Wavelet multi-resolution analysis of recent GOCE/GRACE GGMs

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Abstract

The realization of the GRACE/GOCE missions offer new opportunities for gravity field approximation with higher accuracy at the medium wavebands, while wavelets (WL) provide powerful gravity field analysis tools in the space/frequency domain. This work focuses on the spectral analysis of GOCE, GOCE/GRACE and combined Global Geopotential Models (GGMs) through wavelet decomposition, filtering and reconstruction to improve their performance in the higher bands of the spectrum. The GGMs evaluated refer to the latest DIR-R4/R5, TIM-R4/R5 and GOCO03s models, which are compared with local GPS/Leveling geoid heights and gravity anomalies, while EGM2008 is used as a reference. Through a WL-based multi-resolution analysis, gravity anomalies and geoid heights are analyzed to derive their approximation and detail coefficients for various levels of decomposition, which correspond to different spatial scales. The content and signal power of each level of decomposition is analyzed to conclude on the amount and quality of signal power that GOCE/GRACE GGMs represent compared to EGM2008, especially up to the targeted waveband of 100-150 km. Filtering is investigated as well to remove high-frequency information from the low resolution GOCE models and adjust the WL reconstruction. The model synthesis that follows, through WL coefficient reconstruction, aims at the generation of new synthesized GGMs, where both GOCE and EGM2008 information is used, the latter serving to model the omission error in the GOCE GGMs. The synthesized GOCE GGMs offer an improvement of more than 30 cm compared to the original GOCE GGMs, while they provide a 1-2 cm improvement compared to EGM2008. In terms of the validation with gravity anomalies, a 5 mGal improvement was found, w.r.t. to the original GOCE GGMs, while w.r.t. EGM2008 there was no improvement. Finally, it was concluded that the GOCE GGMs show improved, between 5-22%, correlation with the land topography compared to EGM2008 for spatial scales between 176-704 km.

Keywords

Wavelets, gravity field, multi-resolution analysis, filtering, spectrum coherency, correlation, validation.

1. Introduction

Monitoring and understanding of the Earth's gravity field parameters at various spatial scales has been the focus of many studies during the past decades. The satellite missions of GOCE and GRACE have provided new insights to gravity field monitoring and interpretation. Moreover, significant results related to the time variation and evolution of the gravity field have emerged, the latter being a result of mass/water redistribution in system Earth as well as a response to geodynamic phenomena e.g., mega-earthquakes (Fuchs et al., 2013). GOCE has offered invaluable data on sea-level change, ocean currents and circulation and ice dynamics (Knudsen et al., 2011; Menna et al., 2014). Especially as far as gravity field and geoid approximation is concerned, GOCE has offered improved, compared to EGM2008, representations of the medium band of the spectrum, i.e., degree and order between 160 and 220, by as much as 4-5 cm over Europe (Gruber et al., 2011; Hirt et al., 2011; Vergos et al, 2014).

This work focuses on the evaluation of the spectral content of GOCE/GRACE-based GGMs, both satellite only and combined ones, by assessing their accuracy in terms of both gravity anomalies and geoid heights. Gravity anomaly evaluation is carried out through local gravity measurements covering the entire European continent. Moreover, an extensive network of collocated GPS/Leveling benchmarks, covering continental Greece, are used for the geoid height evaluation.

Contrary to the usual evaluation in terms of the Global Geopotential Model (GGM) combination with EGM2008 (see e.g., Gruber et al., 2011; Vergos et al., 2014), in this work we employ wavelet (WL) decomposition as a multi-resolution analysis (MRA) tool. MRA describes the (infinite) sequence of closed linear subspaces of the space of square integrable functions $V_j \subset L^2(\mathfrak{R})$ and can be applied with or without data. The transition from a MRA model $\{V_j\}$ to an associated wavelet basis function is what constitutes the decomposition scheme for every MRA subspace. Although WL MRA is relatively new as an analysis tool compared to Fourier analysis, wavelets have been developed in order to overcome the deficiencies of the Fourier transform (Mallat, 1989). The advent of the WL transform and WL modelling in geosciences brought flexibility in the analysis process for over a decade, since it allows the decomposition of the signal under study to specific spatial scales that correspond to the levels of Wl decomposition. This is especially important since it allows the study of the properties of each individual level (corresponding to specific spatial scales) without influencing the rest. Within that frame, WL have been employed for local/regional determination of the Earth's gravity field (Panet et al., 2011), the identification of large-scale geoid undulations and their relation to mantle processes (Hayan et al., 2012), the solution of the Altimetry-Gravimetry Boundary Value Problem (Grebenitcharsky and Sideris, 2005) and lately to the analysis of GOCE satellite gravity gradiometry data (Grebenitcharsky and Moore, 2014). In this work WL transform and MRA are used to analyse both gravity anomalies and geoid heights in approximation and detail coefficients for various levels of decomposition. Given the initial resolution of the available data, the decomposition levels can be translated to spatial scales, hence WL MRA allows the study of GOCE/GRACE GGM contribution to various parts of the gravity field spectrum. To improve the performance of GOCE/GRACE GGMs, as to their spectral content in the higher bands of the spectrum, they are combined through wavelet decomposition, filtering and reconstruction with EGM2008. Hence EGM2008 is used to model the omission error in the low-degree GOCE/GRACE GGMs. Both the original and synthesized GOCE/GRACE fields are evaluated with in-situ gravity anomalies and GPS/Leveling observations on benchmarks (BMs).

