GOCE variance and covariance contribution to height system unification

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Abstract

The definition and realization of vertical datum is a key concept in support of not only geodetic works but also for surveying and hydraulic studies to name a few. In the GOCE era, this is customarily done by estimating height and/or geopotential offsets with respect to a conventional reference geopotential value or to available GNSS/Leveling observations on trigonometric BMs and a GOCE-based geoid. This work investigates the influence of GOCE errors in the determination of the Hellenic Local Vertical Datum. This is facilitated through a least-squares adjustment of collocated GNSS/Leveling and GOCE geoid heights over a network of 1542 BMs. TIM-R5, GOCO05s and GOCO05c Global Geopotential Models (GGMs) are used for representing the contribution of GOCE and GRACE to the Earth's gravity field. First, a weighted adjustment is carried out employing the GGMs commission error as indicative of the geoid height variance for all stations. Then, full variance-covariance matrices of the GGMs are employed for utilizing realistic GOCE error information and investigating their influence on the adjustment results. Using the available GNSS/Leveling formal errors, a Variance Component Estimation (VCE) is performed to evaluate height (h, H, N) error matrices and assess the stochastic model for the corresponding observational noise. VCE is used to address the impact of a simplified uniform variance assumption for all geoid height data on the final prediction variances in contrast to using the full covariance matrices. Finally, zero-level geopotential values are estimated for the Greek mainland following weighting schemes as the ones described above.

1 Introduction

The use of heights is of main importance for a wide range of geodetic, surveying and engineering applications. In the case of orthometric heights, height differences are determined nationwide by conventional spirit leveling accompanied by gravity measurements along dedicated traverses. The orthometric heights of all established benchmarks (BMs) are then obtained, through a least-squares (LS) adjustment of the entire vertical network, as height differences w.r.t. a selected BM that serves as the origin point of the country's vertical reference system.

With GOCE having completed its mission at the end of October 2013, there still exists a wide range of applications that GOCE-derived products can have a significant contribution to. It has been very recently concluded that GOCE, apart from a high-accuracy static gravity field (Brockmann et al, 2014; Bruinsma et al., 2013; Mayer-Gürr et al. 2015), can offer unique insights to oceanographic, engineering and geophysical applications (Albertella et al., 2012; Fuchs et al., 2013; Reguzzoni et al., 2013; Tziavos et al., 2013).

In the pure geodetic context, the contribution of GOCE is viewed in improving and assessing local gravity and geoid models also in combination with GPS/levelling measurements (Andritsanos et al., 2015; Tziavos et al., 2016; Vergos et al., 2014, 2015).

Height System Unification (HSU) over Greece is an issue of major importance since neither the mainland nor the islands use a common zero level geopotential value and/or common referend. The Hellenic Vertical Datum (HVD) was established by the Hellenic Geographic Military Service within the period 1963-1986. In principle, the physical heights in the HVD were modeled as Helmert orthometric heights in the mean-tide system. They refer to the TG station at the Piraeus harbor, where local MSL was computed from sea level measurements over the period 1933-1978 (Takos 1989). The true accuracy of the HVD's leveling network is largely unknown, while a common adjustment of the entire vertical network was never performed. Over the Greek islands, the corresponding vertical datums were established by the Hellenic Military Geographic Service through the fixed MSL at a single tide-gauge station in each island. In essence, each island has its own LVD, which is not connected to the mainland and to the origin of the HVD at Piraeus harbor.

Kotsakis et al. (2002) estimated the zero-level geopotential value for selected Greek Islands, while Grigoriadis et al. (2014) for the Greek mainland. In all previous studies, no additional information about the weighting of the geoid heights was taken into account. In this study, four different weighting scenarios were considered in order to perform efficient validation on the heights systems and draw some conclusions on the deformations present in the HVD.

2 Methodology and data

2.1 Data and preprocessing

For the present study, GPS/levelling data that refer to stations belonging to the Hellenic Triangulation Network were available along with geoid heights obtained from GOCE/GRACE-based GGMs, GOCO5C (Fecher et al., 2016), GOCO05S (Mayer-Gürr et al., 2015) and TIM-R5 (Brockmann et al. 2014), and EGM2008 (Pavlis et al., 2012). Regarding the leveling data, these were measured by the Hellenic Military Geographic Service using spirit and trigonometric leveling. There is no scientific documentation available for the vertical datum of Greece and inconsistencies are known to exist between the mainland and the islands. On the other hand, the GPS data originate from measurements carried out using Geodetic GPS receivers in the frame of the HEPOS project (Gianniou 2008). For more information about the GPS/Leveling data and their distribution please consult Tziavos et al. (2016).

