

# Evaluation of GOCE/GRACE GGMs over Attica and Thessaloniki, Greece, and Wo determination for height system unification

<sup>1</sup>Department of Geodesy and Surveying  
Aristotle University of Thessaloniki

<sup>2</sup>Department of Civil Engineering and Surveying and Geoinformatics Engineering,  
Technological and Educational Institute of Athens

G.S. Vergos<sup>1</sup>, V.D. Andritsanos<sup>2</sup>, V.N. Grigoriadis<sup>1</sup>,  
V. Pagounis<sup>2</sup>, I.N. Tziavos<sup>1</sup>

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## Availability of several GOCE/GRACE GGMs for the Earth's potential

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- TIM (R1, R2, R3, R4)
- DIR (R1, R2, R3, R4, R5)
- GOCO (01s, 02s, 03s)
- EIGEN\*s/c (51c, 6s/c, 6s/c2, 6c3stat)
- SPW (R1, R2)
- ITG-GOCE02s, GOGRA02s, JYY\_GOCE02s

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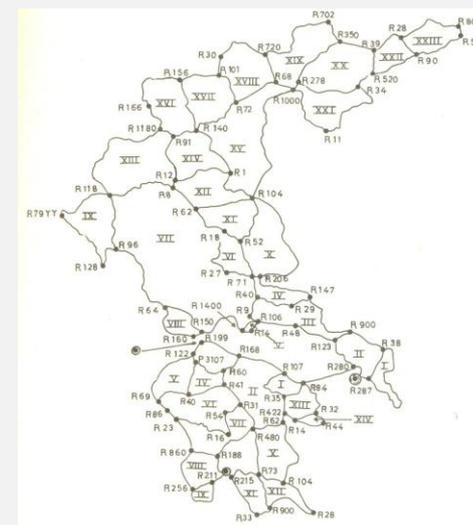
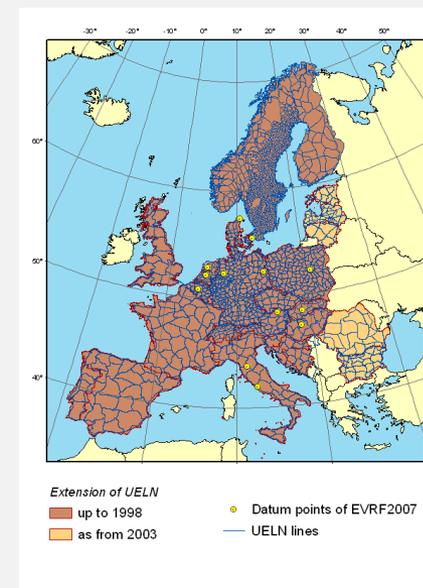
## Validation of the GOCE/GRACE GGMs and $W_0$ determination

- External validation over Greece with collocated GPS/Leveling BMs
- Contribution to  $\hat{W}_0$  LVD determination

Evaluation of the HelLEnic VERTICAL network in  
the frAme of the European SysTems and  
Control Networks InterconnectON –  
Application in the areas of Attica and  
Thessaloniki

## Evaluation of the HelLEnic Vertical network in the frAme of the European SysTems and Control Networks InterconnectON – Application in the areas of Attica and Thessaloniki

- TEI of Athens – Aristotle University of Thessaloniki – National Technical University of Athens
- The Hellenic VN has not been connected with the EVRS
- Validation before the connection
- GRACE and GOCE data provide valuable information



Combined GGM differences to  $n_{\max}=216,000$

$$\Delta N = N^{GPS/Lev} - N^i \Big|_2^{n_1} - N^{EGM2008} \Big|_{n_1+1}^{2160} - N^{RTM} - N_o$$

$N^i \Big|_2^{n_1}$  evaluation of GOCE/GRACE GGMs every 1 degree

$N^{RTM}$  from a 3" DTM over Greece

## Combined GGM differences to $n_{\max}=216,000$

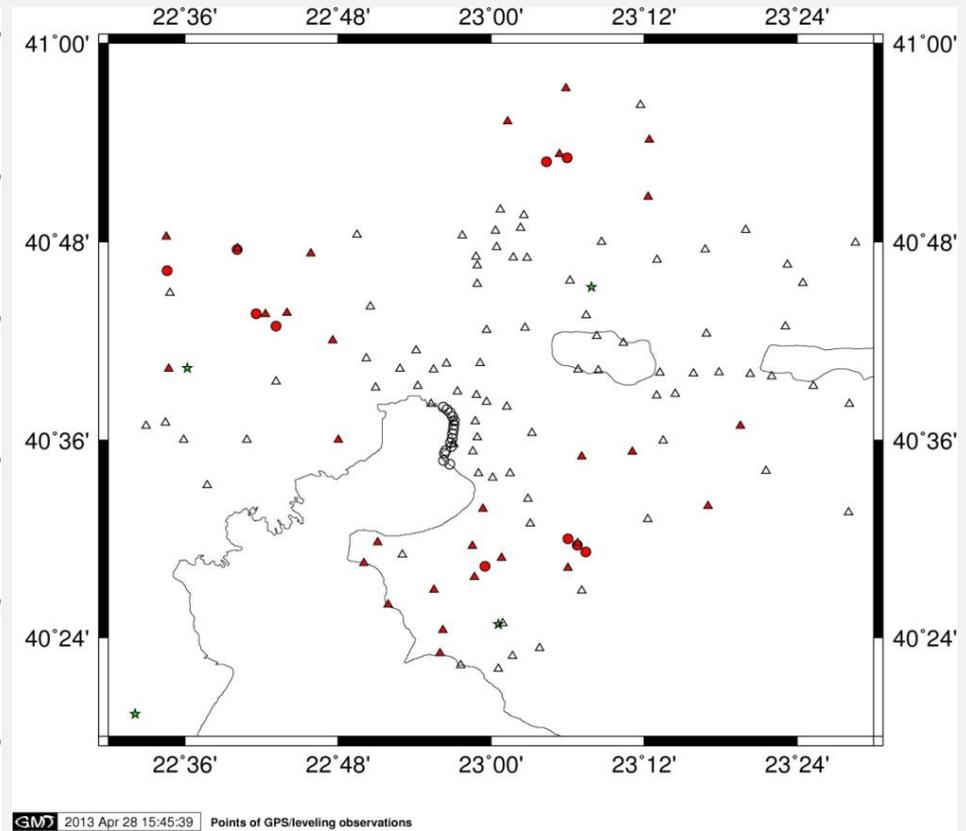
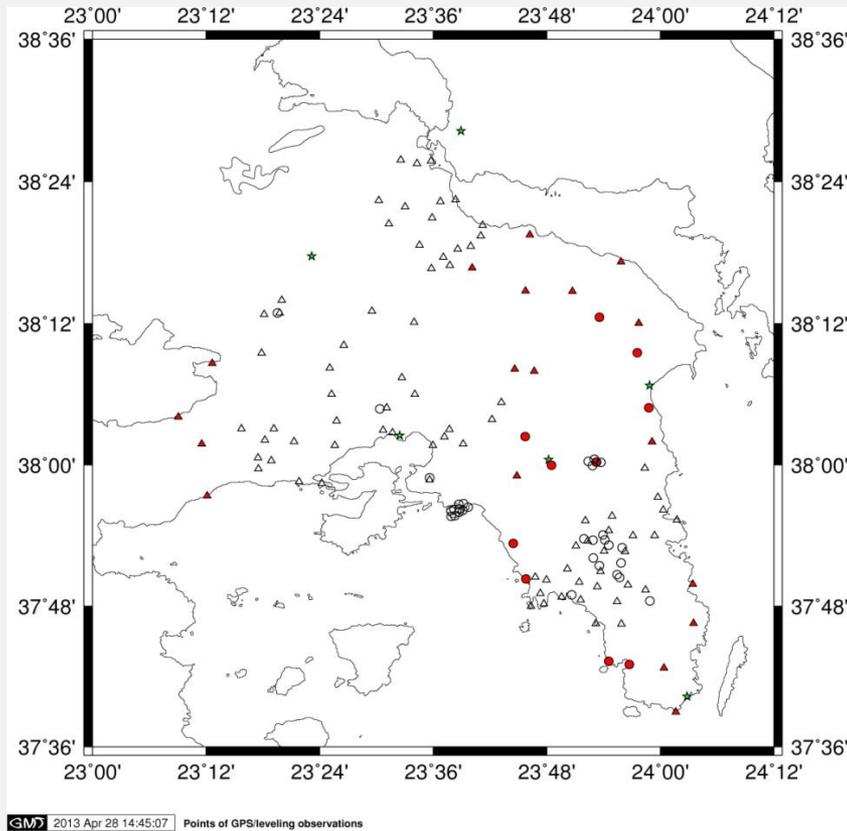
$$\Delta N = N^{GPS/Lev} - N^i \Big|_2^{n_1} - N^{EGM2008} \Big|_{n_1+1}^{2160} - N^{RTM} - N_o$$

