Gravity, Geoid and Height Systems 2016

GOCE variance and covariance contribution to height system unification

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Outline

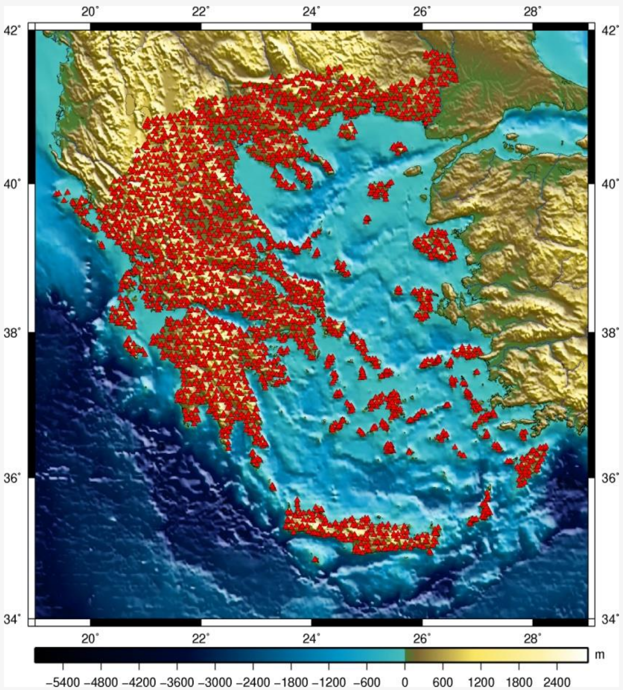
- Objectives
- Area and data description
- Methodology analysis
- Results and discussion
- Conclusions and future work

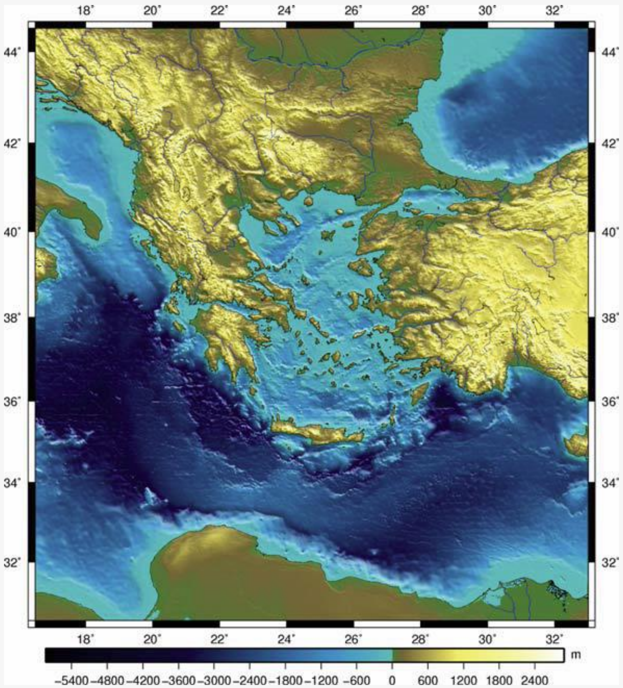
Objectives

- Investigation of the influence of GOCE errors in the determination of the Hellenic LVD
- Use of recent GOCE satellite-only and combined models
- Spectral enhancement approach for data validation
- Various approaches on the weighting of the geoid heights
- Weighting influence to the Wo estimation

Area and Data description

- 1542 GPS/ Leveling benchmarks over the Greek mainland
- Geopotential models (EGM2008, GOCE models – DIR and TIM release 5 and GOCO satellite and combined models release 5)
- RTM effects from DTM of 1” resolution





Methodology analysis

- GPS/Leveling geoid heights were transform to tide – free system (orthometric heights → mean tide)
- GOCE information is taken into account to a maximum degree (175 and n_{max})
- EGM2008 and RTM effects were subtracted → reduced ΔN were modeled

$$\boxed{\Delta N = N^{GPS/Lev} - N^{i}\Big|_{2}^{n_1} - N^{EGM2008}\Big|_{n_1+1}^{2160} - N^{RTM} - N_o}$$

Methodology analysis

- Combined adjustment

Deterministic parametric model

$$\Delta N_i = \boldsymbol{a}_i^T \boldsymbol{x} + v_i^h - v_i^H - v_i^N$$

- Two ways to interpret the parametric model

1. Datum transformation model for geoid undulation

$$\Delta N_i = \Delta\alpha + \Delta X_o cos\varphi_i cos\lambda_i + \Delta Y_o cos\varphi_i sin\lambda_i + \Delta Z_o sin\varphi_i \longrightarrow \text{4-parameters (model A)}$$
$$+ \alpha\Delta f sin^2\varphi_i \longrightarrow \text{5-parameters (model B)}$$

2. Height-dependent corrector surfaces

$$h_i - (1 + \delta s_H)H - (1 + \delta s_N)N = \mu \longrightarrow \text{Scale factor(s) and bias}$$

$$\boldsymbol{a}_i^T \boldsymbol{x} = \mu + \delta s_H H_i + \delta s_N N_i \longrightarrow \text{3-parameters (model C)}$$

$$\boldsymbol{a}_i^T \boldsymbol{x} = \mu + \delta s_H H_i \longrightarrow \text{2-parameters (model D)}$$

$$\boldsymbol{a}_i^T \boldsymbol{x} = \mu + \delta s_N N_i \longrightarrow \text{2-parameters (model E)}$$