Coherence and correlation are powerful tools for testing the relation between land topography and gravity anomalies (Δg) for different spectral bands. They are both employed over the Amazon area and Europe, in combination with WL decomposition, to determine the coherency and correlation of GOCE/GRACE GGMs for different bands of the spectrum.

2. Methodology, GGMs and data availability

2.1 GOCE/GRACE GGM analysis

Wavelets are base functions with localization properties in both space (time) and scale (frequency) domains and allow the synchronous analysis of GGM data at different levels/scales. Therefore wavelet signal processing can be a multiresolution analysis (MRA) at various levels of decomposition (Chui, 1992). The two-dimensional wavelet transform gives coefficients that correspond to different spatial resolutions, related to the signal scales (Grebenitcharsky and Moore, 2014). According to the WL decomposition algorithm, each scale (level L_n) of the signal is analyzed in an approximation coefficient (A_n), which carries the main (large scale) information and three detail coefficients (horizontal, vertical and diagonal $(H,V,D)_i|_{i=1,2,\dots,n}$ that carry the high-frequency (short-scale) information of the signal (Mallat, 1989; 1999). Through the synthesis process, various GGMs can be combined, since each level can be represented by a different GGM, based on the GGM performance at each specific level of analysis. Of course, if after the WL MRA the WL coefficients remain unaltered during the reconstruction process, the the original signal will be reconstructed perfectly due to the orthogonality of the WL base functions.

The synthesis is defined as the algebraic sum of the detail coefficients of each level used $(H,V,D)_i|_{i=1,2,\dots,n}$ and the approximation coefficient of the last level (A_n) as:

$$Synthesis = \mathrm{A}_n + (H,V,D)_\mathrm{n} + (H,V,D)_\mathrm{n-1} + \cdots + (H,V,D)_1 \tag{1}$$

Therefore, for the available GOCE/GRACE GGMs, their spectral content at each level is analyzed and compared among each other and with EGM2008. The goal is to construct combined GGMs where both GOCE/GRACE and EGM2008 information will be used, so that the gravity field signal will be represented with higher accuracy. The choice of the GGM that will be used at each level depends on its resolution and the gravity field content with respect to EGM2008. Then, different GGMs can be combined during the synthesis process in order to determine a combined/hybrid GGM. In that sense, the first levels of the EGM2008 decomposed signal (small spatial scales) can be used to model the omission error in the GOCE GGMs. Likewise, the decomposed GOCE GGM signal can provide gravity information for those levels that correspond to the spatial scales targeted by the GOCE mission (larger than 100 km).Hence, during the synthesis process, various levels from different GGMs can be combined, in order to provide improved representations of the Earth's gravity field. This aims at reducing the omission error in GOCE GGMs and augmenting EGM2008 with gravity information from the GOCE mission.

Moreover, when GOCE/GRACE GGMs are analyzed, the gravity signal of the first levels (high-frequencies) is dominated by noise since these spatial scales are not mapped by the GOCE mission. This is especially profound at the limits of the spatial scales targeted by GOCE, i.e., around 100-120 km. For those levels that the noise is either dominant or contaminates the gravity field signal, increasing the SNR (signal-to-noise-ratio) demands a digital or spatial filter implementation. In this work, both Gaussian and boxcar filters have been used to remove noise. The former is considered very effective for filtering in the space domain and the latter being a smoothing mathematical function, which uses a rectangular window in the frequency domain. After this synthesis and filtering process, the new combined GGMs can be evaluated w.r.t. some external information, e.g., GPS/Leveling geoid heights and gravity anomalies, to quantify the improvement reached.