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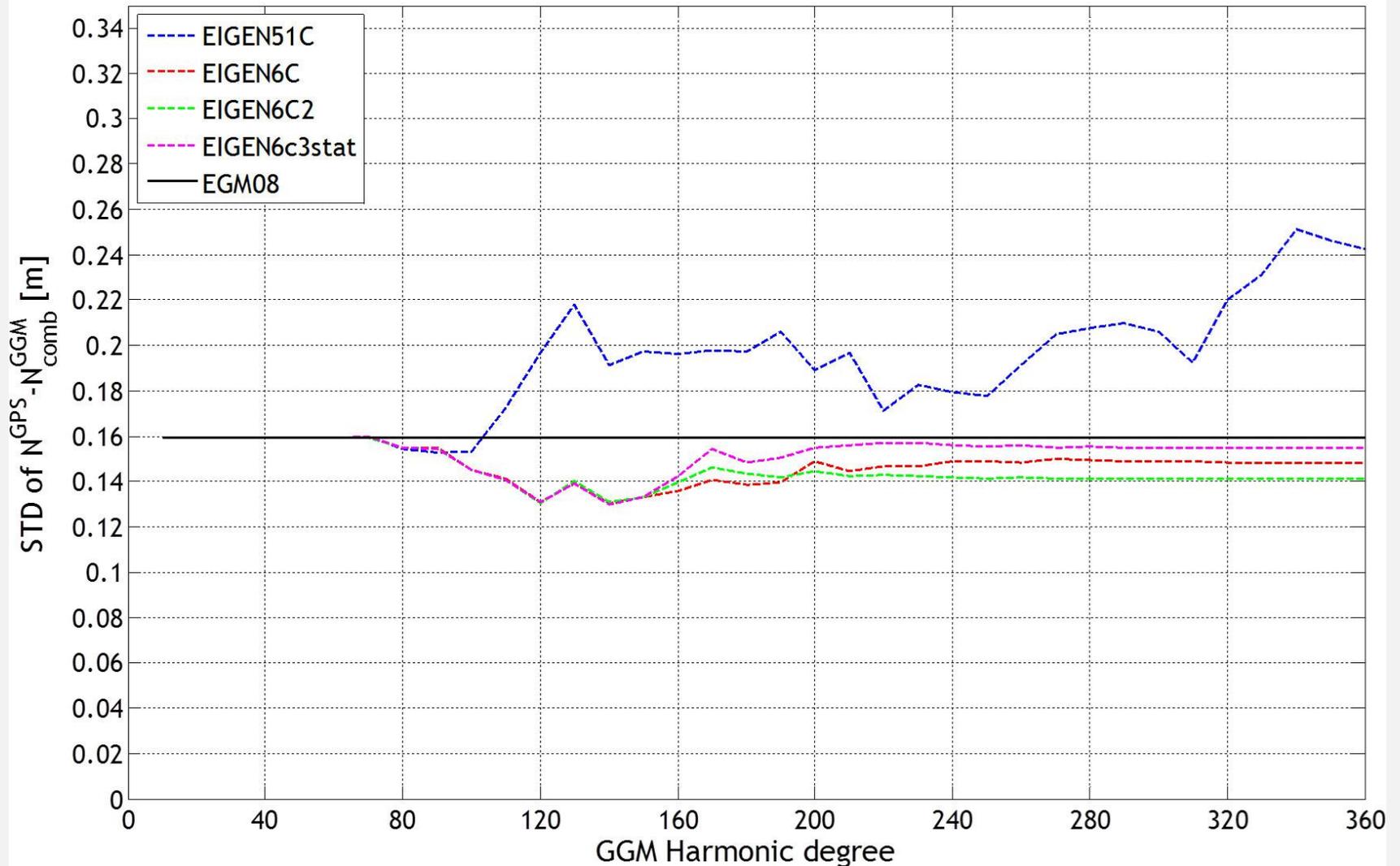
- Mean Tide to Tide Free conversion for orthometric heights
- GGM Zero Tide to Tide Free when necessary
- All computations in GRS80
- $N_o$  relative to the IERS  $W_o$  of  $62636856.0 \text{ m}^2/\text{s}^2$

