 2014). The orthometric heights were measured by spirit and/or precise trigonometric levelling long before the GPS measurements were taken and their precision at that time was given as approximately 1-2cm. Nevertheless, their true precision remains unknown, because the levelling was not accompanied by local gravimetric ground measurements; instead, interpolated values from free air anomaly maps were used (Kotsakis et al. 2012). This results in a hardly quantifiable error due to nonparallel equipotential fields. For this reason, the levelling data represent the most problematic data set used in our study.

3 Omission Error Analysis

For the omission error analysis, we compare the geoid solutions of the three mentioned cases with the local geoid height which we get from GPS-levelling by subtracting the orthometric height H from the ellipsoidal height h (compare Figure 1). This is done for every point i with GPS/levelling observations by the difference

$$\Delta N_i = N_i - (h_i - H_i), \quad (1)$$

where N_i is the selected geoid solution. As the geoid model N and the orthometric height H in general refer to different equipotential reference surfaces there is a height offset in ΔN. It is assumed that the GPS/levelling observations contain the full signal of the Earth gravity field, so the omission error of our geoid models N completely transfers to ΔN along with random and systematic errors in all three quantities involved. Systematic errors can occur due to geometrical distortions in the levelling network, long or medium wavelength effects in the geoid model, datum inconsistencies between geoid and ellipsoidal heights and unmodeled time-dependent variations (Kotsakis et al. 2012). For analyzing the omission error, we eliminate the constant offset and the systematic distortion from the observations by a planar fit to ΔN and by subtracting this plane from the differences.

$$\Delta N_i^{corrected} = N_i - (h_i - H_i) - \Delta N^{correctionSurface}. \qquad (2)$$

We do not apply a higher order correction surface as this could partially remove the omitted signal as well. Because there are outliers in the GPS/levelling data we also apply a simple 2σ criterion during the data processing, which eliminates about 5% of our observation points. Also, these outliers were not used for further analyses.

After removing the offset and the systematic distortions, the random errors as well as the omission errors remain in $\Delta N_i^{corrected}$ and can be interpreted for our three test cases. As we are not interested in single point differences we use the standard deviations over a target area (island or mainland Greece respectively) to evaluate geoid differences (Figure 2). When we neglect the observation errors of GPS and levelling for a moment and assume that the estimation of the correction plane removes systematic distortions, then in case 1 and 2 the remaining $\Delta N_i^{corrected}$ gives us the sum of omission and commission error of our geoid model N determined from spherical harmonics. In case 3, in contrast, $\Delta N_i^{corrected}$ shows mainly modelling errors in the local Hellenic Geoid. According to a variance-covariance propagation of the GOCO05S model, the commission error in Greece accounts for about 1.6cm.

The standard deviations in case 1 range from 5 to 65cm and are much higher than in the other cases (Figure 2) because all gravity field signals above degree 200 are neglected. In both the second and the third case our extreme values account from 3-4cm to about 12cm; thereby the local Hellenic geoid in case 3 generally provides slightly better results.

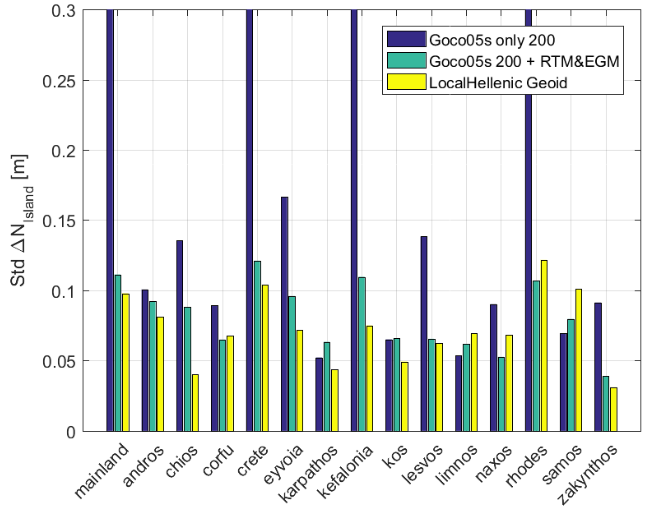

Fig. 2 Standard deviation of $\Delta N_i^{corrected}$ for the three different geoid models: case 1 (blue), case 2 (green) and case 3 (yellow). All bars are presented after parameter estimation of a plane and outlier removal by a 2σ criterion. Cutted bars show 48cm for the mainland, 64cm for Crete, 39cm for Kefalonia and 33cm for Rhodes

As expected, case 1 shows by far the worst results, though there are islands (Karpathos, Limnos and Samos) where case 1 performs slightly better than case 2. In general, it can be seen that large islands, respectively islands with a higher number of measurement points, tend to show higher omission errors here.