Another valuable tool in the evaluation of the GOCE GGMs is in terms of the relationship between the GGM-derived gravity information and land topography. Therefore, a further investigation in terms of the spectral coherence and correlation was realized. Spectral coherence is a measure of the relation between two signals or data sets and if applied to the analysis of gravity field data it can give insight to how well they relate to the Earth's topography. The basic idea behind this evaluation, performed for each level of WL decomposition, is that if a GGM has higher coherence with the topography for a specific level, i.e., for a specific spectral band, then it represents better compared to the other GGMs the Earth's gravity field. In that way, the possible improvement by GOCE can be envisaged at specific targeted wavebands. Coherence is defined as (Bendat and Piersol, 2010):

$$\mathrm{C_{gH}} = \frac{\left|\mathrm{G_{gH}}\right|^2}{\mathrm{G_{gg}G_{HH}}} \tag{2}$$

where, G_{gH} denotes the cross-spectral density between the gravity and topography signals g and H, and G_{gg} and G_{HH} the auto-spectral density. Another useful measure to be employed is the correlation matrix that can show whether and how strongly two signals are related. In our case, we construct the correlation matrix by estimating the correlation coefficients between the various levels of decomposition of both gravity and topography. Then the correlation coefficients are estimated as (Bendat and Piersol, 2010):

$$R_{gH}^{ij} = \frac{C_{gH}^{ij}}{\sqrt{C_{gg}^{ii} C_{HH}^{jj}}} \tag{3}$$

In Eq. (3), R_{gH}^{ij} denotes the correlation between level i of gravity data from the GGMs and level j of the topography, C_{gH}^{ij} is the cross-covariance matrix of the two datasets and C_{gg}^{ii} and C_{HH}^{jj} the covariance matrices. In that way, the correlation matrix has as diagonal elements the coefficients for the same levels of decomposition. Finally, it should be noted that we have used the "awkward" term gravity to describe the GGM contribution in the sense that the gravity signal g can be any functional related to the Earth's gravity field. In our study, as far as coherence and correlation are concerned, we focus on GGM-derived gravity anomalies over the Amazon area and over Europe.

2.2 Available data and external validation

The present study focuses on the GGM evaluation with external data, over the European Continent, within the region bounded between $30^o \leq \varphi \leq$

60^o and $-10^o \leq \lambda \leq 30^o$. For the investigation of the correlation and coherence between the topography and GGM-induced gravity, the study focuses on two regions. The first one is the aforementioned European area while the second one focuses over the Amazon region, bounded between$-10^o \leq \varphi \leq 20^o$ and $275^o \leq \lambda \leq 335^o$. The main reason for focusing in two areas is their different topographic features and gravity field representation. The former stems from the Amazon area being mostly flat, while Europe has highly varying terrain. The latter refers to the fact that Amazon is a poorly surveyed area in terms of terrestrial gravity data, contrary to Europe. Therefore, GOCE GGMs should have improved performance over the Amazon, in terms of the correlation and coherency with land topography compared to EGM2008.

The GOCE/GRACE GGMs evaluated refer to the latest DIR and TIM models employing the fourth (R4) and fifth (R5) release, i.e., an effective data volume of 26.5 months of GOCE observations (R4) and the low orbit data (R5). TIM-R4 (Pail et al., 2011) presents a spherical harmonics expansion of the Earth's potential to a maximum degree n_{max}=250 while TIM-R5 reaches a maximum degree of 280, both employing the time-wise approach and being pure GOCE models. DIR-R4 (Bruinsma et al., 2010; 2013), presents a spherical harmonics expansion of the Earth's potential to a maximum degree 260 while DIR-R5 reached a maximum degree 300, employing the direct approach and in addition to the GOCE observations, 9 years of GRACE data (10 for DIR-R5) and SLR have also been used. Apart from these GGMs, GOCO03s is used as well (Mayer-Gürr et al. 2012), which is based on both GOCE and GRACE data with a maximum degree of expansion to 250. In all cases, the performance of GOCE/GRACE GGMs is evaluated against EGM2008 which is used as reference (Pavlis et al., 2012). From these models, gravity anomalies and geoid heights have been determined, for all areas under study, at grid points with a spatial resolution of $3' \times 3'$ (5.5 km).

As far as the external GPS/Leveling and gravity data are concerned, the former refer to a set of 1542 collocated GPS and Leveling observations over the Hellenic trigonometric network (Vergos et al., 2014) in mainland Greece. The gravity data, refer to the gravity anomaly field derived in the frame of the World Gravity Map project (WGM2012) covering the entire European area (Balmino et al., 2012). WGM2012 is largely based on EGM2008 with the addition of a worldwide 1′×1′ grid of ETOPO1-induced gravity disturbances. Therefore, the improvements brought by GOCE to the gravity anomaly comparison are expected to be marginal. Finally, the coherence and correlation between land topography and the Earth's gravity field are computed for the Amazon and Europe area, where ETOPO1 (Amante and Eakins, 2009) was used for the topographic information. All computations were carried out in the Tide Free (TF) system, while GRS80 has been used as a normal field in the evaluation of the geoid zero-degree term.