## 2 tests areas 100 km × 100 km each – 230 BMs



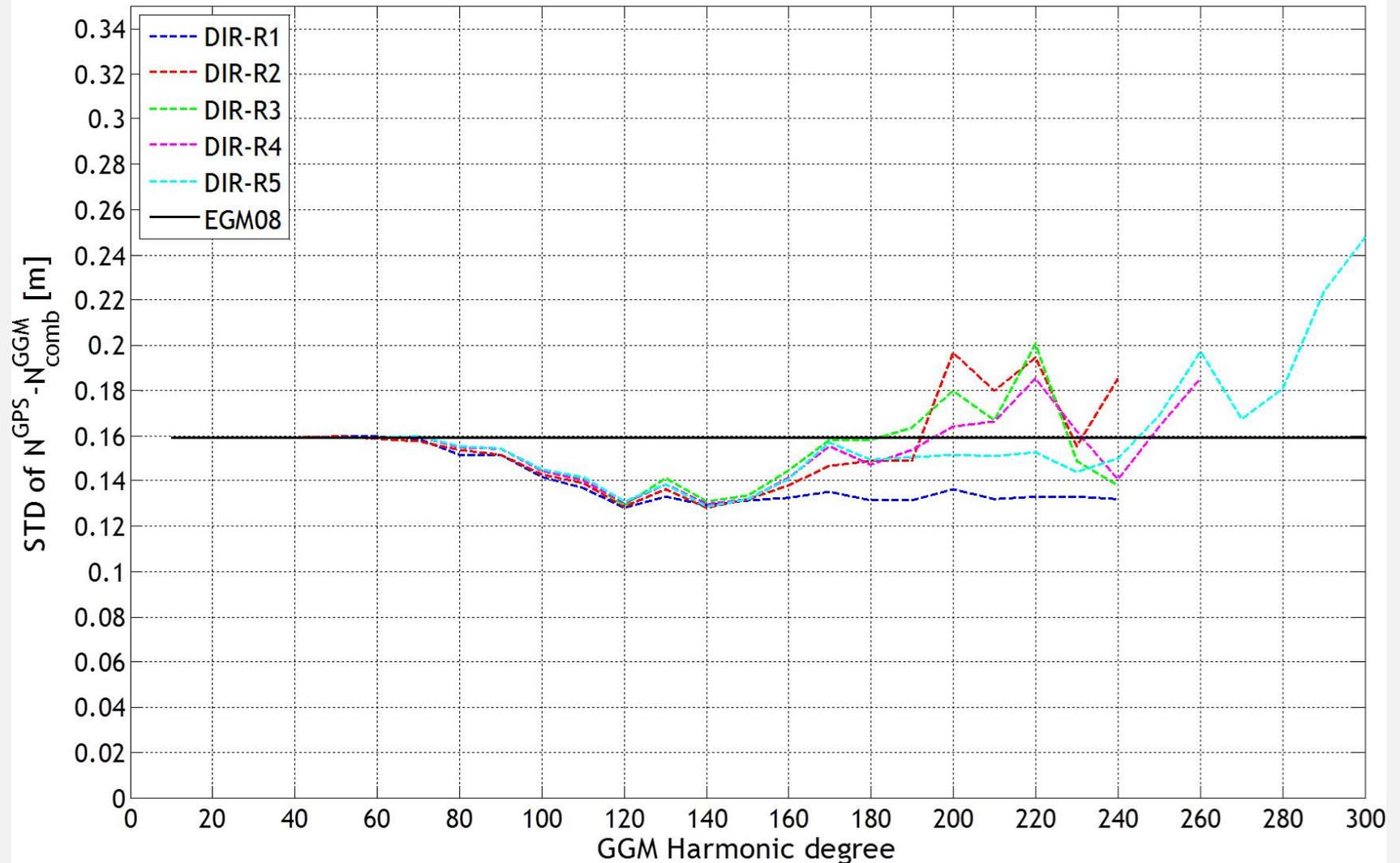
# GOCE/GRACE GGM VALIDATION

Standard Deviations of  $N^{GPS} - N^{GGM}_{comb}$

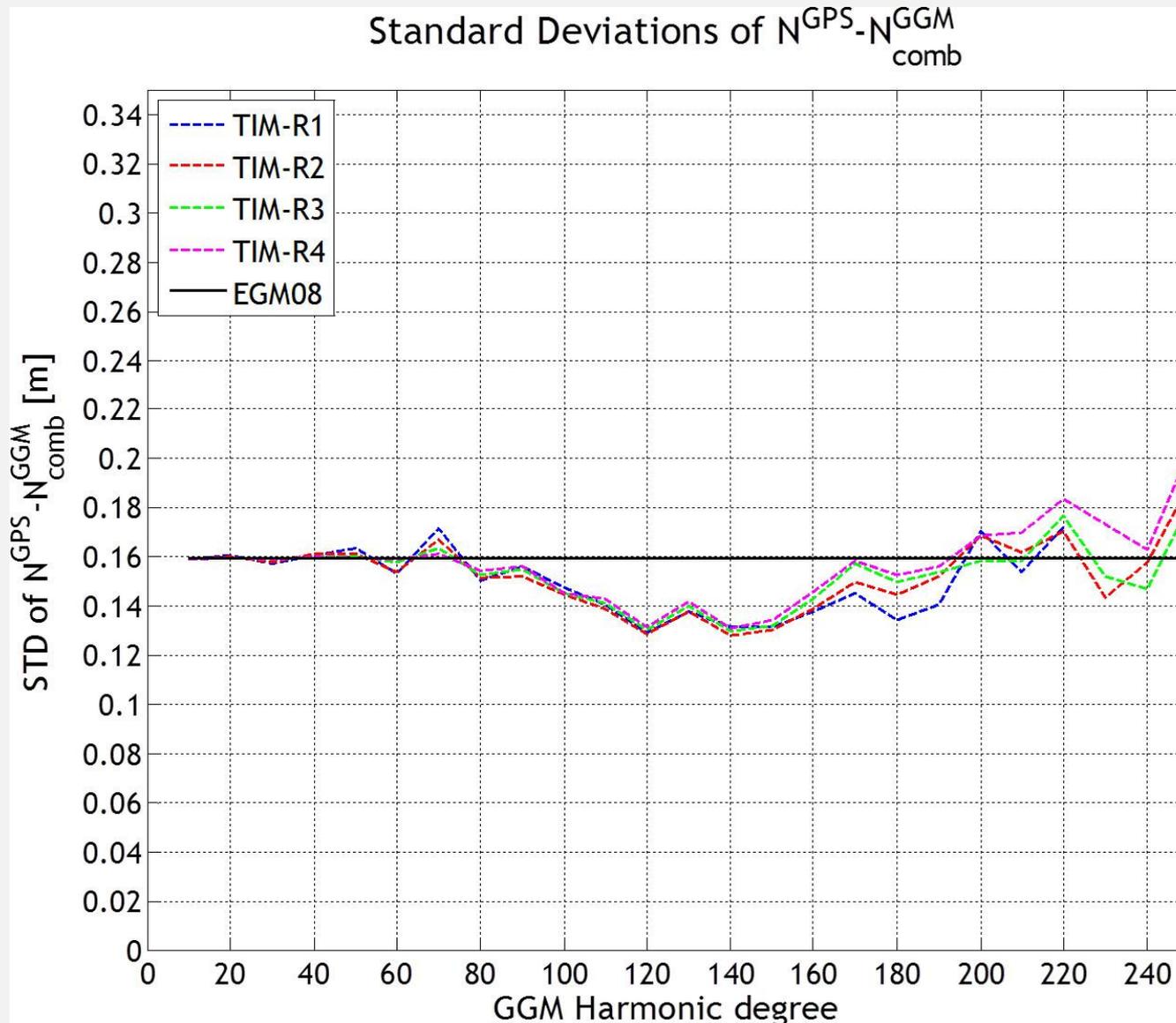