Of course our simplification with error-free observations is not true and, in fact, we already know that our GPS/levelling observations were not optimally done. The visualized difference in Figure 2 shows (in all 3 cases) random errors due to the observation accuracy of GPS and spirit levelling and therefore the omission error (case 1&2) and the modelling errors (case 3) of the high frequency signal are even below the values presented in Figure 2. As a conclusion, the approach in case 2, where we calculate the geoid heights N only by using a global satellite model, point positions, and EGM08 coefficients as input, shows almost the same performance as the far more complex local geoid calculation.

4 Height System Offsets between Islands

In chapter 3 we used the comparison of the selected geoid models to the local geoid from GPS/levelling to evaluate the accuracy of our models at selected islands. Now we take the geoid model differences ΔN again, but do not subtract the correction surface resulting in absolute geoid height offsets ΔN^{mean} for every island. This is done by calculating the mean value over Eq. (1), where n is the number of observation points per island:

$$\Delta N^{mean} = \Sigma(N_i - (h_i - H_i))/n. \quad (3)$$

These ΔN^{mean} values per island represent the mean offset of the Local Vertical Datum (LVD) to the geoid model N and enable the connection of different vertical datums. As in general, there are no well observed tide gauge stations on the Greek islands; it is considered to be more accurate to use mean values over the whole island instead of single reference points (e.g. tide gauges) for the offsets of the LVD. However, with this consideration it is not possible to compare heights between two individual points of different vertical datums as the offsets stay unknown.

In Figure 3 the offsets of the LVD are visualized for every island and the mainland in case 1 (upper value) and case 2 (lower value). While in case 1 the offsets have a wide distribution from -243 to +25cm, they range from -38 to +13cm in case 2. Almost all of the offsets are negative which means that the LVD for that island is below the used geoid model.

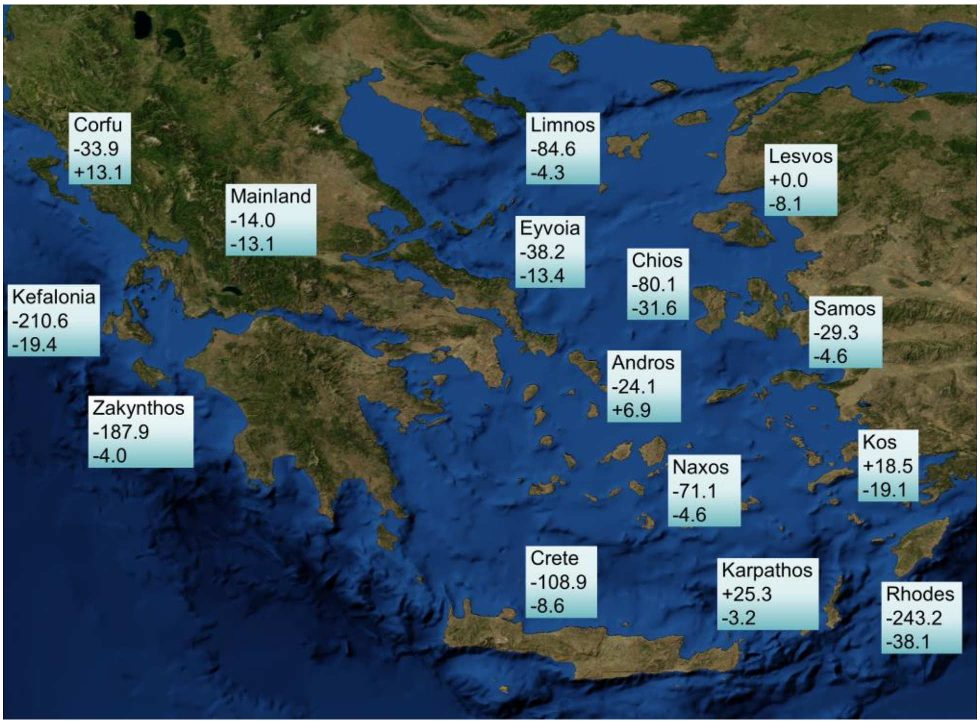

Fig 3 Mean offset of GOCO05s and extended GOCO05s geoid models to point-wise GPS-levelling observations. The two values give the mean offset of the LVD when calculating the model with EGM and RTM above degree 200 (case 2, bottom value) and without them (case 1, upper value)