2.2 Adjustment combination schemes

The residual geoid heights ΔN have been evaluated first following a spectral enhancement approach (Vergos et al., 2015) as:

$$\Delta N = N^{GPS/Lev} - N^{GOCE}\Big|_{2}^{n_1} - N^{EGM2008}\Big|_{n_1+1}^{2190} - N^{RTM} - N_o\,, \tag{1}$$

where $N^{GPS/Lev}$ are the GPS/Leveling geoid heights, N^{GOCE} and $N^{EGM2008}$ the GOCE-based and EGM2008 GGM-derived geoid heights respectively and N_o the zero-degree geoid (see Moritz et al., 1967 – Eq. 2.182) with GRS80 used as the reference ellipsoid. The Residual Terrain Model effects on geoid heights (N^{RTM}), was computed from an SRTM-based 3 arcsec digital terrain model (Tziavos et al., 2010), so that the geoid spectrum represented is equivalent to d/o 216,000. Therefore, the geoid omission error is very small (mm-level), so it can be neglected in the formed differences. Finally, all computations have been performed in the Tide Free system, while the necessary conversions were performed according to Ekman (1989). The evaluation scheme has been carried out for d/o 175 of each GGM up to their n_{max}. The choice of the 175 d/o of expansion is made according to previous studies on the optimal combination synthesis, where the spectral range that the GOCE GGMs perform better than EGM2008 (Tziavos et al., 2016; Vergos et al. 2015). For a more elaborate discussion of the followed methodology and conventions Carrion et al. (2015), Tocho and Vergos (2015), Tziavos et al. (2016) and Vergos et al. (2015) should be consulted.

ΔN may be described by various parametric models following a least-squares adjustment procedure. For the validation of the GOCE/GRACE GGMs six models were selected. The well-known four- (MODEL A) and five-parameter (MODEL-B) similarity transformation models (Heiskanen and Mortiz, 1967), a model that corresponds to a height-dependent corrector surface with a simple bias and two scale terms (MODEL C), a bias and an orthometric height scale term (MODEL D) and a bias and a geoid height scale term (MODEL E) (Kotsakis and Katsampalos, 2010). Finally, a 3rd order polynomial (MODEL F) can be used as a corrector surface in the computations as outlined in Vergos and Sideris (2002).

2.3 Error consideration

Crucial role in the height adjustment has the proper propagation of errors v, so that the final estimate will be reliable. The errors may be divided into an orthometric, ellipsoid and geoid height error component. Each unknown error component can be described by its second-order stochastic model of the form (Kotsakis and Sideris, 1999):

$$E\{\mathbf{v}_h\mathbf{v}_h^T\}=\mathbf{C}_h,\quad E\{\mathbf{v}_H\mathbf{v}_H^T\}=\mathbf{C}_H,\quad E\{\mathbf{v}_N\mathbf{v}_N^T\}=\mathbf{C}_N \tag{2}$$

For the orthometric heights, the covariance (CV) matrix $\mathbf{C}_H$ is usually known from the adjustment of the leveling network, while $\mathbf{C}_h$ can be computed from the adjustment of the GPS surveys performed at the leveled benchmarks. Unfortunately, this is not the case in the present study. Since no reliable information about leveling and GPS error is available, only assumptions on their statistical information will be made. In the gravimetric geoid case, the covariance matrix $\mathbf{C}_N$ is computed using four different scenarios. These error scenarios that will be used in the weighting of the adjustment and the Variance Component Estimation (VCE) that follows can be summarized as:

Scenario 1. Equally weighted heights

The following stochastic model will be adopted for the random noise effects in the three height data sets:

$$E\{\mathbf{v}_h\mathbf{v}_h^T\}=\sigma_h^2\mathbf{Q}_h, \quad E\{\mathbf{v}_H\mathbf{v}_H^T\}=\sigma_H^2\mathbf{Q}_H, \quad E\{\mathbf{v}_N\mathbf{v}_N^T\}=\sigma_N^2\mathbf{Q}_N \tag{3}$$

where the cofactor matrices $\mathbf{Q}_h$, $\mathbf{Q}_H$, and $\mathbf{Q}_N$ are assumed equal to the identity matrix, and the three variance components are treated as unknown parameters controlling the validity of the a priori random error models. The assumption of the equally weighted height is not the case in real applications but we chose this scenario in order to investigate the adequacy of the tested parametric models (MODELS A to F) to the height fitting.

