# Combination of GOCE SGG data and surface gravity anomalies for local/regional geoid determination


**R.S. Grebeticharsky[1], G.S. Vergos[2], D. Piretzidis[3], I.N. Tziavos[2]**

[1]RSG Consult Ltd., Bulgaria, rgrebenitcharsky@gmail.com
[2]Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Greece, vergos@topo.auth.gr
[3]Department of Geomatics Engineering, University of Calgary, Canada


## Introduction

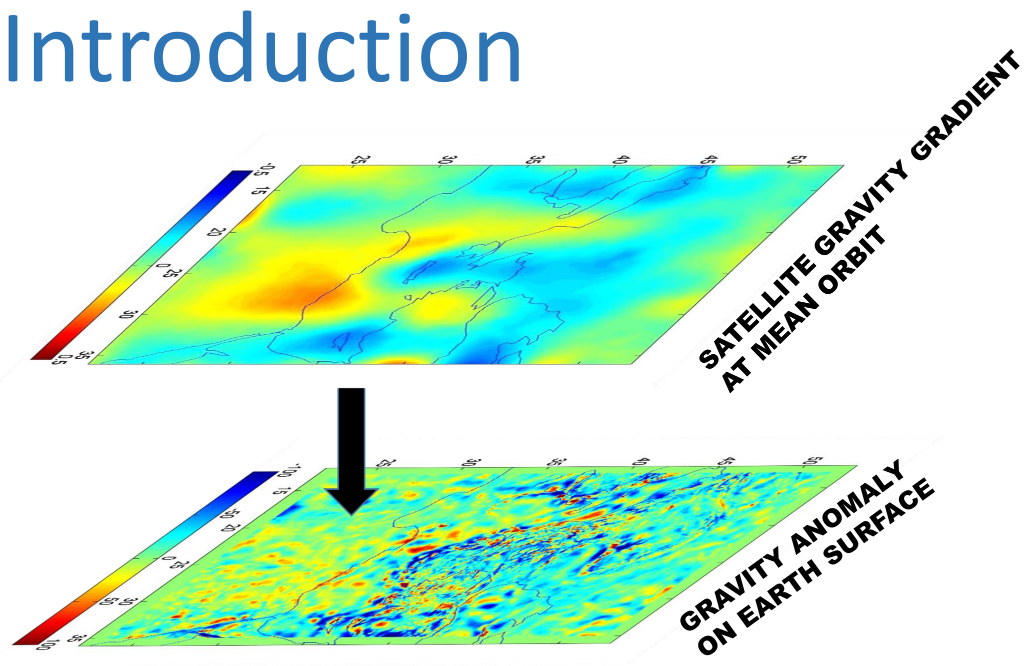


*Figure 1: MAIN TASK DESCRIPTION*

MAIN TASK: Combination of SGGs at mean orbit and ground gravity anomaly data on the Earth's surface trough GOCE gradient downward continuation for local/regional geoid modeling:

- Using a Monte Carlo Method (Simulated Annealing) for solution of inverse problems
- Application of Quasi-random generator (QG)
- Validation of geoid model by GPS&GNSS/Leveling

## Background

**MOTIVATION: TO COMBINE SGG AT MEAN ORBIT WITH GRAVITY ANOMALY DATA ON THE EARTH SURFACE FOR REGIONAL GEOID MODELLING**

REGIONAL GEOID MODELING: for performing calibration of SGGs with external gravity information on a grid to use all available data at highest possible resolution

- HOW? By application of a Monte-Carlo method (Simulated Annealing). Simulated annealing (SA) is a probabilistic method proposed by Kirkpatrick, Gelett and Vecchi (1983) for finding the global minimum of a cost function that may possess several local minima. It works by emulating the physical process whereby a solid is slowly cooled so that when eventually its structure is 'frozen', this happens at a minimum energy configuration.
- WHY? SA allows solving inverse problems like downward continuation of SGGs from mean orbit to the Earth's surface using an iterative Monte Carlo procedure based on quasi-random generator.
- WHAT TO DO? 1)Quasi-random generation of gravity anomalies on earth surface in the form of grid; 2) Upward continuation to mean orbit by MIMOS – Forward step; 3) Comparison of SGG observations with upward SGGs and 'freeze' those satisfying SA criteria; 4) Repetition of 1-3 till all generated SGGs meet the SA criteria.