3. GGM external validation with MRA

Table 1 and Table 2 summarize the statistics of the differences between the GPS/Levelling geoid heights, the WGM2010 gravity anomalies and the GOCE/GRACE GGMs. In these Tables, the normal faced lettering denotes the differences before any analysis of the GGM data has been performed, i.e., the original ones to the GGM n_{max}. EGM2008, which will be used as the reference GGM, provides a std at the 13.4 cm and 3.2 mGal, while the GOCE/GRACE GGMs reach the 44-46 cm and 21-22 mGal, respectively. The latter is expected due to the omission error in both geoid heights and gravity anomalies, due to the small n_{max}.

The goal now is to try and enhance this performance of the GOCE/GRACE GGMs through WL-based MRA. To that respect the derived geoid heights and gravity anomalies from all GGMs have been decomposed through a discrete wavelet transform (DWT) into 12 levels (L_1, L_2, …, L_{12}), where each level was analyzed in an approximation coefficient and three detail coefficients. For all GGMs, L_1 corresponds to spatial scales between 5.5-11 km (smallest scales), L_2 to 11-22 km, L_3 to 22-44 km, L_4 to 44-88 km, L_5 to 88-176 km, L_6 to 176-352 km, L_7 to 352-704 km, L_8 to 704-1,408 km, L_9 to 1,408-2,816 km, L_{10} to 2,816-5,632 km, L_{11} to 5,632-11,264 km, and L_{12} to 11,264-22,528 km (largest scales). Given the decomposition, the signals are then synthesized for the GOCE/GRACE GGMs, by replacing some of their levels with those of EGM2008, so that a new combined GGM would be generated, where both GOCE/GRACE and EGM2008 information is used. When the new GGM is denoted as, e.g., *GOCO03s (*L_5*,* L_6*,* L_7*)* this means that the first four levels come from EGM2008, levels 5-7 from GOCO03s and then levels 8-12 from EGM2008 again. L_5, L_6, L_7 span the spatial scales between 88 km and 704, corresponding to harmonic degrees ~28-225, therefore they represent the main waveband that GOCE should primarily provide its highest improvement.

For the WL-based MRA implementation, various mother WLs have been tested, from the simple Haar WL, to Coiflet and Daubechies (db), while for the latter two various orders have been investigated. It was finally decided that db10, i.e., the daubechies WL with ten vanishing moments, would be used for the WL analysis, since it provided the best results to the GPS/Leveling geoid heights. After the decomposition of the GGMs followed the reconstruction of their levels, by combining their detail coefficients, and then the synthesis, as outlined in the preceding section and Eq.1. In Tables 1 and 2 we summarize the synthesized results from two scenarios, where GOCE/GRACE provides the information for L_5, L_6, L_7 and L_6, L_7. In the first case, the synthesized GGMs (herein denoted as SynthGOCO03s, SynthTIM-R4, SynthTIM-R5, SynthDIR-R4 and SynthDIR-R5) provide improved differences with the GPS/Leveling data by ~20 cm in terms of the std compared to the original ones. SynthGOCO03s reduced the std to 26 cm, while SynthTIM-R4/SynthTIM-R5 and SynthDIR-R4/SynthDIR-R5 are at the 24 cm and 22 cm. Still, these are about 10 cm worse than EGM2008, signalling that some of the GOCE levels used for the synthesis are of lower quality and contain more noise than signal. When only L_6, L_7 from GOCE are used (corresponding to spatial scales 176-352 km and 352-704 km, respectively) then the situation improves drastically. SynthGOCO03s now reaches the 12.4 cm when compared to the GPS/Leveling geoid heights, while SynthTIM-R4 and SynthDIR-R4 provide even better results at the 12.2 and 12 cm. The R5 versions of the TIM and DIR models manage to improve the results of their R4 counterparts to 12.1 and 11.8 cm. These are 1-1.6 cm better than the performance of EGM2008, signalling the improvement offered by GOCE in the specific spectral range. Moreover, it shows that the WL-based MRA is an effective tool in order to analyse the GOCE GGMs and model their omission error in the shorter spatial scales.

TABLE 1

The same holds for the external evaluation with gravity anomalies, since the Wavelet MRA Synthesis with L_5, L_6, L_7 improves the original ones by as much as 13-14 mGal. What is striking in the gravity anomaly differences is the significant improvement brought by TIM-R5 and DIR-R5. The std of the differences before any analysis is ~3 mGal lower than that of the R4 GGMs, while after the synthesis they give improved std by ~1 mGal. This signals the value of the low orbit GOCE data, which is especially evident in the gravity anomalies compared to geoid heights due to the larger spectral content of the former in the smaller scales. When the synthesis is based only on L_6, L_7 from GOCE the std of the differences is the same as that of EGM2008 (~3 mGal). It is not unexpected that the synthesized GOCE/GRACE GGMs do not manage to improve the results of EGM2008, since most of the WGM2012 gravity data have been used in the compilation of EGM2008.