Methodology analysis

- Wo estimation procedure

$$W_o^{LVD} = W_o - \frac{\sum_1^m (h_i - H_i - N_i) g_i}{m}$$

For GOCE-based models:

$$N = N_o + N^{GOCE}\Big|_2^{175} + N^{EGM2008}\Big|_{176}^{2160}$$

or

$$N = N_o + N^{GOCE}\Big|_2^{n_{max}}$$

For EGM2008:

$$N = N_o + N^{EGM2008}\Big|_2^{2160}$$

W_o: 62 636 853.4 m^2s^{-2} [IAG Resolution No.1/2015]

Methodology analysis

- Weighting consideration

$$\mathbf{C}_i = \sigma_i^2 \mathbf{Q}_i, \quad i: h, H, N \qquad \mathbf{P} = \begin{bmatrix} \mathbf{Q}_h^{-1} & 0 & 0 \\ 0 & \mathbf{Q}_H^{-1} & 0 \\ 0 & 0 & \mathbf{Q}_N^{-1} \end{bmatrix}$$

- Assumptions on the ellipsoid and orthometric height weights

$$\mathbf{Q}_h = \mathbf{I} \quad \text{or} \quad \mathbf{Q}_h = 0.01 \cdot \mathbf{I} \qquad \mathbf{Q}_H = \mathbf{I} \quad \text{or} \quad \mathbf{Q}_H = 0.04 \cdot \mathbf{I}$$

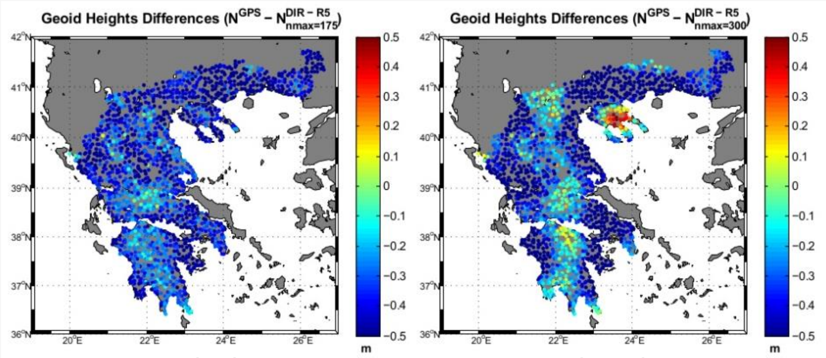
- Four different weighting methodologies for the geoid height

1. Equally weighted heights $\mathbf{Q}_h = \mathbf{Q}_H = \mathbf{Q}_N = \mathbf{I}$
2. Weights based on geoid model cumulative errors
3. Weights from propagated geoid model variances
4. Weights using full variance-covariance matrix

Results

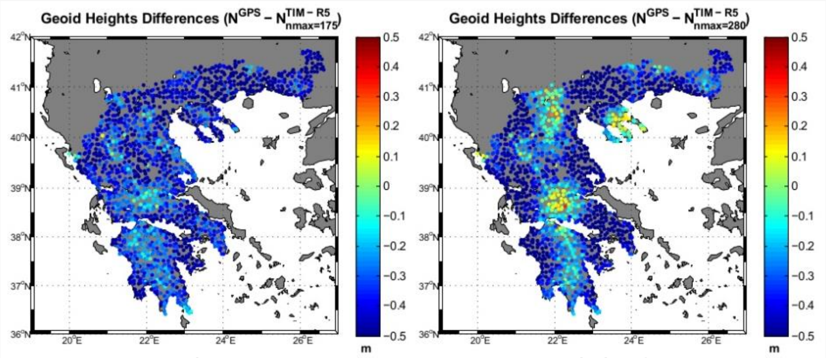
- Differences before the parametric adjustment