Scenario 2. Geoid height weights based on geoid model cumulative errors

In this case, the stochastic model of the orthometric and ellipsoid heights is chosen to be a standard value based on some a-priori information about the accuracy of the respective observations. Weights of 1/0.01 and 1/0.04 are adopted for the ellipsoid heights and for the orthometric heights respectively. This accuracy ratio (1/4) is close to real world applications, where the ellipsoid heights are estimations with increased accuracy with respect to the orthometric heights of base trigonometric networks, derived from older measurement adjustments.
In this scenario, the geoid height stochastic model is derived using synthetic information of the cumulative geoid model error. The synthesis is based on the error degree variances of GOCE model till d/o 175 or n_{max} and the residual geoid error computed using error degree variances of EGM2008. The stochastic model of the current scenario is provided by the cofactor matrix of the geoid:

$$\mathbf{Q}_N = \varepsilon_{cml}^2 \cdot \mathbf{I} = \left(\varepsilon_{N_{GGM\_nmax}}^2 + \varepsilon_{N_{08\_nmax\_to\_2190}}^2\right) \cdot \mathbf{I} \tag{4}$$

where $\varepsilon_{N_{GGM}}$ is the cumulative error of the respective GOCE-based model and $\varepsilon_{N_{08}}$ is the contribution of the EGM2008 to the total error.

Scenario 3. Geoid height weights from propagated error variances

The stochastic model of the geoid heights in this scenario is based on the propagated error variances of the GOCE geoid models. The stochastic model is constructed using the propagated error of the respective GOCE model ($\sigma_{prop_{N_{GGM\_nmax}}}^2$) till a specific degree (175 or n_{max}) and the residual of the cumulative error of EGM2008 geopotential model according to the equation:

$$\mathbf{Q}_N = \sigma_{prop}^2 \cdot \mathbf{I} = \left(\sigma_{prop_{N_{GGM\_nmax}}}^2 + \varepsilon_{N_{08\_nmax\_to\_2190}}^2\right) \cdot \mathbf{I} \tag{5}$$

Scenario 4. Geoid height weights using full geoid variance-covariance matrix

In this final scenario, the full variance-covariance matrix of the GOCO05x models is used till a maximum degree of expansion (n_{max} = 175, 280 or 720) and the contribution of EGM2008 is taken into account from this degree and above. The cofactor matrix of the geoid heights is provided by:

$$\mathbf{Q}_N = \mathbf{C}_{prop_{N_{GGM\_n\max}}}^{full} + \varepsilon_{N_{08\_n\max\_to\_2190}}^2 \cdot \mathbf{I} \tag{6}$$

where $\mathbf{C}_{prop_{N_{GGM\_nmax}}}^{full}$ is the full variance – covariance matrix of the geopotential model. The full covariance information is available only up to n_{max} and only for the GOCO05x models.

3 Results and discussion

The differences ΔN at the 1542 GPS/Levelling benchmarks using the spectral enhancement method are presented in Figure 1. As it can be seen from Figure 1, the standard deviation (std) of the differences between GPS/Levelling geoid heights and GGM geoid heights is at the level of ±12.7 to ±13.0 cm when the assimilation degree of the GOCE model reaches d/o 175 and decreases when the complete signal of the GOCE model is used. An interesting exception is the case of GOCO05C where the std of the differences stays at the level of ±13.4 cm probably due to the use of surface gravity data to the coefficients estimation. TIM-R5 performs slightly better, in terms of std of the differences, than the other models using an assimilation degree 175. Using the full spectra of TIM-R5 (expansion degree 280) some geographically correlated errors appeared before any parametric fitting. This fact confirms the weak character of the higher harmonics of GOCE models above degree 200. Still EGM2008 contribution performed better than GOCE models in the band between 200 and 280 degree. In addition, the geographically correlated errors remain smaller than the ones in the case of DIR-R5 to its full d/o 300, confirming the error augmentation as the assimilation degree grows.

Figure 1

3.1 Parametric models adjustment

The effect of the parametric model used in the adjustment of the differences is examined. Tables 1 and 2 present the statistics for TIM-R5 model which proved the best in the assimilation test of the previous section. In Table 1 MODEL C gave the best statistical results in terms of the std of the differences. An improvement of 1.6 cm is presented using this mixed bias, geoid and orthometric height factor model. Considering Figure 2, major differences remain after the parametric adjustment at the mountainous areas of Greece, focusing on the problematic character of the orthometric heights in steep terrain.