TABLE 2

From these results, it becomes apparent that indeed GOCE manages to improve the geoid and gravity field representation in the medium bands of the spectrum and that the followed methodology manages to provide reasonable results and can be employed in related studies where a synthesized geoid and/or gravity field is needed. A further test performed, was to examine the behavior of L_5 from the GOCE/GRACE GGMs, given that its inclusion in the synthesis worsens the results. L_5 corresponds to spatial scales between 88-176 km, therefore given that GOCE was to provide useful signal up to spatial scales of 100 it means that part of the signal in L_5 is beyond the mesuring waveband of GOCE (ESA, 1999). To that respect, and in order to remove some of the noise present in L_5 of the decomposed GOCE/GRACE GGM signal, a simple Gaussian and a boxcar type of filter have been tested. Both will act as low-pass filters, where we intend to remove some of the high-frequencies in the waveband between 88-176 km. Various cut-off wavelengths between 90 km and 140 km, with an increment of 5 km have been tested, and finally it was decided to keep the one corresponding to a spatial scale of 120 km. For all these tests, after the filtering, the signals were synthesized and comparisons with the GPS/Leveling and gravity data have been performed. The cut-off wavelength of 120 km was the one that offered the best results in the comparisons with the external data. For smaller wavelengths, the results deteriorate due to the presence of noise, while for larger wavelengths signal was removed along with noise deteriorating the results. Tables 3 and 4 presents the statistics of the differences between the filtered and then synthesized GOCE/GRACE GGMs (denoted as, e.g., *GOCO03s-f* where *f* stands for filtered) with the GPS/Leveling and gravity data. For GOCO03s, the std of the differences with the GPS/Leveling data reduces from 25.9 cm to 17.6 cm and 19.2 cm for the Gaussian and boxcar filters. A smaller improvement is found for the TIM-R4 and DIR-R4 models, for which the std decreases by ~4-5 cm. Noticeably though, while the synthesis of TIM-R4 (L_5, L_6, L_7) is better than that of GOCO03s (23.9 cm compared to 25.9 cm), after the filtering with the Gaussian filter GOCO03s is better by 1.6 cm. This may be due to the fact that the latter is based on fewer GOCE data, hence the L_5 geoid signal is poorly modeled compared to TIM-R4. In that way, filtering benefits more the GGM with the least amount of information in that waveband, by removing the noise, while in TIM-R4 it removes not only the noise but some useful geoid signal that is present. With TIM-R4 and DIR-R4 probably a more elaborate filtering process is needed, so that the noise and signal can be better decomposed and separated. This is out of the scope of the present work and is actually the field of future investigation. For DIR-R4, after filtering L_5 the std drops to 18.7 and 21.2 cm, showing slightly better results than TIM-R4. Once again, more interesting results from the filtering are acquired for the R5 GGMs, since for TIM-R5 the std drops to 16.3 and 18.0 cm and for DIR-R5 to 15.7 and 17.7 cm, for the Gaussian and boxcar filter respectively. This is a 3 cm improvement compared to the R4 GGMs and is due to the fact that the R5 models contain the low orbit GOCE data, hence the SNR is larger at L_5. The same results are acquired for the external validation with gravity anomalies, where an improvement of 2-3 mGal is achieved, compared to the unfiltered synthesis, but this is still worse than the synthesis where only L_6 and L_7 are used. Concluding on the filtering process investigated, the results acquired are indeed improved compared to no filtering L_5 at all, but in no case did we achieve the results when using for the synthesis only L_6 and L_7. If some useful signal is to be derived from

L_5, so that the results will be further improved, then more elaborate filtering and/or error modeling is needed to de-correlate signal from noise.

TABLE 3

TABLE 4

The final part of the GOCE/GRACE GGM evaluation was carried out over the Amazon area and Europe by investigating the spectral coherence and correlation between the GGM-derived gravity anomalies and land topography. To derive the coherence between the GGM gravity anomalies and topography/bathymetry, WL MRA has been used again so that both signals have been decomposed in twelve levels and then the signal of each level has been reconstructed. Then, employing Eq. 2 the spectral coherence has been evaluated for each level between the topography signal and gravity anomalies from EGM2008, TIM-R4/R5, DIR-R4/R5 and GOCO03s.