DIR – R5



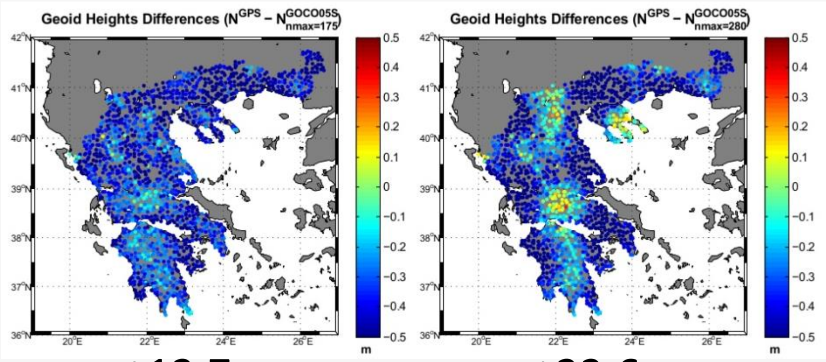
σ = ±12.9 cm   σ = ±25.9 cm

TIM – R5



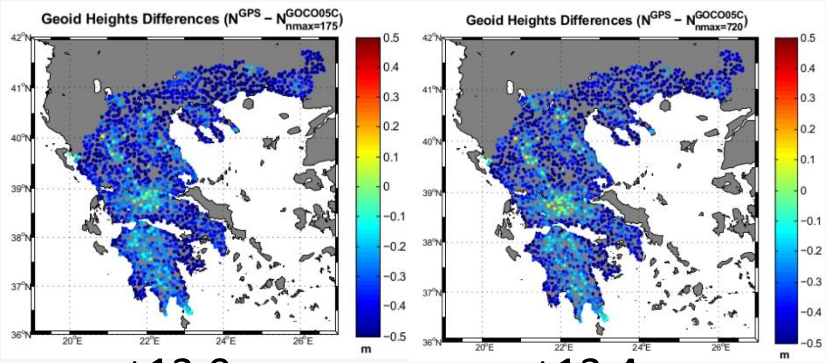
σ = ±12.7 cm   σ = ±22.6 cm

GOCO05s



σ = ±12.7 cm   σ = ±22.6 cm

GOCO05c



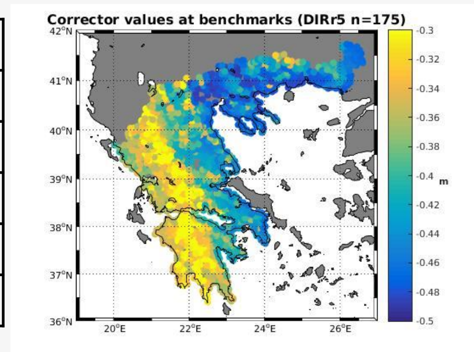
σ = ±13.0 cm   σ = ±13.4 cm

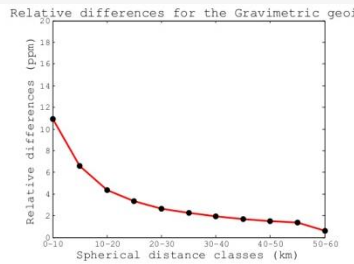
Results

- Case A: Equally weighted heights $\mathbf{Q}_h = \mathbf{Q}_H = \mathbf{Q}_N = \mathbf{I}$

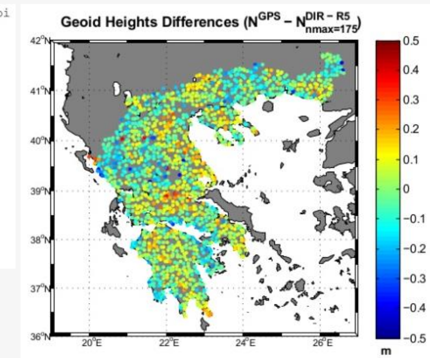
DIR – R5 (nmax = 175)

Parametric model	σ (cm)
MODEL A	±11.9
MODEL B	±11.9
MODEL C	±11.3
MODEL D	±12.1
MODEL E	±12.2



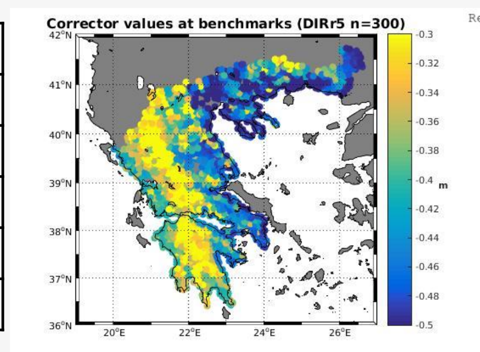


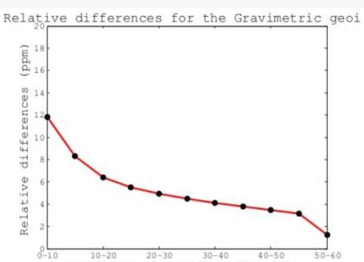
$$a_{ij} = \frac{\Delta N_{ij}}{S_{ij}} [ppm]$$

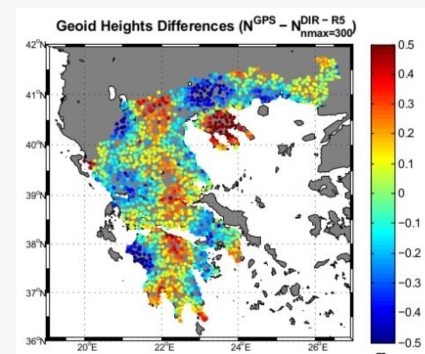


DIR – R5 (nmax = 300)

Parametric model	σ (cm)
MODEL A	±25.5
MODEL B	±25.5
MODEL C	±24.4
MODEL D	±24.8
MODEL E	±25.6





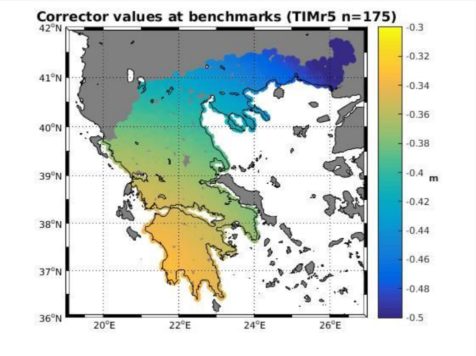


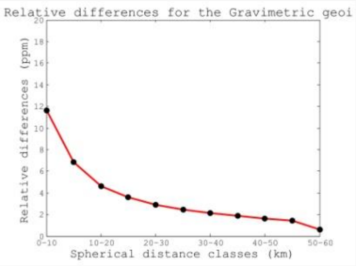
Results

- Case A: Equally weighted heights $\mathbf{Q}_h = \mathbf{Q}_H = \mathbf{Q}_N = \mathbf{I}$