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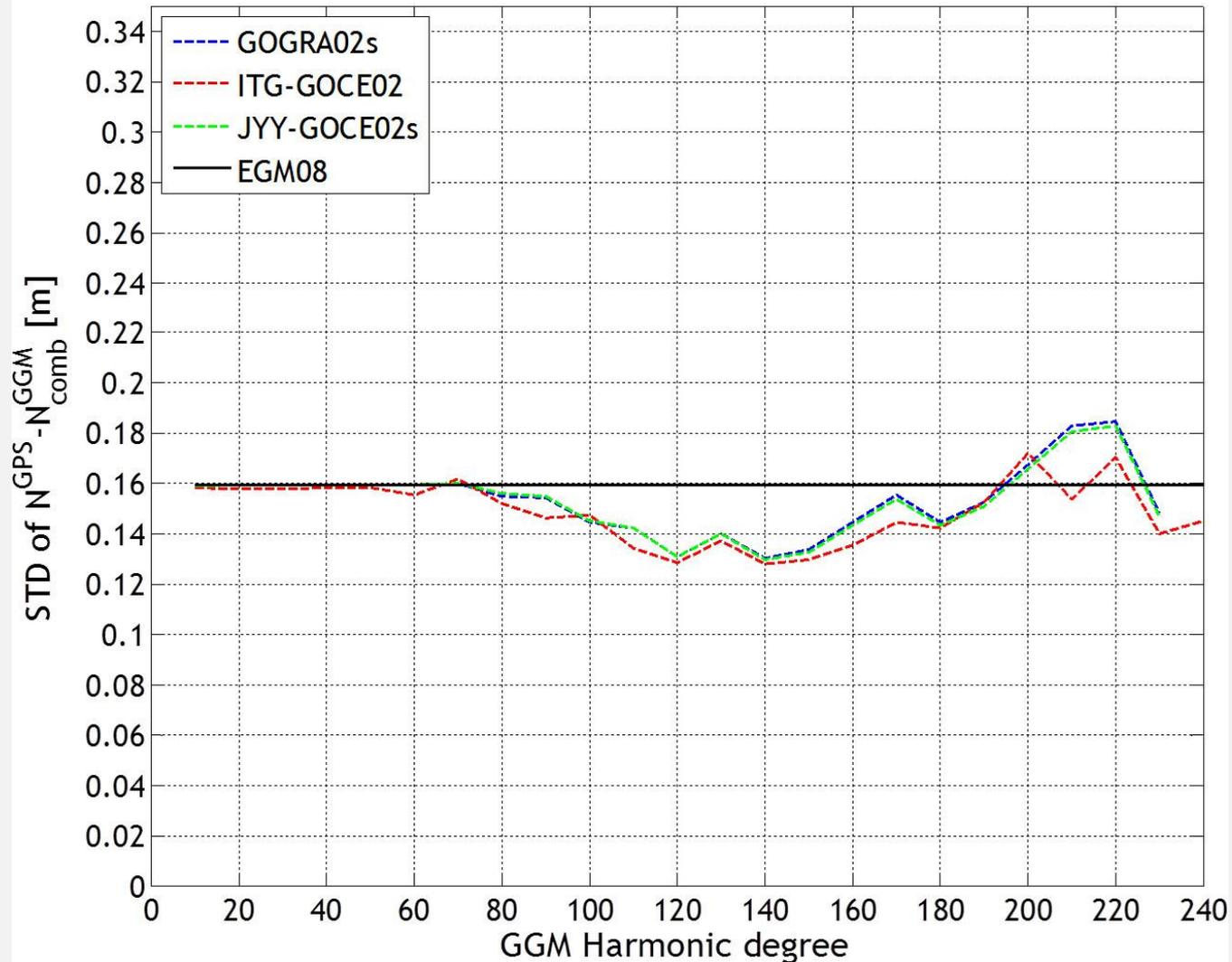


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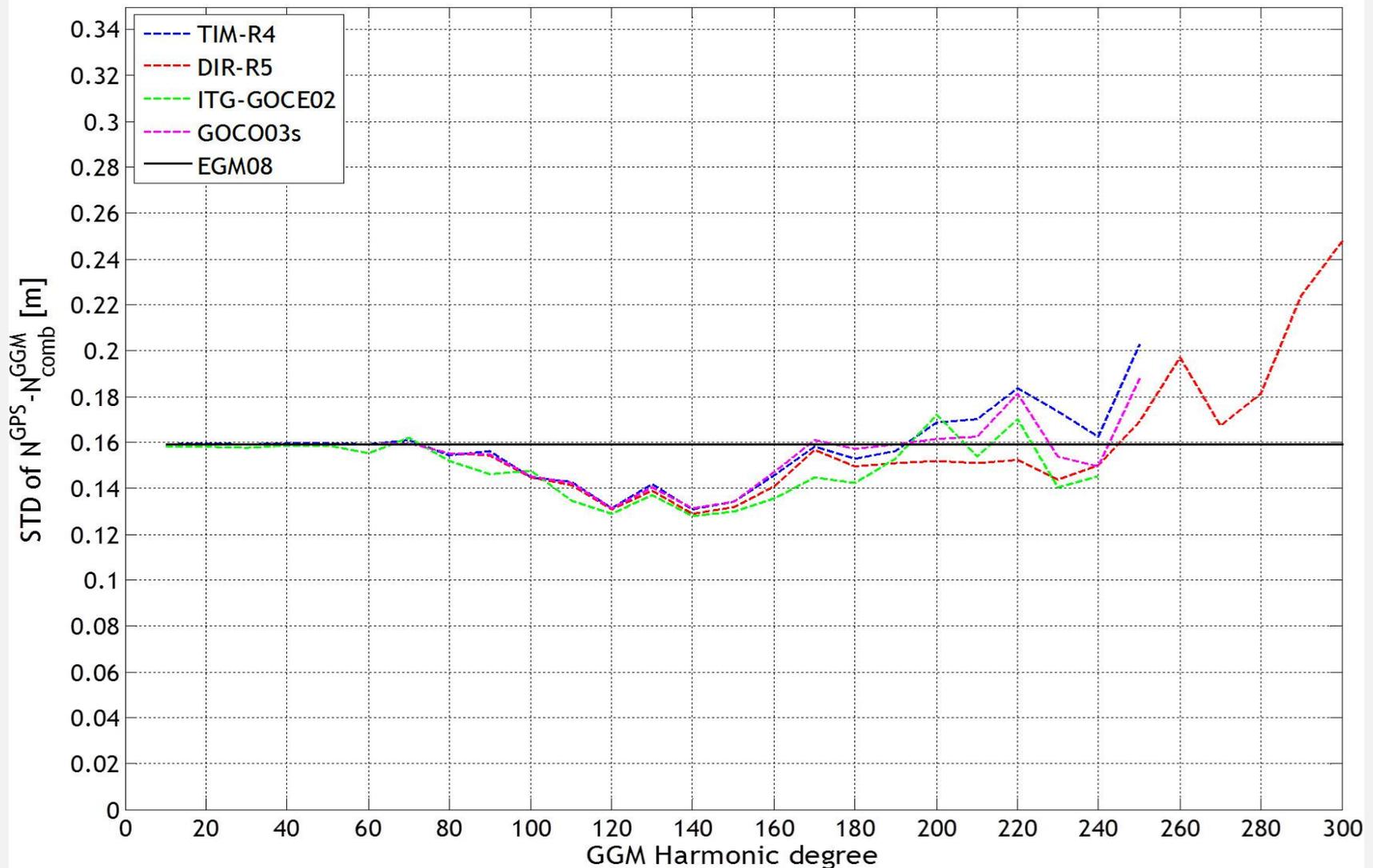
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# GOCE/GRACE GGM VALIDATION