For Amazon, the same results with the external evaluation have been derived, with the higher coherence found in L_6 and L_7 for the GOCE GGMs and lower coherence for L_5 and L_4. These are depicted in Figure 1, where the coherence for L_3, L_5, L_6 and L_7 is shown for the area over Amazon. As expected, the coherence for the GOCE/GRACE GGMs for L_3 is very low with practically no coherence up to 30-35 km and then only up to 20%, which is probably just noise. EGM2008 on the other hand has a more or less uniform coherence between 55% and 72%. In L_5 the situation starts to improve for the GOCE/GRACE GGMs, with higher coherence up to ~42%, while in all cases their coherence is below that of EGM2008. Notice that the filtering cut-off wavelength set in the previous test to 120 km coincides in the coherence plot with the point that the coherence starts to raise for the GOCE/GRACE GGMs. Therefore, in the second half of the GOCE/GRACE L_5 there seems indeed to be some useful signal that still remains to be accounted for within the present methodological scheme. The situation changes completely for L_6 and L_7, where the GOCE /GRACE GGMs are equivalent to EGM2008 and in most cases better than that. Between the GOCE/GRACE GGMs it is interesting to notice that for L_5, the release 5 versions of TIM and DIR have better coherency than GOCO, which is due to the more GOCE data used in their development. For larger wavelengths, GOCO performs equally well given that these lower harmonic degrees of the spectrum are sufficiently modeled by the release 5 of GOCE data along with the information form GRACE which was used in its development. Comparing the R4 and R5 versions of the TIM and DIR models, Figure 2 shows their coherence for L_5 over Amazon. From this Figure it is readily seen how the addition of the low-orbit GOCE data manage to provide improved results by about 10-20%. This is especially evident at spatial scales between 140-160 km, where the difference between TIM-R4 and TIM-R5 is at the 20% level. Finally, DIR-R5 manages to provide improved results especially for scales between 80-100 km, where it is better by as much as 35%.

Figure 1

Figure 2

Over Europe the results are slightly different, given that EGM2008 is dominant. For L_3 EGM2008 has a mean coherence of 70% while it reaches the 85% level as well. For L_5 EGM2008 retains a high coherence between 50% and 80% while the GOCE GGMs have a maximum of 50% and a mean of ~30%. This is due to fact that Europe is a well surveyed area in terms of terrestrial gravity data, hence EGM2008 manages to represent very well the gravity field over this region. From this analysis it can be concluded that indeed in poorly surveyed areas, GOCE GGMs can be expected to contribute significantly in gravity field mapping.

Likewise, the correlation between the topographic signal and that of the GGM-derived gravity anomalies for the various levels of decomposition has been analyzed. Table 5 presents the correlation coefficients for EGM2008 and the GOCE/GRACE GGMs only for Amazon given the aforementioned discussion. The correlation results for Europe are again in favor of EGM2008, since most of the terrestrial gravity anomalies over the entire continent have been used in its development. A similar picture with the coherence analysis is found. For the very first levels (L_1 to L_4), EGM2008 dominates since the GOCE/GRACE GGMs present little or no correlation at all (up to 33% for L_4). From L_5 onwards the contribution of GOCE is evident, since the latter GGMs manage to perform equally well with EGM2008 (for L_5) and outperform it for L_6, L_7 and L_8. For L_6 and L_7 they present a higher correlation between 3-5% compared to EGM2008, which increases to 22% for L_8. The correlation found for L_9 (corresponding to spatial scales between 1408-2816km) is somewhat puzzling, since it is lower than that for L_8 and L_{10}. Analyzing the topography and gravity anomaly signals for L_9, it was found that the former has two (positive) dominant features in the EW direction with a low over the Amazon basin, probably coming from the Andes to the East and a merge of the Brazilian and Guyana shields in the West. On the other hand, the gravity anomaly signal shows very little variation for these spatial scales over the area under study, with two predominant positive features in the NS direction and a low over the Amazon basin. This inconsistency can be due to the fact that the gravity signal from the Andes is more high frequency in nature and is represented in the lower levels (it is mostly seen in L_4-L_7) compared to the Brazilian and Guyana shield topography that dominate the area under study in the southern and northern parts respectively. In any case, the contribution of GOCE to the medium wavebands of the spectrum is once again evident, while if a higher resolution digital terrain model have been used, then the superiority of the GOCE/GRACE GGMs would have been more evident in the coherence and correlation analysis.

TABLE 5

4. Conclusions

A detailed evaluation, employing WL-based MRA, has been carried out for the recent GOCE and GOCE/GRACE GGMs both in terms of geoid heights and gravity anomalies. From the external evaluation that referred to geoid heights, it was concluded that the combined GGMs improve the estimated geoid heights, compared to local GPS/Leveling data, since the std is reduced from ±0.45 m to ±0.22 m for the DIR-R4/R5 and TIM-R4/R5 models. When only L_6 and L_7 have been used from the GOCE/GRACE GGMs, then the results improved further to

±0.118 cm for DIR-R5, being 1.6 cm better than EGM2008. Contrary to that, the evaluation with gravity anomalies revealed that in the best case scenario the GOCE/GRACE GGMs manage to reach the agreement of EGM2008, something attributed to the fact that most of the data used for the compilation of WGM2012 have been included in EGM2008. Hence this gravity information is not a totally independent dataset due to the high correlation with EGM2008.