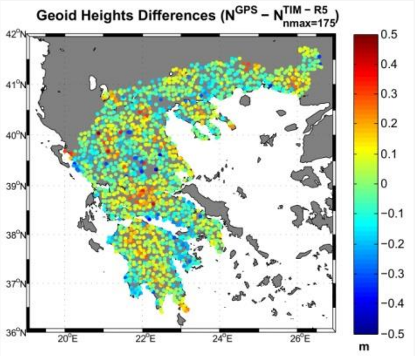
TIM – R5 (nmax = 175)

Parametric model	σ (cm)
MODEL A	±11.8
MODEL B	±11.8
MODEL C	±11.1
MODEL D	±11.9
MODEL E	±12.1



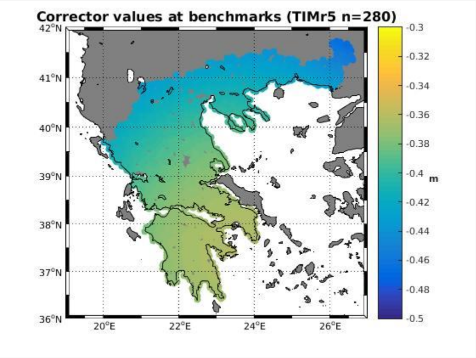


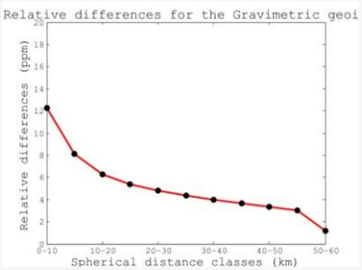
$$a_{ij} = \frac{\Delta N_{ij}}{S_{ij}} [ppm]$$

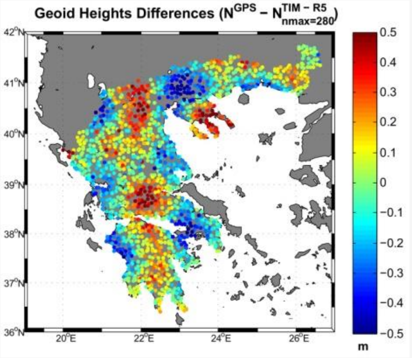


TIM – R5 (nmax = 280)

Parametric model	σ (cm)
MODEL A	±22.4
MODEL B	±22.4
MODEL C	±21.4
MODEL D	±21.2
MODEL E	±22.4





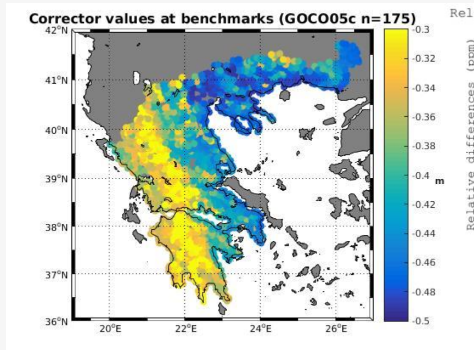


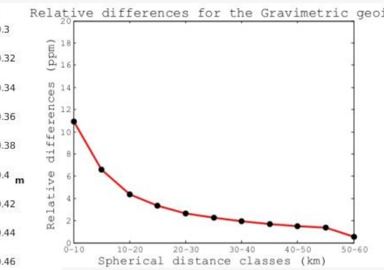
Results

- Case A: Equally weighted heights $\mathbf{Q}_h = \mathbf{Q}_H = \mathbf{Q}_N = \mathbf{I}$

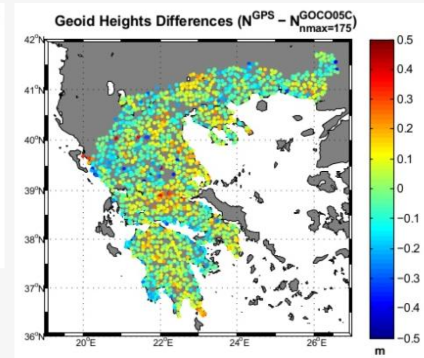
GOCO05c (nmax = 175)

Parametric model	σ (cm)
MODEL A	±12.0
MODEL B	±12.0
MODEL C	±11.2
MODEL D	±12.1
MODEL E	±12.2



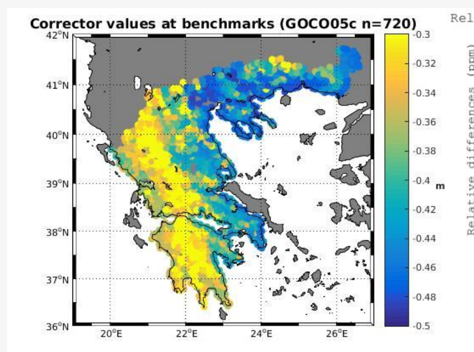


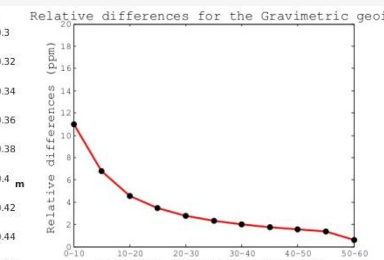
$$a_{ij} = \frac{\Delta N_{ij}}{S_{ij}} [ppm]$$

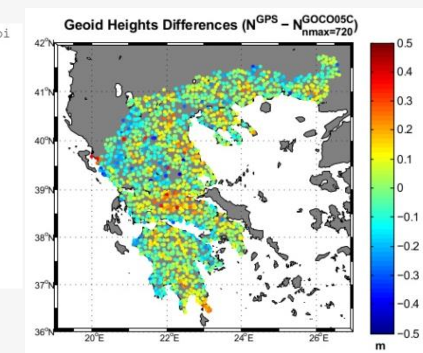


GOCO05c (nmax = 720)