Standard Deviations of  $N^{GPS} - N_{comb}^{GGM}$



# GOCE/GRACE GGM VALIDATION

Geoid height differences between GOCE/GRACE GGMs and GPS/Lev BMs. Unit [m]

GGM	d/o	range	mean	std
EGM2008	2190	1.107	-0.505	±0.159
DIR-R5	140+EGM08	0.736	-0.469	±0.128
TIM-R4	140+EGM08	0.729	-0.505	±0.131
GOCO03s	140+EGM08	0.734	-0.473	±0.131
ITG-GOCE02s	140+EGM08	0.728	-0.469	±0.127
EIGEN6c3	140+EGM08	0.731	-0.464	±0.129

Basic physical model (Helmert ortho heights):

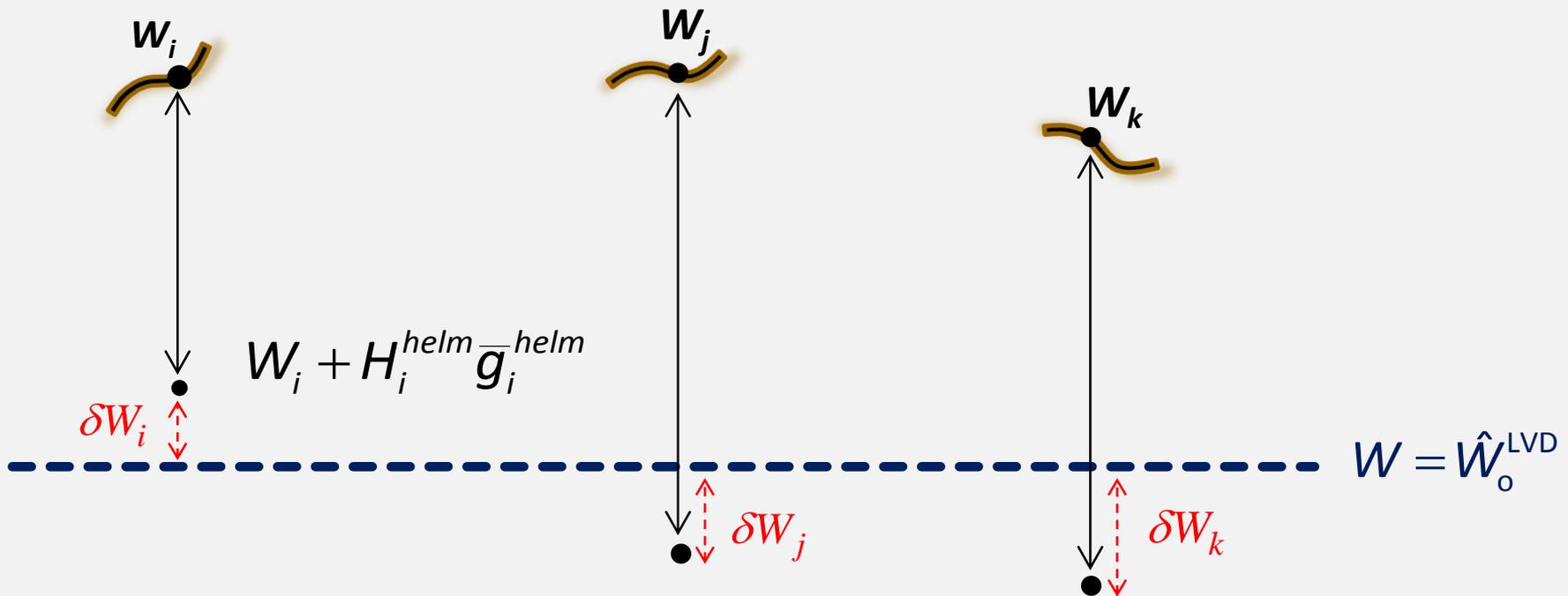
$$H_i^{helm} = \frac{W_o^{LVD} - W_i}{g_i + 0.0424 \cdot 10^{-5} H_i^{helm}} = \frac{W_o^{LVD} - W_i}{\bar{g}_i}$$

LS estimate of LVD's zero-height level

$$\hat{W}_o^{LVD} = \frac{\sum_i p_i y_i}{\sum_i p_i} \longrightarrow W_i + H_i^{helm} \bar{g}_i^{helm}$$

$\longrightarrow$  'weights'

## Method's rationale



Estimate the LVD zero-height level such that:  $\sum_i p_i \delta W_i^2 = \min$

The Hellenic (*local*) VD was established by HMGS within the period 1963-1986

In principle, physical heights in the HVD were modeled as Helmert orthometric heights

An unknown  $W_0$  value is associated with the HVD

The Helmert orthometric heights refer to the TG station at the Piraeus port (Athens) – MSL measurements were performed over the period 1933-1978

The true accuracy of the HVD's leveling network is largely unknown

## Un-weighted LS estimate

○ EGM2008 (2190)  $\hat{W}_0^{LVD} = 62636860.77 \pm 0.4 \text{m}^2 / \text{s}^2$