**FOPWARD STEP : MULTIPLE INPUT OUTPUT SYSTEMS (MIMOS) FOR UPWARD CONTINUATION (TO THE MEAN ORBIT) (SIDERIS 1996 - MODIFIED)**

$$FFT\left\{T_{ij}^{MO}\right\} = FFT\left\{\mathbf{K}_{ij}^{MO,ES}\left(LT^{ES}\right)\right\}.*FFT\left\{LT^{ES}\right\}, \quad i,j = x,y,z$$

where : $FFT$ - Fast Fourier Transform;

$T_{ij}^{MO}$ - the gravity gradients at mean orbit along the axis i, j

$LT^{ES}$ - Functional of disturbing gravity potential (gravity anomaly) on Earth surface

$\mathbf{K}_{ij}^{MO,ES}$ - Kernel for transformation of the functional of disturbing potential $LT^{ES}$ on Earth Surface to the gravity gradient $T_{ij}^{MO}$ at mean orbit (Eshagh, 2010)

where: ES – Earth's surface; MO – mean satellite orbit at 260 km
{.*} – element by element multiplication

## Experiment description and results

- Application area: 22 < latitude < 53; 12 < longitude < 36
- Applied on: *2D regional 6' x 6' grid for* ***Tzz*** *(directly observed) and* ***Tzz,lp*** *derived from Laplace's equation with directly observed* ***Txx*** *and* ***Tyy****:*

  *Tzz,lp = -Txx - Tyy*
- Gridding – standard procedure with Delaunay triangulation
- Time interval covered: 01/11/2009 – 30/04/2011 (16 months) with 421 days of SGG data
- Data type and source: EGG_NOM_2, GOCE HLPF, ESA/ESRIN
- SGG description: normal potential gradients are evaluated at the LNOF
- transformed to the GRF. Disturbing potential gradients are computed and then filtered with an FIR band-pass filter (5-100 mHz) with a Hamming window for spectral leakage. The filtered $T_{ij}$ are then reduced to a mean orbit.
- Reference Gravity Field Model: GOCE DIR R4
- Ground data: EGM 2008 Gravity Anomaly Model generated on Earth's surface
- Type of model SGG data: DIR R4 GPM generated directly at mean orbital altitude
- Number of separate runs: 31
- Number of iterations necessary to reach SA criteria per every run: 500
- Validation of determined geoid models using GPS & GNSS/Leveling bench marks
- Number of GPS/Leveling benchmarks over land only: 1542