Table 1

The incorporation of the complete signal of GOCE models gave worst statistical results as seen in Table 2. An improvement of 1.4 cm in terms of the std of the differences is presented using a mixed bias and orthometric height scale factor model (MODEL D).

Table 2

The statistics of the best performed parametric MODEL C using the tested GOCE GGMs are presented in Table 3. When the assimilation degree of each GOCE model reaches the degree 175 similar results are obtained. A std of ±11 cm is computed with the best results when TIM–R5 and GOCO05S are utilized. If the complete signal of GOCE GGMs is used, degradation in the statistics is noticed. This degradation is addressed to the erroneous effects of the higher coefficients of GOCE models w.r.t. the EGM2008 coefficients. It is to be noted that satellite-only GOCE models lack high frequency information found in GOCO05C since surface gravity data were not included in the computation of their coefficients. This is the main reason why the std of the differences remains at the order of ±11.5 cm. The values of the corrector surfaces computed from the parametric model adjustment reveal a South-to-North and East-to-West trend and a correlation with geoid and orthometric heights in the case of MODEL C and D.

Table 3

Figure 2

3.2 Weighting effect

The weighting effect on the adjustment results is studied using the four abovementioned scenarios. The statistics after the parametric adjustment remain exactly the same as in Scenario 1 showing minimal effect of the weighting in the final results. The major differences using Scenario 2 – 4 weighting procedure can be seen in the estimation of the parameters of each corrector surface as well as in the accuracy of this estimation and the a-posteriori variance of the adjustment (see Table 4). In Table 4 the degradation due to the higher coefficients of the GOCE models is also identified in the a-posteriori std estimation of the parametric adjustment. A 6 cm a-posteriori std is computed when equally weighting heights are used. Nevertheless, this is not the case in real applications. The introduction of more realistic information of the height error led to worst results. It is of great importance that with the incorporation of more realistic errors for the geoid heights (cumulative errors, propagated errors and full variance / covariance matrix) the statistical results are improved in the case of the a-posteriori std.

Table 4

3.3 Variance component estimation

The variance component estimation of the various heights used in the adjustment was performed using the MINQUE method (Rao, 1971; Rao and Kleffe, 1988). Two different cases of initial values were chosen, as seen in Table 5. The variance component estimation results presented in Table 5 and Figure 3 confirm the statement that with the introduction of more realistic weighting scenario, the estimations of height variance components are smaller, signaling the importance of introducing real error information in such height adjustment schemes.

Figure 3

3.4 Estimation of the zero-level geopotential value

The estimation of the zero-level geopotential value W_o was carried out according to the methodology described in Grigoriadis et al. (2014) and Kotsakis et al. (2012) but with weighting schemes based on Scenarios 2, 3 and 4. Table 6 provides the results of the computations carried out for determining W_o for the Greek mainland. From the given results, it may be noticed that no significant change in the results is observed with the substitution of the weight for geoid heights obtained from cumulative geoid errors with that from the error covariance matrix of the GOCO family of models. On the other hand, the computations with weights that are based on the full variance/covariance matrix of the GOCO models did not lead to a solution apart from the combination of the GOCO05s with EGM08. This is due to the fact that it was not possible to invert the computed covariance matrix and hence compute the corresponding weights. An explanation to this problem could be the size of the study area. It should be noted that this problem did not occur in the previous section, where heights were used in the adjustment procedure.

By further examining the results presented above, additional conclusions may be drawn with respect to the different models used and their degree of expansion. The combination of GOCE-based models with EGM08 (up to degree and order

2160) leads to similar results. There is also an increase in the W_o of the order of 0.2 m^2/s^2 when comparing to the solution computed only with EGM08. Thus it is obvious that the GOCE-based models and the spectral patching applied have a significant impact on the computed results. The selection though of the best value to be adopted for the Greek mainland is currently not possible due to the accuracy of the source data used as well as to inhomogeneities present in the Greek vertical datum (see also Andritsanos et al., 2015).