○ EGM2008 (260)  $\hat{W}_0^{LVD} = 62636859.43 \pm 0.4 \text{m}^2 / \text{s}^2$

○ DIR-R4 (260)  $\hat{W}_0^{LVD} = 62636859.90 \pm 0.4 \text{m}^2 / \text{s}^2$

○ TIM-R4 (250)  $\hat{W}_0^{LVD} = 62636860.50 \pm 0.4 \text{m}^2 / \text{s}^2$

○ DIR-R5 (300)  $\hat{W}_0^{LVD} = 62636860.01 \pm 0.4 \text{m}^2 / \text{s}^2$

# Numerical Results I

Height residuals relative to  $\hat{W}_o^{LVD}$  from EGM2008 (2190) [Units: m]

$$e_i = H_i^{helm} - \frac{\hat{W}_o^{LVD} - W_i}{\bar{g}_i^{helm}}$$

GGM	max	min	mean	std
EGM2008 (2190)	0.481	-0.386	0.000	±0.172
EGM2008 (260)	1.398	-1.041	-0.137	±0.411
TIM-R4	1.385	-0.833	-0.027	±0.414
DIR-R4	1.240	-1.019	-0.089	±0.363
DIR-R5	1.352	-1.044	-0.078	±0.395

## Un-weighted LS estimate & combined GOCE/GRACE GGMs

○ *EGM2008 (2190)*  $\hat{W}_o^{LVD} = 62636860.77 \pm 0.4 \text{m}^2 / \text{s}^2$

○ *EGM2008 (260)*  $\hat{W}_o^{LVD} = 62636859.43 \pm 0.4 \text{m}^2 / \text{s}^2$

○ *DIR-R4 (140)+EGM08*  $\hat{W}_o^{LVD} = 62636860.39 \pm 0.4 \text{m}^2 / \text{s}^2$

○ *TIM-R4 (140)+EGM08*  $\hat{W}_o^{LVD} = 62636860.43 \pm 0.4 \text{m}^2 / \text{s}^2$

○ *DIR-R5 (140)+EGM08*  $\hat{W}_o^{LVD} = 62636860.42 \pm 0.4 \text{m}^2 / \text{s}^2$

## Un-weighted LS estimate & combined GOCE/GRACE GGMs

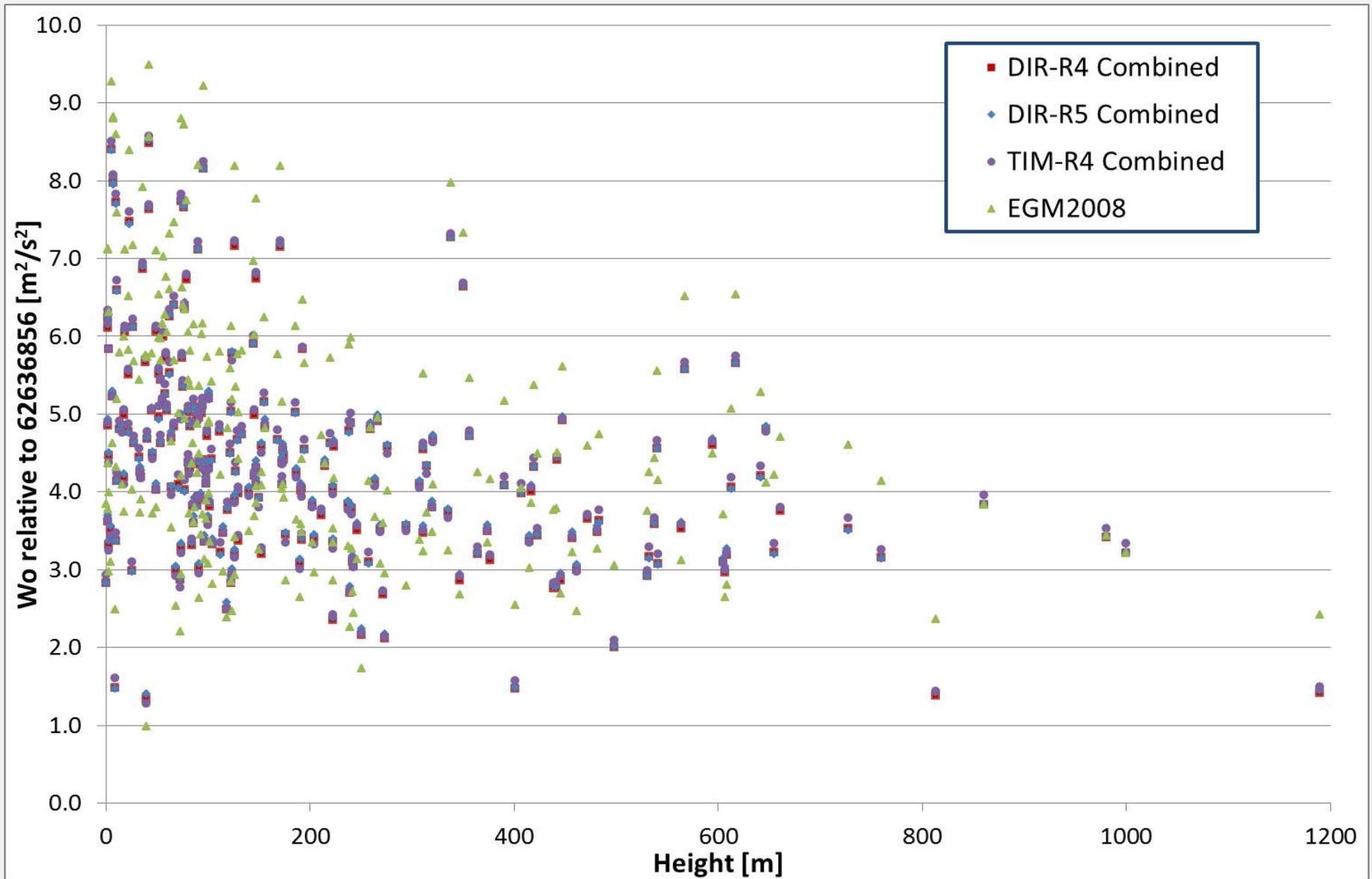
○ *EGM2008 (2190)*  $\hat{W}_0^{LVD} = 62636860.77 \pm 0.4 \text{m}^2 / \text{s}^2$

○ *DIR-R4 (140)+EGM08*  $\delta \hat{W}_0^{LVD} \approx 3.8 \text{cm}$

○ *TIM-R4 (140)+EGM08*  $\delta \hat{W}_0^{LVD} \approx 3.4 \text{cm}$

○ *DIR-R5 (140)+EGM08*  $\delta \hat{W}_0^{LVD} \approx 3.5 \text{cm}$

# Numerical Results II



## Un-weighted LS estimate with a height dependent parameter & combined GOCE/GRACE GGMs

$$H_i^{helm} = \frac{W_o^{LVD} - W_i}{g_i + 0.0424 \cdot 10^{-5} H_i^{helm}} = \frac{W_o^{LVD} - W_i}{\bar{g}_i^{helm}} + \lambda H_i^{helm}$$