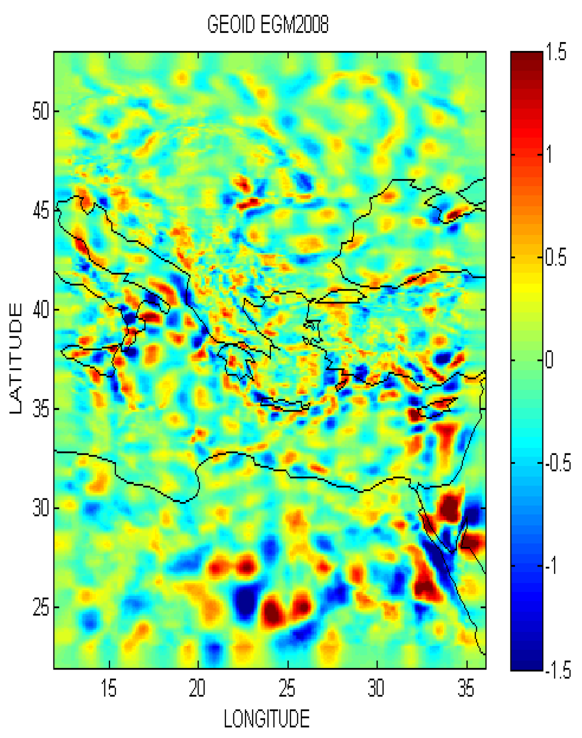


***Figure 2:*** *EGM 2008 geoid [m]*

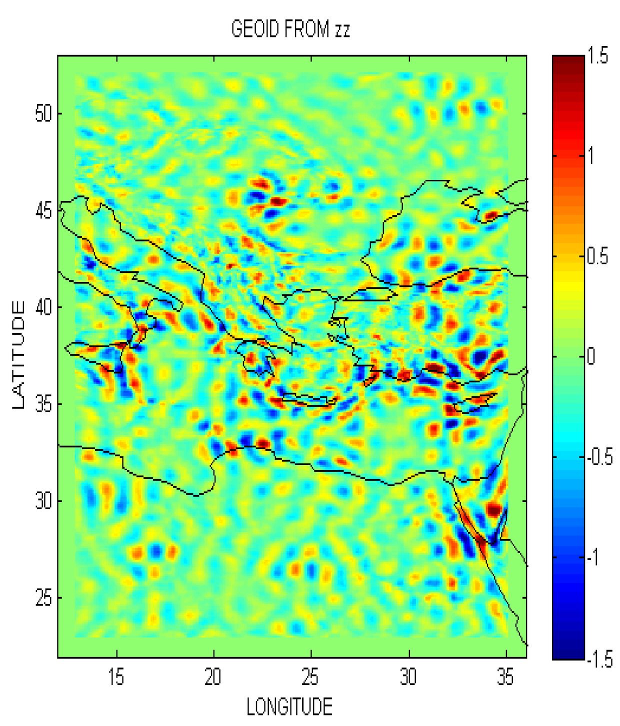


***Figure 3.*** *Geoid determined using Tzz [m]*

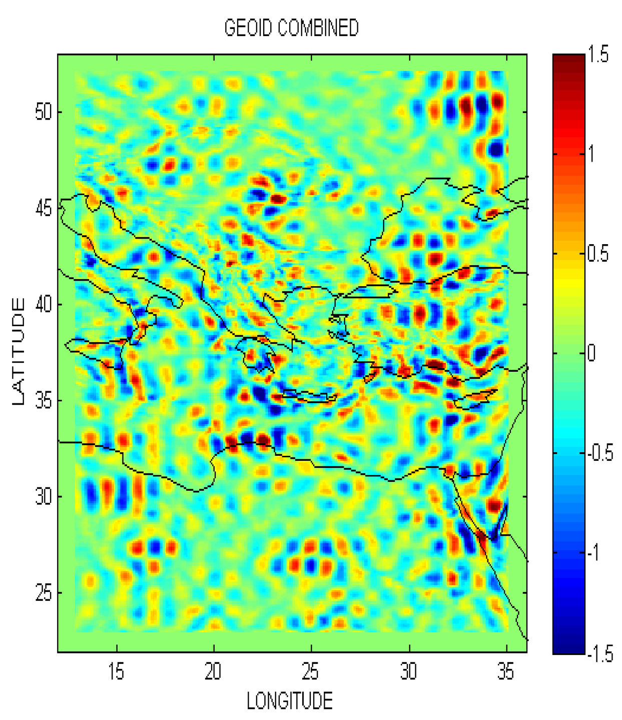


***Figure 4:*** *Geoid determined using Tzz,lp [m]*

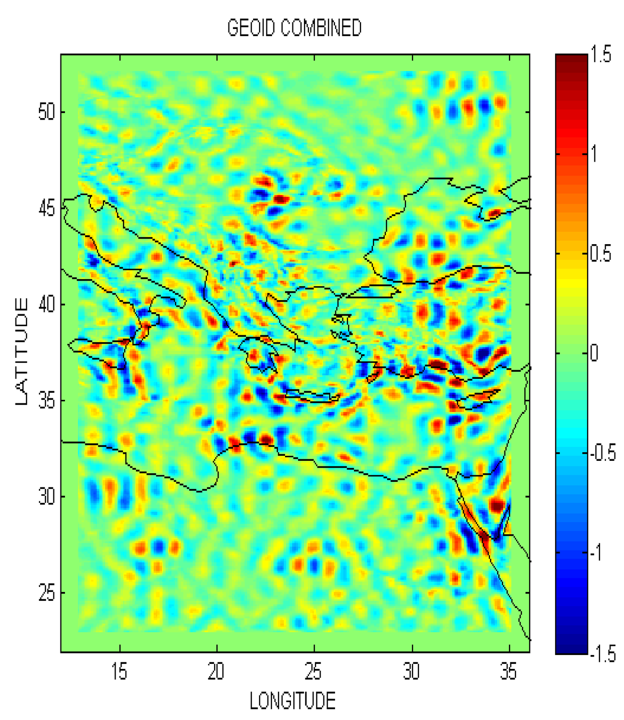


***Figure 5****: Combined Geoid (Tzz & Tzz,lp) [m]*

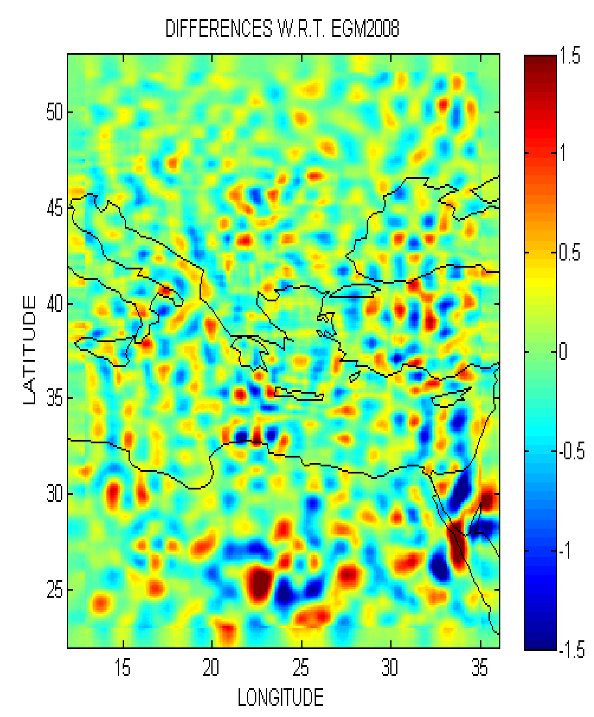