Parametric model	σ (cm)
MODEL A	±12.7
MODEL B	±12.7
MODEL C	±11.5
MODEL D	±12.2
MODEL E	±12.4



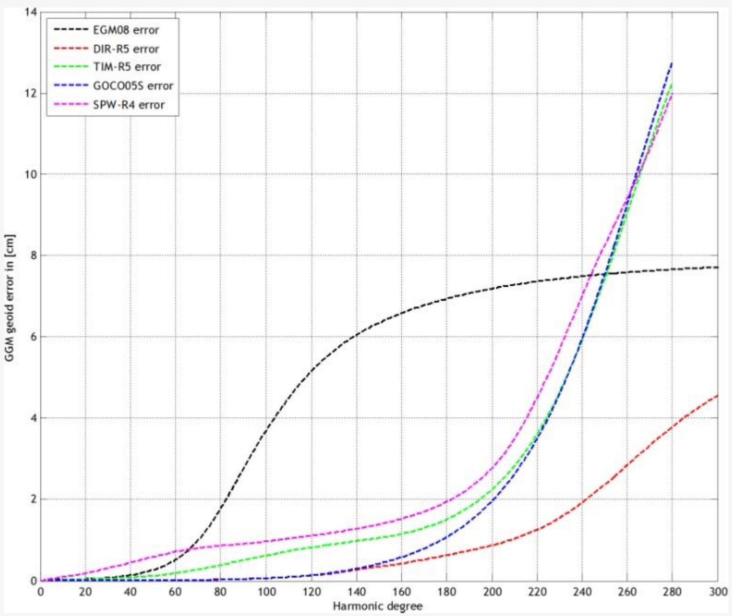




Results

- Case B: Weights based on geoid model cumulative errors

$$\mathbf{Q}_N = \varepsilon^2_{cml} \cdot \mathbf{I} = (\varepsilon^2_{N_{GGM}\, to\, nmax} + \varepsilon^2_{N_{08}\, nmax\, to\, 2190}) \cdot \mathbf{I}$$



$$\varepsilon^2_{cml} = R^2 \sum_{m=0}^{n} (\sigma^2_{\bar{c}_{nm}} + \sigma^2_{\bar{s}_{nm}})$$

- Same statistics in the differences with case A
- Almost identical parameters estimation

DIR – R5 (175) - MODEL C

Case A

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.16356\ (\mathrm{m}) \\ 0.000106764529786 \\ -0.007659611\boxed{11327} \end{bmatrix}$$

Case B

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.16356\ (\mathrm{m}) \\ 0.000106764529786 \\ -0.007659611\boxed{111324} \end{bmatrix}$$

- Similar parameters accuracy estimation

$$\hat{\sigma}_{\mu} = \pm 0.0185\ \mathrm{m} \qquad \hat{\sigma}_{\delta S_H} = \pm 6.51 \cdot 10^{-6} \qquad \hat{\sigma}_{\delta S_N} = \pm 4.96 \cdot 10^{-4}$$

- Different a-posteriori variance estimation due to the stochastic model used in each case

Case A

$$\hat{\sigma}^2 = 0.00425\ \mathrm{m}^2$$

Case B

$$\hat{\sigma}^2 = 0.2530\ \mathrm{m}^2$$

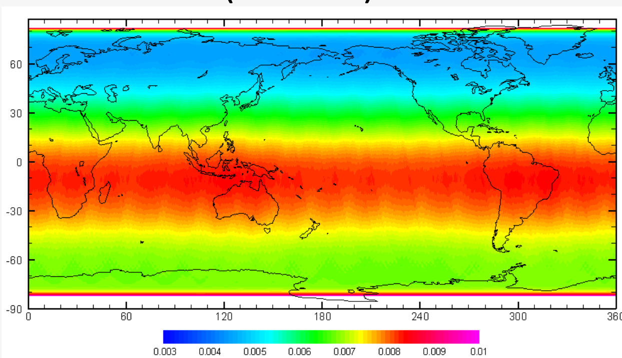
- Same behavior to all geopotential and parametric models

Results

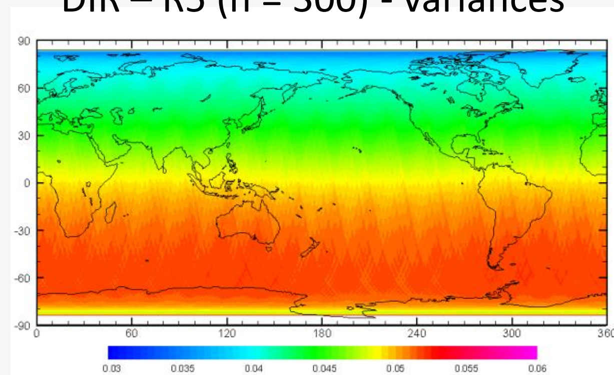
- Case C: Weights from propagated geoid model variances

$$\mathbf{Q}_N = \sigma^2_{prop} \cdot \mathbf{I} = \left(\sigma^2_{prop_{N_{GGM}\,to\,nmax}} + \varepsilon^2_{N_{08}\,nmax\,to\,2190}\right) \cdot \mathbf{I}$$