4 Conclusions

Considering the parametric model adjustment, using the 175-degree GOCE data gave significantly better results than the use of the full signal of GOCE GGMs patched with EGM2008, in terms of the std and the range of the differences. The study of four different scenarios in the weighting of the parametric model adjustment showed minimal effect in the statistics of the differences. On the other hand, the estimation of the a-posteriori variance of the adjustment is affected by the adopted stochastic model. With the incorporation of more realistic errors for the geoid heights, the statistical results are improved in the case of the a-posteriori std. This statement is also in line with the results of the VCE procedure, where the height variance component estimates obtain smaller values when a more realistic error value is introduced in the adjustment. Last, the GOCO-based models employed have a significant impact on the computation of the W_o but not the weighting schemes described in this study. The evaluation though of the W_o values obtained requires more accurate data.

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Figure 1: The differences at GPS/Levelling benchmarks. Below each model name are the differences using GOCE assimilation degree 175 (left) and the full degree of the model (right). The std of the differences in each case is also given.

Figure 2: Values of the corrector surface (MODEL C) for TIM-R5 (expansion degree 175).

Figure 3: Variance Component Estimation results for the different weighting schemes.

Table 1: The statistics of the parametric model adjustment of the GOCE TIM – R5 model when the assimilation reaches the degree 175. [m]

TIM – R5 (175)	**max**	**min**	**mean**	**std**
Before	0.081	-0.865	-0.393	0.127
After – MODEL A	0.468	-0.450	0.000	0.118
After – MODEL B	0.464	-0.451	0.000	0.118
After – MODEL C	0.403	-0.436	0.000	0.111
After – MODEL D	0.401	-0.424	0.000	0.119
After – MODEL E	0.480	-0.444	0.000	0.121
After – MODEL F	0.459	-0.462	0.000	0.117

Table 2: The statistics of the parametric model adjustment of the GOCE TIM – R5 model when the assimilation reaches the degree 280. [m]

TIM – R5 (280)	**max**	**min**	**mean**	**std**
Before	0.337	-1.114	-0.395	0.226
After – MODEL A	0.711	-0.700	0.000	0.224
After – MODEL B	0.708	-0.693	0.000	0.224
After – MODEL C	0.666	-0.717	0.000	0.214
After – MODEL D	0.644	-0.731	0.000	0.212
After – MODEL E	0.725	-0.692	0.000	0.224
After – MODEL F	0.648	-0.663	0.000	0.213

Table 3: The statistics after the parametric adjustment (MODEL C) using GOCE GGMs till degree 175 and the maximum degree of expansion. [m]

Geoid model	max	min	std
DIR – R5 (175)	0.397	-0.429	0.113
DIR – R5 (300)	0.999	0.772	0.244
TIM – R5 (175)	0.403	-0.436	0.111
TIM – R5 (280)	0.666	-0.717	0.214
GOCO05S (175)	0.403	-0.435	0.111
GOCO05S (280)	0.664	-0.739	0.210
GOCO05C (175)	0.411	-0.422	0.112
GOCO05C (720)	0.393	-0.432	0.115

Table 4: Effect of the various weighting scenarios in the a-posteriori std of the parametric adjustment – parametric MODEL C. E.W.: Equally Weighted heights, C.E.: Cumulative Errors weighting scenario, P.E.: Propagated Errors weighting scenario, F.V.C.: Full Variance / Covariance weighting scenario. [m]

Geoid model	E.W.	C.E.	P.E.	F.V.C.
DIR – R5 (175)	0.0652	0.5030	0.4786	-
DIR – R5 (300)	0.1412	1.0669	0.7985	-
TIM – R5 (175)	0.0644	0.4948	0.4335	-
TIM – R5 (280)	0.1213	0.8147	0.5851	-
GOCO05S (175)	0.0643	0.4953	0.4967	0.4938
GOCO05S (280)	0.1216	0.8086	0.7651	0.6100
GOCO05C (175)	0.0648	0.5001	0.5009	0.5002
GOCO05C (720)	0.0663	0.4466	0.5127	0.5074

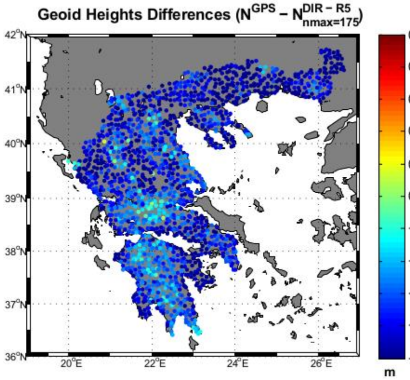
Table 5: Variance component estimation results using various weighting scenarios and initial values for the calculation. E.W.: Equally weighting heights, C.E.: Cumulative errors based heights. [m^2]