When filtering L_5 of the GOCE/GRACE GGMs the results improved between 4-8 cm and 2-3 mGal, showing that indeed L_5 contains useful geoid/gravity signal that can contribute to the overall GGM performance. To that respect, more advanced filtering options will be investigated to separate the noise from the signal and improve the GGM performance. Overall, the proposed WL MRA methodology, for the analysis and synthesis of GOCE/GRACE GGMs, provides promising results since spatial scales in the GGMs that are modeled with lower accuracy can by successfully replaced with other sources of information which are of higher accuracy/quality. It is worth mentioning that with the presented WL-based analysis there is no mixing of the spatial scales of the gravity field signal. This is so in the sense that WLs can isolate specific portions of the gravity field signal through the analysis in various levels of decomposition, the latter corresponding to specific spatial scales. Then each level can be manipulated separately from the rest, allowing the study of the gravity field signal properties for the specific spatial scales. This cannot be done with, e.g., Fourier transform based methods, where the entire spectrum of the gravity field signal is studied in its entity. From our analysis, the main problem with this WL-based approach is that the dyadic nature of WLs allow the isolation of specific spatial scales for each level (given the resolution of the original signal). If one would like to study, e.g., the GOCE GGM signal for spatial scales between 60 and 140 km only, then this approach cannot be followed, since the specific range belongs to two different levels (L_4 and L_5) so each level should be studied separately.

From the analysis of the spectral coherency and correlation between topography and the GGM-derived gravity anomalies it was concluded that EGM2008 has significantly better results for the first few levels. This is expected since the GOCE/GRACE GGMs investigated are satellite-only ones. The contribution of the satellite missions in seen again in L_5, L_6, L_7 and L_8, where the GOCE/GRACE GGMs show improved result compared to EGM2008 by 3-22%.

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Table 1: Statistics of the differences between GPS/levelling and geoid heights from the GGMs before (normal font) and after (italics) the WL MRA synthesis. Units: [m]

	min	max	mean	std
EGM08	-0.853	0.104	-0.372	±0.134
GOCO03s	-1.735	1.110	-0.359	±0.464
GOCO03s (L_5,L_6, L_7)	*-1.083*	*0.453*	*-0.387*	*±0.259*
GOCO03s (L_6, L_7)	*-0.855*	*0.093*	*-0.378*	*±0.124*
TIM-R4	-1.597	1.155	-0.358	±0.450
TIM-R4 (L_5,L_6, L_7)	*-1.151*	*0.399*	*-0.381*	*±0.239*
TIM-R4 (L_6, L_7)	*-0.838*	*0.053*	*-0.383*	*±0.122*
TIM-R5	-1.569	1.123	-0.394	±0.469
TIM-R5 (L_5,L_6, L_7)	*-1.142*	*0.408*	*-0.393*	*±0.242*
TIM-R5 (L_6, L_7)	*-0.831*	*0.047*	*-0.383*	*±0.121*
DIR-R4	-1.540	1.105	-0.366	±0.442
DIR-R4 (L_5,L_6, L_7)	*-1.048*	*0.401*	*-0.392*	*±0.223*
DIR-R4 (L_6, L_7)	*-0.802*	*0.064*	*-0.394*	*±0.120*
DIR-R5	-1.530	1.122	-0.388	±0.454
DIR-R5 (L_5,L_6, L_7)	*-1.031*	*0.388*	*-0.389*	*±0.217*
DIR-R5 (L_6, L_7)	*-0.811*	*0.032*	*-0.381*	*±0.118*

Table 2: Statistics of the gravity anomaly differences between WGM2012 and the GGMs before (normal font) and after (italics) the WL MRA synthesis. Units: [mGal]

	min	max	mean	std
EGM08	-49.66	128.50	0.31	±3.24
GOCO03s	-204.97	272.23	0.11	±22.49
GOCO03s (L_5,L_6, L_7)	*-89.09*	*129.87*	*0.32*	*±9.38*
GOCO03s (L_6, L_7)	*-51.19*	*123.95*	*0.31*	*±3.52*
TIM-R4	-206.98	269.35	0.11	±22.14
TIM-R4 (L_5,L_6, L_7)	*-90.52*	*134.80*	*0.31*	*±8.85*
TIM-R4 (L_6, L_7)	*-50.17*	*123.54*	*0.30*	*±3.48*
TIM-R5	-196.90	272.70	0.11	±19.34
TIM-R5 (L_5,L_6, L_7)	*-59.78*	*129.77*	*0.49*	*±7.66*
TIM-R5 (L_6, L_7)	*-40.53*	*124.22*	*0.43*	*±3.37*
DIR-R4	-201.93	271.43	0.11	±21.93
DIR-R4 (L_5,L_6, L_7)	*-87.10*	*129.69*	*0.29*	*±8.47*
DIR-R4 (L_6, L_7)	*-51.12*	*123.89*	*0.28*	*±3.44*
DIR-R5	-203.66	270.65	0.11	±19.10
DIR-R5 (L_5,L_6, L_7)	*-65.17*	*129.42*	*0.54*	*±7.38*
DIR-R5 (L_6, L_7)	*-41.02*	*125.50*	*0.41*	*±3.35*