In chapter 3 we showed that the GOCO05S model performs much better when adding high frequency parts from EGM and RTM information. This allows us to calculate the omission error in case 1 by using the more accurate case 2 results as reference. The omission error of GOCO05S up to degree and order 200 is then a simple difference of the two values in Figure 3. Regarding the islands, it varies from about 8cm for Lesvos to almost 2m for Rhodes. Most of the islands show omission errors far above the average of 30cm, while the impact for the mainland is much smaller (3cm). The reason is the small size of the islands compared to the resolution of GOCE (about 100km for degree 200). Even the biggest island, Crete, has only an extension of up to 55km in the north-south direction. When a target area is smaller than the resolution of GOCE the satellite-only gravity field is not able to calculate a representative mean value (especially when there is variable topography) and this can result in increased omission errors (Figure 3).

The bottom values in Figure 3 are then used for the computation of height offsets between the data sets (islands and mainland) as shown in Figure 4. The offsets of the LVD in case 2 are presented as absolute values of the pairwise differences which gives us a 15x15 matrix where the colour indicates the height system offset between two data sets. The result is a symmetric matrix with values up to 50cm with the maximum being the result of the difference between the highest and the lowest offset (Corfu and Rhodes). Dark blue values show data sets with similar offsets of the vertical datum while brighter values (e.g. column or line of Rhodes) indicate that a LVD has a large discrepancy to the others. Pairwise differences provide an easy way for height system unification to visualize height systems offsets.

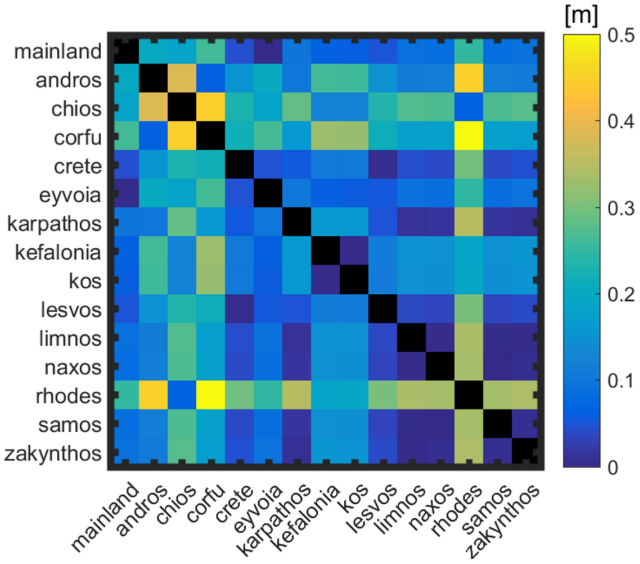

Fig. 4 Pairwise difference of the height offset between chosen islands/mainland. Calculation based on the GOCO05S model with EGM & RTM (case 2)

5 Summary and Conclusions

When combining the results from chapter 3 and 4 it becomes clear that a satellite-only model (case 1) is not suitable to calculate geoid heights for the Greek islands. The small size of the islands leads to large omission errors because GOCE is not able to distinguish the island from the surrounding sea. However, the omission error for the smaller islands (all except Crete, Rhodes, Kefalonia) is homogenous and similar to all points on the island, which can be seen by the small standard deviation in Figure 2.

Case 2 shows in both investigations large differences compared to case 1, which again demonstrates that the gravity field signal above degree 200 should not be neglected. The big differences between the smaller and the bigger islands in case 1 are reduced though not eliminated in case 2 when using the GOCO05S model with EGM and RTM information. And the geoid differences are quite similar between case 2 and 3, which is a good indicator that a satellite-only model with corrections is able to adapt to local characteristics.

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