Initial values	E.W.	C.E.	P.E.	F.V.C.
$\sigma_h^2 = \sigma_H^2 = \sigma_N^2 = 1$	$\hat{\sigma}_h^2 = 0.00438$ $\hat{\sigma}_H^2 = 0.00438$ $\hat{\sigma}_N^2 = 0.00438$	$\hat{\sigma}_h^2 = 0.04119$ $\hat{\sigma}_H^2 = 0.21030$ $\hat{\sigma}_N^2 = 0.06968$	$\hat{\sigma}_h^2 = 0.03450$ $\hat{\sigma}_H^2 = 0.17642$ $\hat{\sigma}_N^2 = 0.09215$	$\hat{\sigma}_h^2 = 0.00146$ $\hat{\sigma}_H^2 = 0.02779$ $\hat{\sigma}_N^2 = 6.79 \cdot 10^{-8}$
$\sigma_h^2 = 0.01$ $\sigma_H^2 = 0.04$ $\sigma_N^2 = 1$		$\hat{\sigma}_h^2 = 0.06572$ $\hat{\sigma}_H^2 = 0.26653$ $\hat{\sigma}_N^2 = 0.10652$	$\hat{\sigma}_h^2 = 0.05627$ $\hat{\sigma}_H^2 = 0.22830$ $\hat{\sigma}_N^2 = 0.09155$	$\hat{\sigma}_h^2 = 0.04253$ $\hat{\sigma}_H^2 = 0.17217$ $\hat{\sigma}_N^2 = 0.01257$

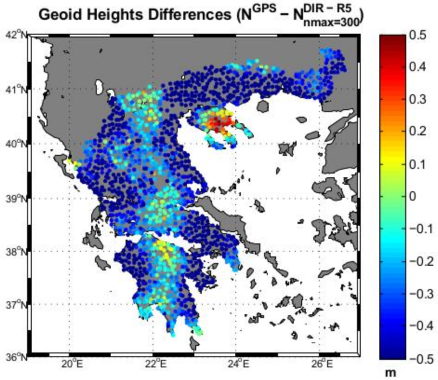
Table 6: Zero-level geopotential values for different weighting scenarios [m^2s^{-2}]

GM	Max Degree	C.E.		P.E.	
		Wo	**σ**	**Wo**	**σ**
EGM08	2160	62636859.664	0.035	n/a	n/a
DIR-R5/EGM08	175/2160	62636859.814	0.034	n/a	n/a
GOCO05c/EGM08	175/2160	62636859.809	0.035	62636859.809	0.035
GOCO05s/EGM08	175/2160	62636859.843	0.034	62636859.844	0.034
TIM-R5/EGM08	175/2160	62636859.859	0.034	n/a	n/a
GM	**Max Degree**	**F.V.C.**			
		Wo		**σ**	
GOCO05s/EGM08	175/2160	62636859.801		0.039	