***Figure 6****: Combined geoid diff. w.r.t. EGM2008 [m]*

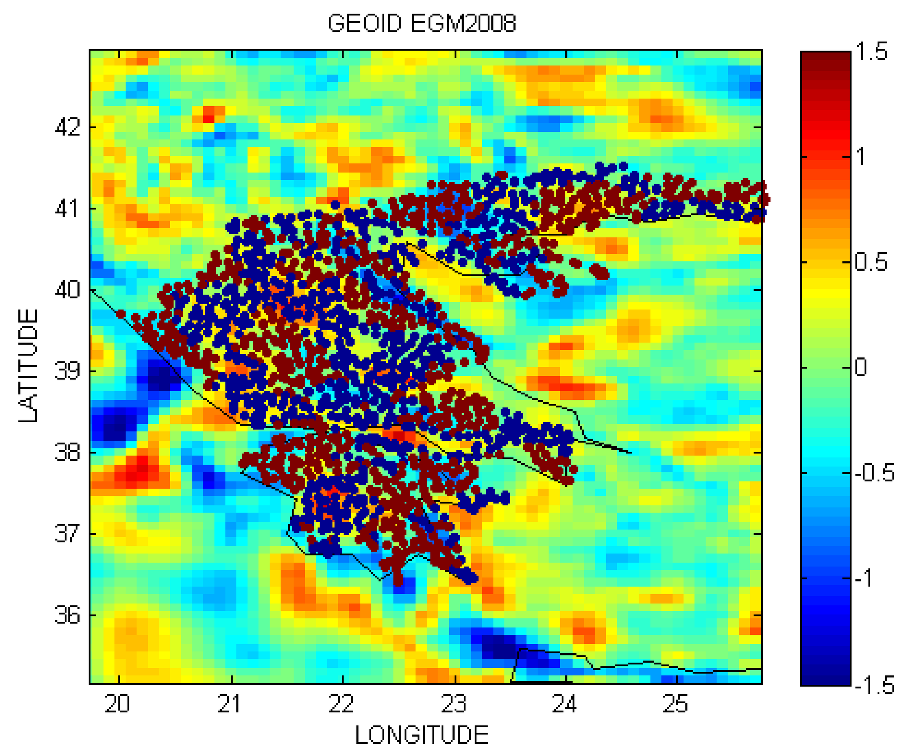


***Figure 7:*** *Combined geoid: improvement w.r.t. GPM DIR R2- blue: GPS/Lev BMs no improvement; brown: improved GPS/Lev BMs*

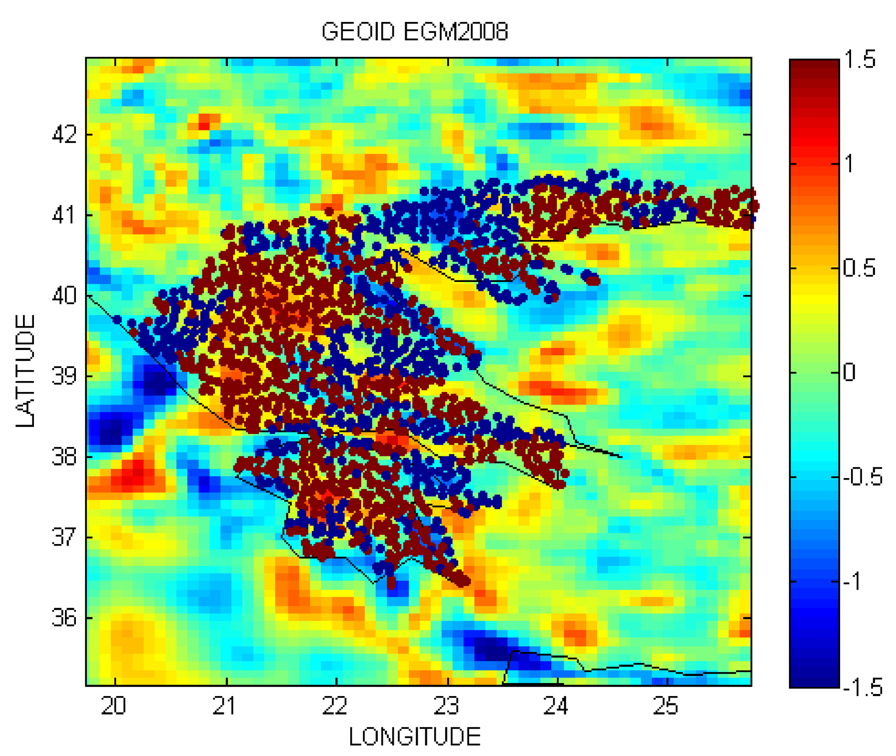


***Figure 8:*** *Combined geoid: improvement w.r.t. EGM 2008 blue: GPS/Lev no improvement; brown: improved GPS/Lev*

| | GPS/LEV | EGM | GOCE: Tzz | GOCE: Txx&Tyy | GOCE: COMBINED |
|---|---|---|---|---|---|
| MIN | -2.141 | -1.118 | -1.352 | -1.230 | -1.120 |
| MAX | 0.648 | 1.210 | 1.222 | 1.130 | 1.160 |
| MEAN | -0.841 | -0.011 | -0.070 | -0.029 | -0.052 |
| RMS | 0.956 | 0.397 | 0.393 | 0.376 | 0.349 |
| STD | 0.455 | 0.397 | 0.387 | 0.376 | 0.345 |