Table 3: Statistics of the differences between GPS/levelling and geoid heights from the synthesized GGMs after filtering GOCE/GRACE L_5. Normal lettering for Gauss and italics for boxcar filtering. Units: [m]

	min	max	mean	std
GOCO03s-f (L_5,L_6, L_7)	-0.870	0.206	-0.377	±0.176
GOCO03s-f (L_5,L_6, L_7)	-0.898	0.244	-0.373	±0.192
TIM-R4-f (L_5,L_6, L_7)	-0.927	0.279	-0.377	±0.190
TIM-R4-f (L_5,L_6, L_7)	-1.011	0.314	-0.373	±0.213
TIM-R5-f (L_5,L_6, L_7)	-0.907	0.202	-0.384	±0.163
TIM-R5-f (L_5,L_6, L_7)	-0.874	0.237	-0.379	±0.180
DIR-R4-f (L_5,L_6, L_7)	-0.925	0.279	-0.378	±0.187
DIR-R4-f (L_5,L_6, L_7)	-1.027	0.316	-0.373	±0.212
DIR-R5-f (L_5,L_6, L_7)	-0.875	0.189	-0.381	±0.157
DIR-R5-f (L_5,L_6, L_7)	-0.874	0.228	-0.376	±0.177

Table 4: Statistics of the differences between WGM2012 and gravity anomalies from the synthesized GGMs after filtering GOCE/GRACE L_5. Normal lettering for Gauss and italics for boxcar filtering. Units: [mGal]

	min	**max**	**mean**	**std**
GOCO03s-f (L_5, L_6, L_7)	-72.77	127.91	0.31	±6.48
GOCO03s-f (L_5, L_6, L_7)	-74.65	130.64	0.32	±6.94
TIM-R4-f (L_5, L_6, L_7)	-74.88	132.14	0.29	±6.36
TIM-R4-f (L_5, L_6, L_7)	-75.29	135.47	0.29	±6.81
TIM-R5-f (L_5, L_6, L_7)	-51.01	130.51	1.09	±5.73
TIM-R5-f (L_5, L_6, L_7)	-53.43	134.46	1.35	±6.19
DIR-R4-f (L_5, L_6, L_7)	-73.01	130.56	0.29	±6.25
DIR-R4-f (L_5, L_6, L_7)	-73.96	134.30	0.29	±6.72
DIR-R5-f (L_5, L_6, L_7)	-53.83	129.66	1.09	±5.63
DIR-R5-f (L_5, L_6, L_7)	-55.70	132.90	1.35	±6.14

Table 5: Correlation between Topography and GGM-derived gravity anomalies over Amazon.

	EGM2008	GOCO03s	TIM-R4	TIM-R5	DIR-R4	DIR-R5
L_1	28.80%	0.10%	2.10%	1.80%	0.30%	0.50%
L_2	59.80%	0.20%	1.40%	0.90%	0.20%	0.21%
L_3	71.90%	3.30%	5.50%	8.58%	2.50%	8.27%
L_4	74.20%	28.20%	32.10%	31.70%	30.10%	33.47%
L_5	65.50%	60.80%	65.20%	63.90%	63.00%	63.90%
L_6	62.40%	63.10%	66.20%	69.87%	68.70%	69.87%
L_7	67.70%	70.20%	71.50%	74.10%	70.10%	74.07%
L_8	42.10%	44.50%	46.20%	64.30%	45.10%	64.16%
L_9	23.00%	29.20%	25.50%	26.00%	26.50%	29.80%
L_{10}	30.10%	35.00%	39.10%	53.20%	39.80%	53.60%
L_{11}	51.80%	52.10%	46.80%	56.10%	52.30%	64.20%
L_{12}	97.90%	98.10%	98.20%	99.00%	98.50%	99.63%

Figure 1: Spectral coherency for various levels of decomposition between topography and GGM-derived gravity anomalies over the Amazon area.

Figure 2: Spectral coherency for L_5 between topography and TIM-R4/R5 and DIR-R4/R5 GGM-derived gravity anomalies over the Amazon area.