DIR - R5

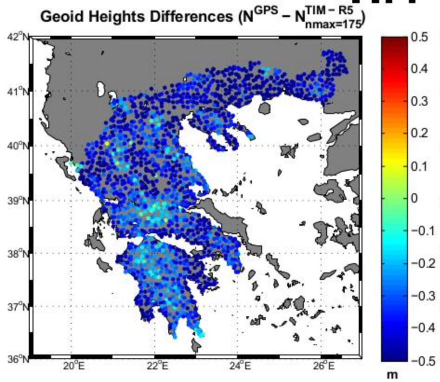


σ = 12.9 cm

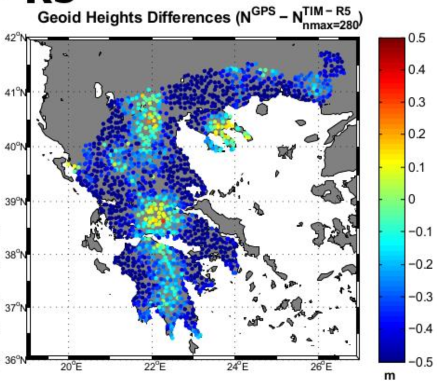


σ = 25.9 cm

TIM - R5

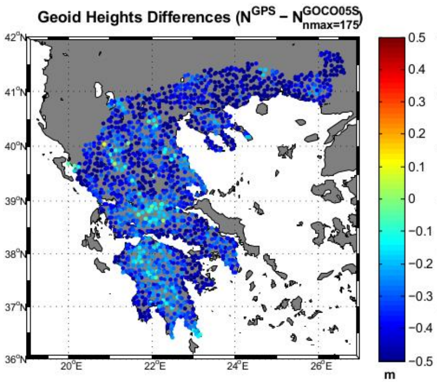


σ = 12.7 cm

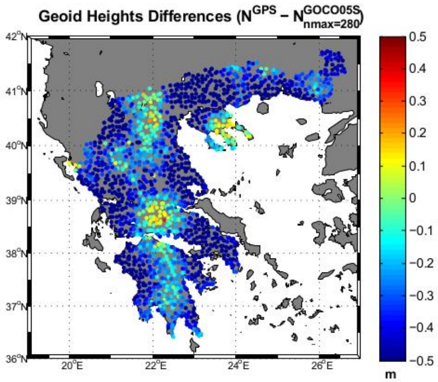


σ = 22.6 cm

GOCO05s

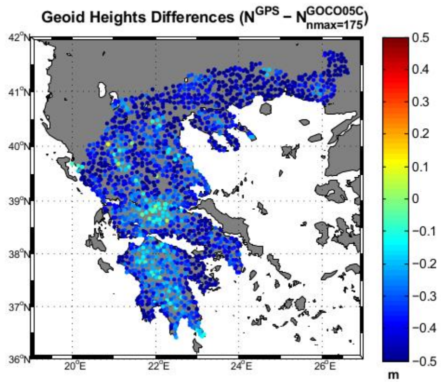


σ = 12.7 cm

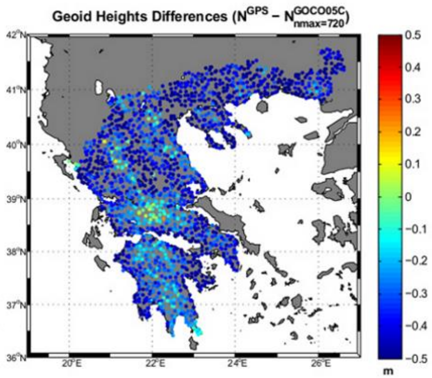


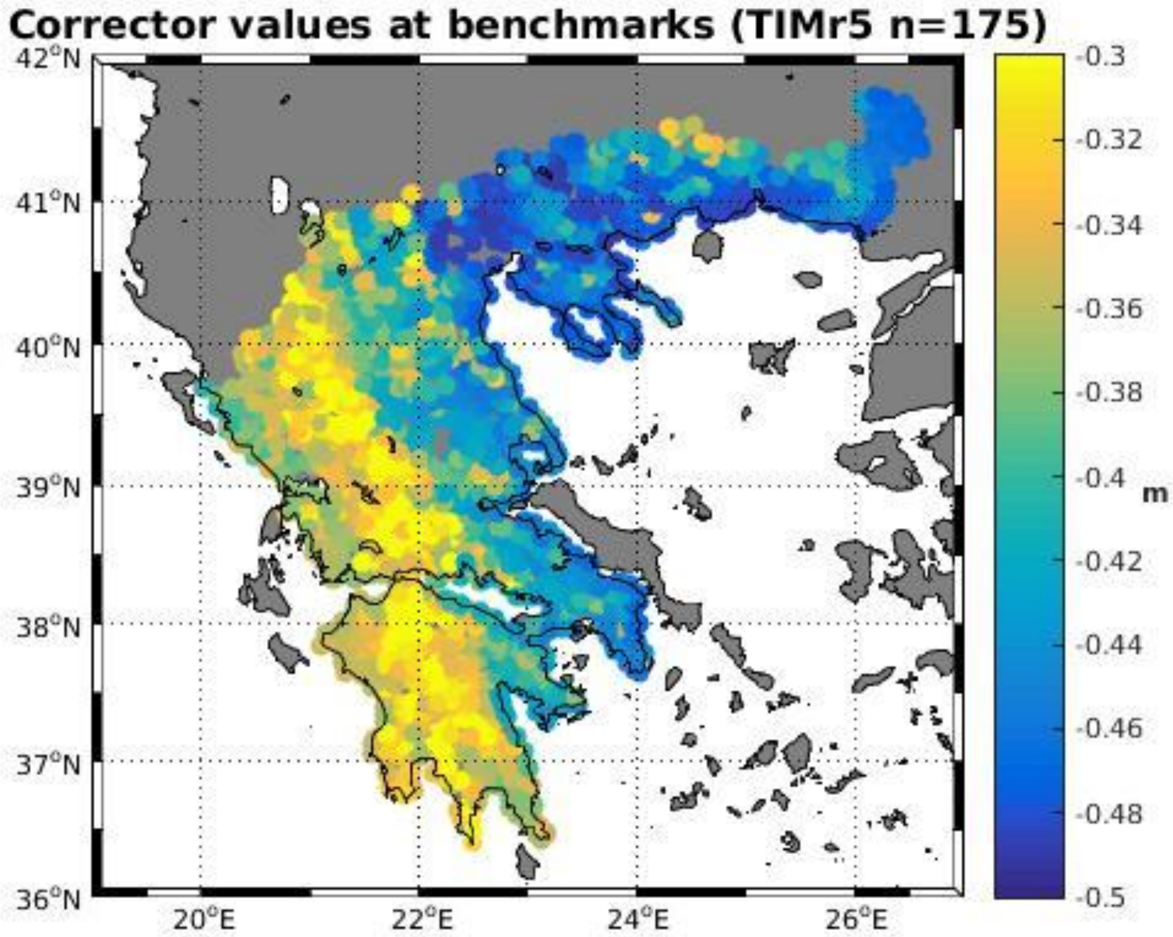
σ = 22.6 cm