***Table 2:*** *Statistics of geoid residuals at GPS/Lev BMs w.r.t. GPM DIR R4. Unit: [m]*

| | GPM | EGM | GOCE: Tzz | GOCE: Txx&Tyy | GOCE: COMBINED |
|---|---|---|---|---|---|
| MIN | -2.141 | -2.553 | -2.727 | -2.775 | -2.665 |
| MAX | 0.648 | 1.147 | 1.499 | 1.307 | 1.267 |
| MEAN | -0.841 | -0.831 | -0.771 | -0.812 | -0.789 |
| RMS | 0.956 | 1.072 | 1.020 | 1.035 | 1.007 |
| STD | 0.455 | 0.678 | 0.668 | 0.641 | 0.627 |

***Table 3:*** *Statistics of GPS/Lev BMs' differences w.r.t. GPM DIR R4, EGM 2008, Tzz Geoid, Tzz,lp Geoid, Combined Geoid. Unit: [m]*

| | GOCE Tzz: IMPROVED GEOID | | | | |
|---|---|---|---|---|---|
| | w.r.t EGM | | w.r.t. GPM | | w.r.t. EGM &GPM |
| MIN/#POINTS | -0.785 | 884 | -0.921 | 801 | 543 |
| MAX/STATUS | 0.661 | BETTER | 1.352 | BETTER | BETTER |
| MEAN/ IN % | -0.060 | 57.3% | -0.001 | 52.0% | 35.2% |
| RMS/#POINTS | 0.257 | 658 | 0.354 | 741 | 998 |
| STD/STATUS | 0.250 | WORSE | 0.354 | WORSE | WORSE |
| | GOCE Txx&Tyy: IMPROVED GEOID | | | | |
| | w.r.t EGM | | w.r.t. GPM | | w.r.t. EGM &GPM |
| MIN/#POINTS | -1.215 | 863 | -1.050 | 739 | 497 |
| MAX/STATUS | 0.982 | BETTER | 1.160 | BETTER | BETTER |
| MEAN/ IN % | -0.042 | 56.0% | 0.018 | 47.9% | 32.2% |
| RMS/#POINTS | 0.320 | 679 | 0.345 | 803 | 1045 |
| STD/STATUS | 0.317 | WORSE | 0.344 | WORSE | WORSE |
| | GOCE COMBINED: IMPROVED GEOID | | | | |
| | w.r.t EGM | | w.r.t. GPM | | w.r.t. EGM &GPM |
| MIN/#POINTS | -1.025 | 889 | -0.940 | 788 | 506 |
| MAX/STATUS | 0.756 | BETTER | 1.160 | BETTER | BETTER |
| MEAN/ IN % | -0.063 | 57.7% | -0.004 | 51.1% | 32.8% |
| RMS/#POINTS | 0.278 | 653 | 0.319 | 754 | 1036 |
| STD/STATUS | 0.271 | WORSE | 0.319 | WORSE | WORSE |

***Table 4:*** *Statistics of improvement w.r.t EGM 08, GPM DIR R4 and EGM&GPM*

## Conclusions

- The developed procedure for the downward continuation of GOCE SGG from mean orbit to the Earth surface using Simulated Annealing Monte Carlo method can be successfully applied for regional/local geoid modeling, providing more high frequency signal comparing to the GOCE only models (see figures 3-5).
- The residuals w.r.t. DIR R4 GPM (Table 2) at GPS/Lev BMs have much reduced mean value (~ 80 cm) compared to GPS/Lev derived geoid and decreasing STD (behavior similar to EGM); The combined geoid has the lowest STD and RMS while Tzz,lp geoid has the smallest mean value.
- The residuals of GPS/Leveling geoid at BMs w.r.t. the other models (Table 3) have the smallest mean value and RMS, while the lowest STD is again for the combined geoid; Table2 and Table 3 show the overall best performance for the combined geoid.
- In terms of improvement at GPS/Lev BMs (Table 4) the greatest improvement is w.r.t. EGM (for the combined geoid ~60%), and w.r.t. GPM DIR R4 (for Tzz geoid - ~50%).


IUGG 2015 General Assembly, Session G02 "Static Gravity Field Models and Observations", G02p-413
June 22-July 2, 2015, Prague, Czech Republic