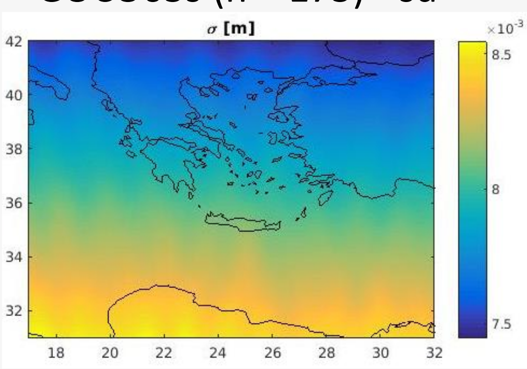
DIR – R5 (n = 175) - variances



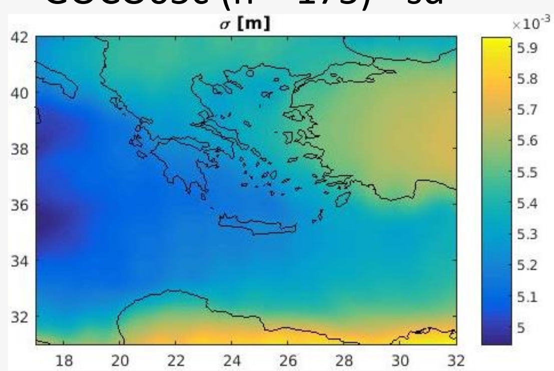
DIR – R5 (n = 300) - variances



GOCO05s (n = 175) - sd



GOCO05c (n = 175) - sd



The variance matrix was generated by bilinear interpolation at the 1542 benchmarks locations

Results

- Case C: Weights from propagated geoid model variances

$$\mathbf{Q}_N = \sigma^2_{prop} \cdot \mathbf{I} = (\sigma^2_{prop_{N_{GGM}\,to\,nmax}} + \varepsilon^2_{N_{08}\,nmax\,to\,2190}) \cdot \mathbf{I}$$

- Identical results in the statistics of the differences
- Minor differences in the parameters estimation

GOCO05s (175) – MODEL C

Case A

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.18289\ (\mathrm{m}) \\ 0.0001045785897\boxed{93} \\ -0.0070174242367\boxed{69} \end{bmatrix}$$

Case B

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.18289\ (\mathrm{m}) \\ 0.000104578589793 \\ -0.00701742423\boxed{6780} \end{bmatrix}$$

Case C

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.18288\ (\mathrm{m}) \\ 0.000104577983220 \\ -0.007017482757325 \end{bmatrix}$$

- Similar parameters accuracy estimation

$$\hat{\sigma}_{\mu} = \pm 0.0182\ \mathrm{m} \qquad \hat{\sigma}_{\delta S_H} = \pm 6.42 \cdot 10^{-6} \qquad \hat{\sigma}_{\delta S_N} = \pm 4.90 \cdot 10^{-4}$$

- Different a-posteriori variance estimation

Case A

$$\hat{\sigma}^2 = 0.00413\ \mathrm{m}^2$$

Case B

$$\hat{\sigma}^2 = 0.2453\ \mathrm{m}^2$$

Case C

$$\hat{\sigma}^2 = 0.2467\ \mathrm{m}^2$$

Results

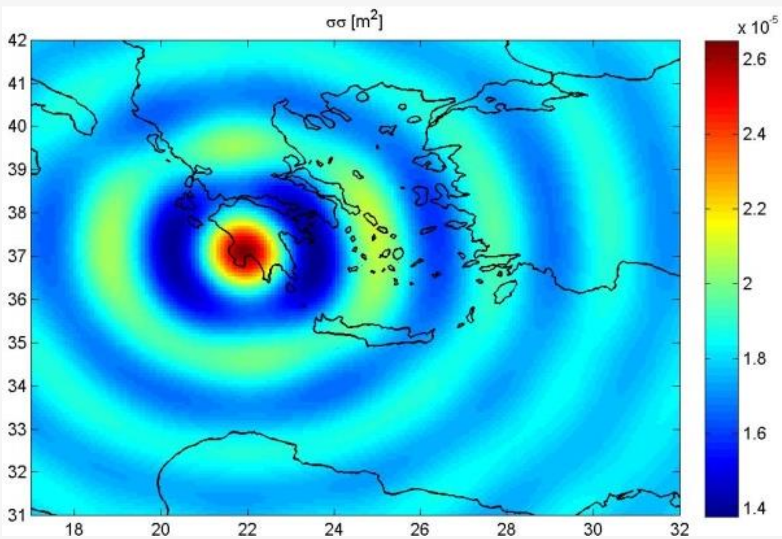
- Case D: Weights using full variance-covariance matrix

$$\mathbf{Q}_N = \mathbf{C}^{full}_{prop_{N_{GGM}\,to\,nmax}} + (\varepsilon^2_{N_{08}\,nmax\,to\,2190}) \cdot \mathbf{I}$$

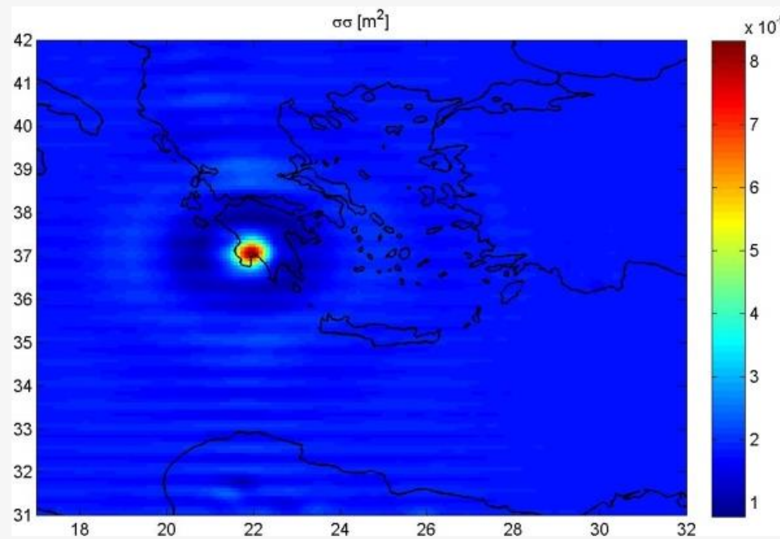
- No covariance information from nmax to 2190 → only cumulative errors used