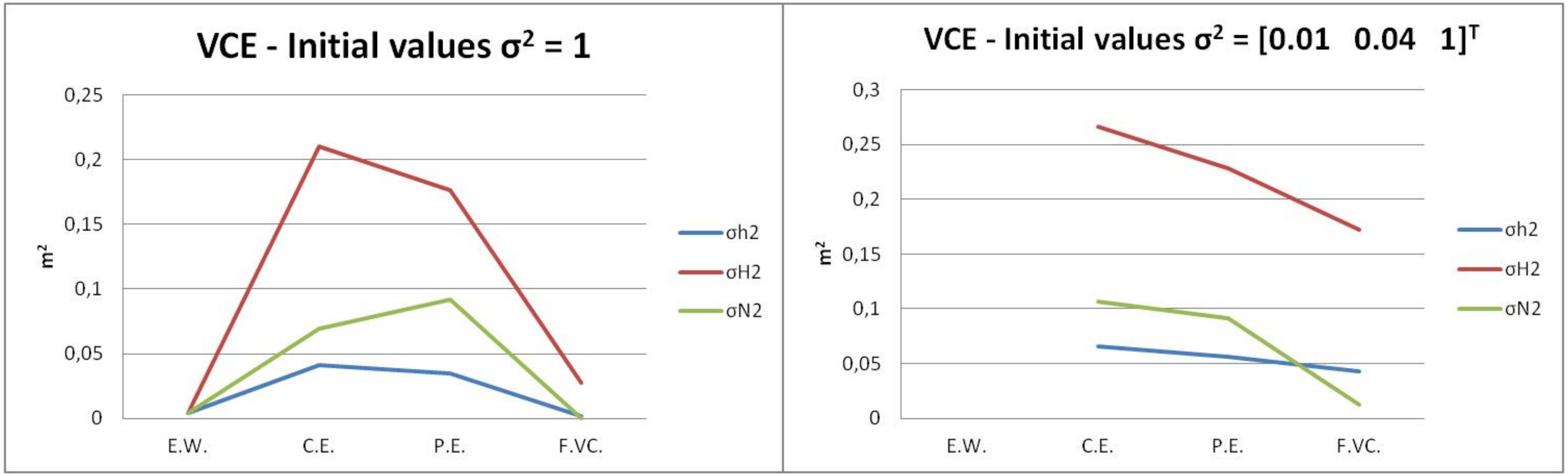
GOCO05c



σ = 13.0 cm




Corrector values at benchmarks (TIMr5 n=175)
42°N
41°N
40°N
39°N
38°N
37°N
36°N
20°E
22°E
24°E
26°E
-0.3
-0.32
-0.34
-0.36
-0.38
-0.4
-0.42
-0.44
-0.46
-0.48
-0.5
m


VCE - Initial values σ^2 = 1
0,25
0,2
0,15
0,1
0,05
0
m²
E.W.
C.E.
P.E.
F.VC.
σh2
σH2
σN2
VCE - Initial values σ^2 = [0.01 0.04 1]^T
0,3
0,25
0,2
0,15
0,1
0,05
0
m²
E.W.
C.E.
P.E.
F.VC.
σh2
σH2
σN2