GOCO05c – Covariance at point no 941

nmax = 175



nmax = 720



- Covariance row/column constructed by bilinear interpolation from specific covariance file

Results

- Case D: Weights using full variance-covariance matrix

$$\mathbf{Q}_N = \mathbf{C}^{full}_{prop_{N_{GGM}\ to\ nmax}} + \left(\varepsilon^2_{N_{08}\ nmax\ to\ 2190}\right)\cdot\mathbf{I}$$

- Identical results in the statistics of the differences
- Minor differences in the parameters estimation

GOCO05c (nmax = 720) – MODEL C

Case A	Case B	Case C	Case D
$\hat{\mathbf{x}} = \begin{bmatrix} -0.18493\ (\mathrm{m}) \\ 0.000131944657027 \\ -0.007113661382659 \end{bmatrix}$	$\hat{\mathbf{x}} = \begin{bmatrix} -0.18493\ (\mathrm{m}) \\ 0.000131944657027 \\ -0.007113661382667 \end{bmatrix}$	$\hat{\mathbf{x}} = \begin{bmatrix} -0.18518\ (\mathrm{m}) \\ 0.000132107484412 \\ -0.007107377090662 \end{bmatrix}$	$\hat{\mathbf{x}} = \begin{bmatrix} -0.18605\ (\mathrm{m}) \\ 0.000133008275695 \\ -0.007085682762273 \end{bmatrix}$

- Similar parameters accuracy estimation

$\hat{\sigma}_\mu = \pm 0.0006\ \mathrm{m}$ $\quad\hat{\sigma}_{\delta s_H} = \pm 0.21\cdot 10^{-6}$ $\quad\hat{\sigma}_{\delta s_N} = \pm 0.16\cdot 10^{-4}$

- Different a-posteriori variance estimation

Case A	Case B	Case C	Case D
$\hat{\sigma}^2 = 0.0044\ \mathrm{m}^2$	$\hat{\sigma}^2 = 0.1994\ \mathrm{m}^2$	$\hat{\sigma}^2 = 0.2629\ \mathrm{m}^2$	$\hat{\sigma}^2 = 0.2574\ \mathrm{m}^2$

Results

- $W_o^{LVD} - W_o^{IAG}$ [Mainland of Greece]

Weighting Scheme	EGM08	GOCO05C / EGM08	GOCO05s / EGM08	DIR R5 / EGM08	TIM R5 / EGM08
None (average)	**6.264 ±0.035**	**6.409 ±0.035**	**6.443 ±0.034**	**6.414 ±0.034**	**6.459 ±0.034**
$1\,/\,\left(\sigma_h^2 + \sigma_{\mathrm{H}}^2 + \sigma_{\mathrm{N}}^2\right)$ [commission error]	*No change in results*				
$1\,/\,\left(\sigma_h^2 + \sigma_{\mathrm{H}}^2 + \sigma_{\mathrm{N}}^2\right)$ [GOCE variances + EGM08 com error]		*No change in results*			
$1\,/\,\left(\sigma_h^2 + \sigma_{\mathrm{H}}^2 + \sigma_{\mathrm{N}}^2\right)$ [GOCE var/covar + EGM08 com error]		-	6.401 ±0.039		

Units: m^2s^{-2}

Results

- Variance Component Estimation

$$\mathbf{C}_i = \sigma_i^2 \mathbf{Q}_i, \quad i: h, H, N$$

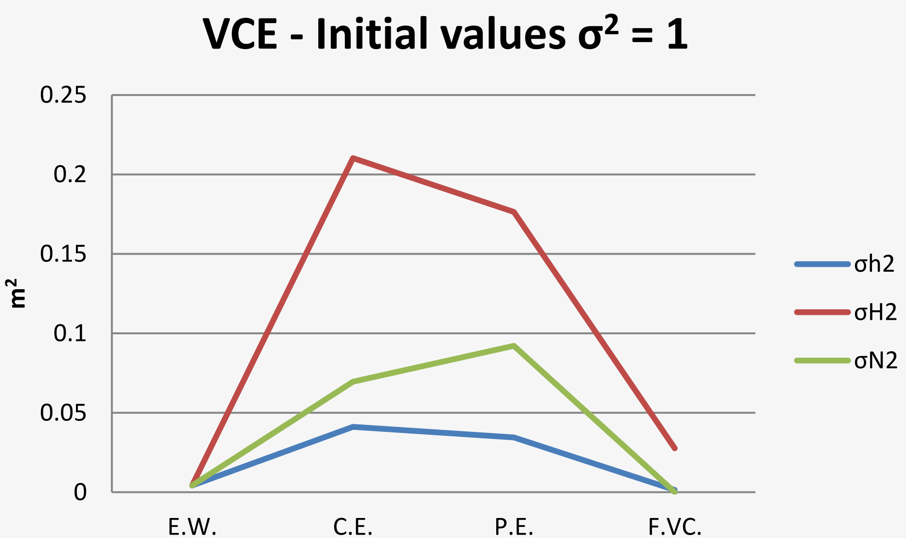
- Unknown variance components using iterative MINQUE (Rao, 1971, 1977)

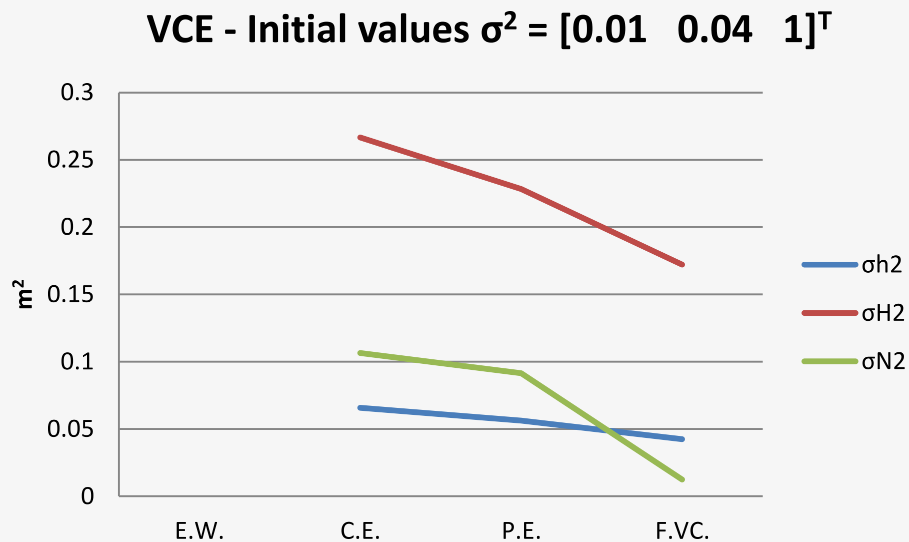
GOCO05c (nmax = 720)

Initial values	Equally weighted heights	Geoid variances from model cumulative errors	Geoid variances from propagated errors	Full variance-covariance geoid information
$\boldsymbol{\sigma_h^2 = \sigma_H^2 = \sigma_N^2 = 1}$	$\hat{\sigma}_h^2 = 0.00438\, m^2$ $\hat{\sigma}_H^2 = 0.00438\, m^2$ $\hat{\sigma}_N^2 = 0.00438\, m^2$	$\hat{\sigma}_h^2 = 0.04119\, m^2$ $\hat{\sigma}_H^2 = 0.21030\, m^2$ $\hat{\sigma}_N^2 = 0.06968\, m^2$	$\hat{\sigma}_h^2 = 0.03450\, m^2$ $\hat{\sigma}_H^2 = 0.17642\, m^2$ $\hat{\sigma}_N^2 = 0.09215\, m^2$	$\hat{\sigma}_h^2 = 0.00146\, m^2$ $\hat{\sigma}_H^2 = 0.02779\, m^2$ $\hat{\sigma}_N^2 = 6.76 \cdot 10^{-8}\, m^2$
$\boldsymbol{\sigma_h^2 = 0.01\, m^2}$ $\boldsymbol{\sigma_H^2 = 0.04\, m^2}$ $\boldsymbol{\sigma_N^2 = 1}$		$\hat{\sigma}_h^2 = 0.06572\, m^2$ $\hat{\sigma}_H^2 = 0.26653\, m^2$ $\hat{\sigma}_N^2 = 0.10652\, m^2$	$\hat{\sigma}_h^2 = 0.05627\, m^2$ $\hat{\sigma}_H^2 = 0.22830\, m^2$ $\hat{\sigma}_N^2 = 0.09155\, m^2$	$\hat{\sigma}_h^2 = 0.04253\, m^2$ $\hat{\sigma}_H^2 = 0.17217\, m^2$ $\hat{\sigma}_N^2 = 0.01257 m^2$

Results

- Variance Component Estimation





E.W.: Equally weighted heights
C.E.: Weights from cumulative geoid errors
P.E.: Weights from propagated geoid errors
C.E.: Full variance-covariance geoid matrix

Conclusions and future work

- The weighting effect
  a) on the **residual geoid modeling** using parametric models,
  b) on the **variance component estimation** and
  c) on the **local Wo estimation** is investigated.

- Minor differences in the parameter estimation were revealed.

- The different a-posteriori variance of each solution is depending on the specific scenario used for the stochastic model (weighting cases).

Conclusions and future work

- The adequateness of the parametric model utilized has to be confirmed applying statistical tests for the parameters.

- The use of the full variance/covariance matrix for the geoid heights led to a decrease in variance component estimation values.

- In all weighting schemes VCE led to an increased error for the orthometric heights, signaling that the Greek LVD is outdated and needs modernization (geoid-based datum?)

Conclusions and future work

- W_o estimation with GOCO05s, GOCO05c, TIM-R5, DIR-R5 patched with EGM08 leads to similar results for the Greek Mainland (differences up to 0.05 m^2s^{-2})

- W_o estimation with GOCE-based models patched with EGM08 versus EGM08 leads to significantly different results (0.15 – 0.19 m^2s^{-2})

- Use of error variances or error variances/covariances matrix of the GOCO models in the adjustment weighting scheme made no change to the W_o estimates.

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