## Un-weighted LS estimate with a height dependent parameter & combined GOCE/GRACE GGMs

- *EGM2008 (2190)*  $\hat{W}_o^{LVD} = 62636860.77 \pm 0.4 \text{ m}^2 / \text{s}^2$
- *DIR-R4 (140)+EGM08*  $\hat{W}_o^{LVD} = 62636860.92 \pm 0.008 \text{ m}^2 / \text{s}^2$   
 $\lambda = (2.559 \pm 0.009) \times 10^{-4} \text{ m}^2 / \text{s}^2$
- *TIM-R4 (140)+EGM08*  $\hat{W}_o^{LVD} = 62636860.96 \pm 0.008 \text{ m}^2 / \text{s}^2$   
 $\lambda = (2.558 \pm 0.009) \times 10^{-4} \text{ m}^2 / \text{s}^2$
- *DIR-R5 (140)+EGM08*  $\hat{W}_o^{LVD} = 62636860.95 \pm 0.008 \text{ m}^2 / \text{s}^2$   
 $\lambda = (2.552 \pm 0.009) \times 10^{-4} \text{ m}^2 / \text{s}^2$

## Un-weighted LS estimate with a height dependent parameter & combined GOCE/GRACE GGMs

○ DIR-R4 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.92 \pm 0.008 \text{m}^2 / \text{s}^2$

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○ DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.95 \pm 0.008 \text{m}^2 / \text{s}^2$

Great consistency for the GOCE/GRACE GGMs in  $\hat{W}_0^{LVD}$

$$\delta \hat{W}_0^{LVD} \approx 0.3 - 0.4 \text{ cm}$$

## Un-weighted LS estimate with a height dependent parameter & combined GOCE/GRACE GGMs

$$e_i = H_i^{helm} - \frac{\hat{W}_0^{LVD} - W_i}{\bar{g}_i^{helm}} - \hat{\lambda} H_i^{helm}$$

Height residuals relative to  $\hat{W}_0^{LVD}$  from EGM2008 (2190) [Units: m]

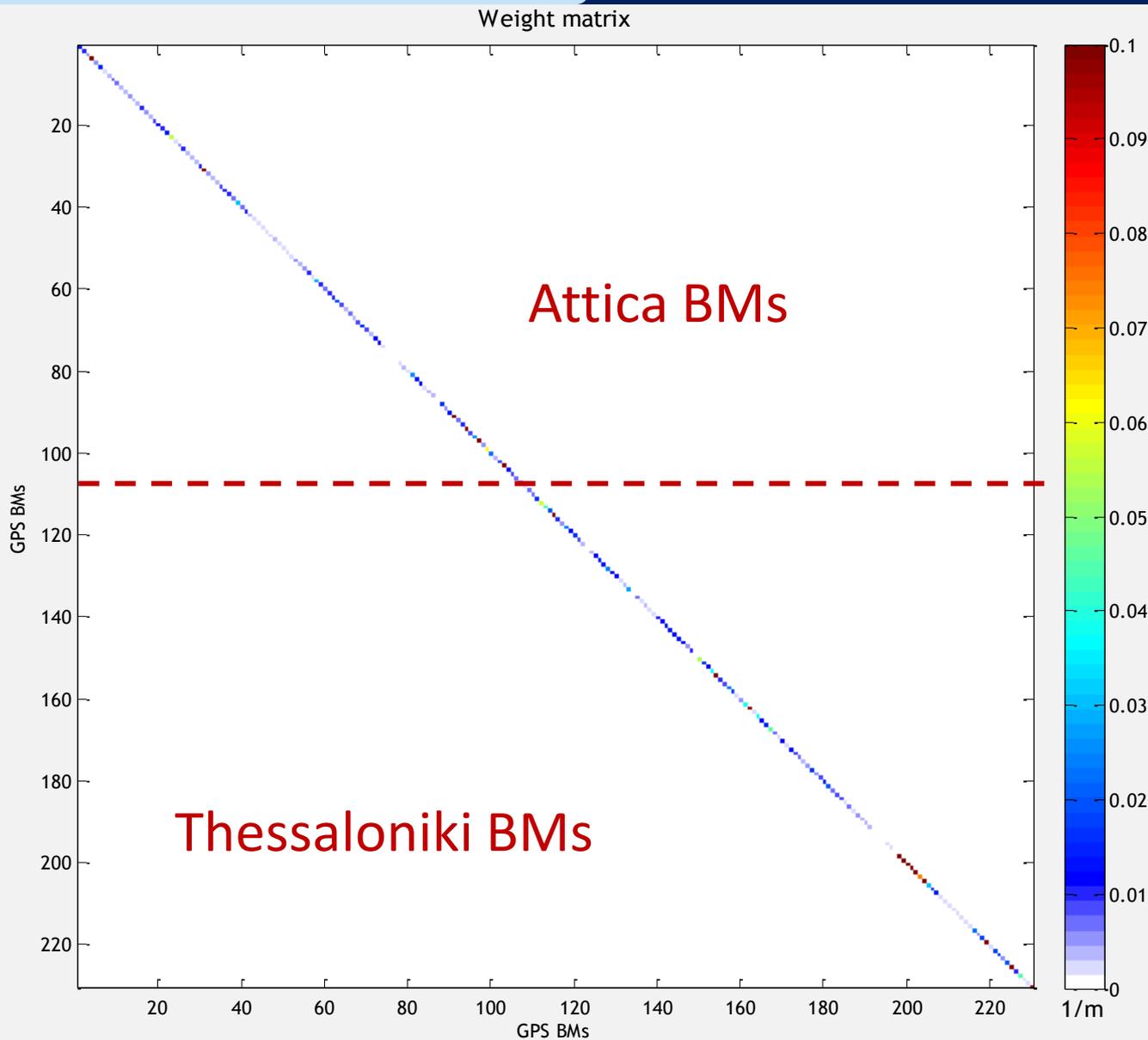
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EGM2008 (2190)	0.481	-0.386	0.000	±0.172
TIM-R4	0.398	-0.346	-0.019	±0.128
DIR-R4	0.390	-0.340	-0.016	±0.126
DIR-R5	0.393	-0.334	-0.019	±0.125

**Weighted LS estimate with  $p_i = 1/H_i^{helm}$ , a height dependent parameter & combined GOCE/GRACE GGMs**

### Weighted LS estimate with $p_i = 1/H_i^{helm}$ , a height dependent parameter & combined GOCE/GRACE GGMs

- DIR-R5 combined with EGM2008 provides the overall best results w.r.t. the differences with the GPS/Lev BMs.
- It will be the one used to carry-out the weighted LS adjustment

# Numerical Results IV



## Weighted LS estimate with $p_i = 1/H_i^{helm}$ , a height dependent parameter & combined GOCE/GRACE GGMs

○ DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.95 \pm 0.008 \text{m}^2 / \text{s}^2$

○ DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.94 \pm 0.002 \text{m}^2 / \text{s}^2$

$$p_i = 1/H_i^{helm}$$

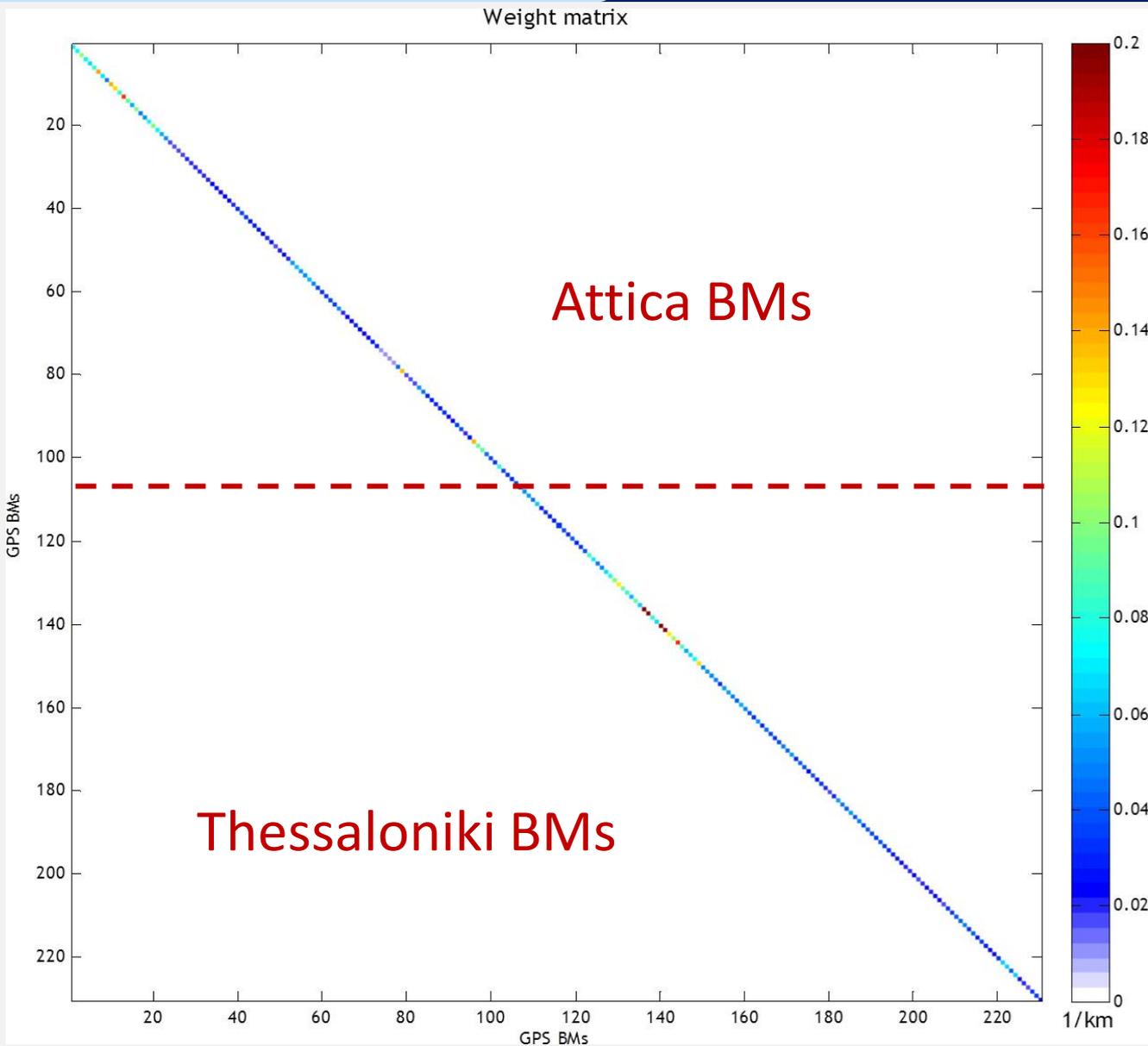
$$\delta \hat{W}_0^{LVD} \approx 0.1 \text{cm}$$

## Weighted LS estimate with $p_i = 1/L_i$ , a height dependent parameter & combined GOCE/GRACE GGMs

- $L_i$  [km] denotes the distances from the Piraeus and Thessaloniki TGs

GGM	max	min	mean	std
Attica	103.774	6.097	33.729	$\pm 20.416$
Thessaloniki	56.553	2.814	26.540	$\pm 12.428$

# Numerical Results IV



## Weighted LS estimate with $p_i = 1/H_i^{helm}$ , $p_i = 1/L_i$ a height dependent parameter & combined GOCE/GRACE GGMs

○ DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.95 \pm 0.008 \text{m}^2 / \text{s}^2$

○ DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.94 \pm 0.002 \text{m}^2 / \text{s}^2$   
 $p_i = 1/H_i^{helm}$

○ DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.92 \pm 0.002 \text{m}^2 / \text{s}^2$   
 $p_i = 1/L_i$

$$\delta \hat{W}_0^{LVD} \approx 0.1 - 0.3 \text{cm}$$

Our final  $\hat{W}_0^{LVD}$  estimate for Greece is

○ DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.95 \pm 0.008 \text{m}^2 / \text{s}^2$

Our final  $\hat{W}_0^{LVD}$  estimate for Greece is

- DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} = 62636860.95 \pm 0.008 \text{m}^2 / \text{s}^2$

Estimating  $\hat{W}_0^{LVD}$  separately for Attica and Thessaloniki

- DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} (\text{Attica}) = 62636860.31 \pm 0.021 \text{m}^2 / \text{s}^2$

- DIR-R5 (140)+EGM08  $\hat{W}_0^{LVD} (\text{Thess.}) = 62636860.79 \pm 0.017 \text{m}^2 / \text{s}^2$

$$\delta \hat{W}_0^{LVD} \approx 4.8 \text{cm} \quad \delta \text{MSL} \approx 2.5 \text{cm}$$

- Latest GOCE/GRACE GGM validation over Greece and Wo determination using various data/weight combinations

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- The latest releases of the GOCE/GRACE GGMs are better as much as 3 cm compared to EGM2008

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- Latest GOCE/GRACE GGM validation over Greece and  $W_0$  determination using various data/weight combinations
- The latest releases of the GOCE/GRACE GGMs are better as much as 3 cm compared to EGM2008
- DIR-R5 is better up to d/o 245 with significant improvement in the spectral range between d/o 185-230
- $\hat{W}_0^{LVD}$  from GOCE GGMs is more robust compared to EGM2008, with  $\delta\hat{W}_0^{LVD}$  between the various GGMs at 0.3-0.4 cm and for DIR-R5, between the various weight schemes, at 0.1 cm

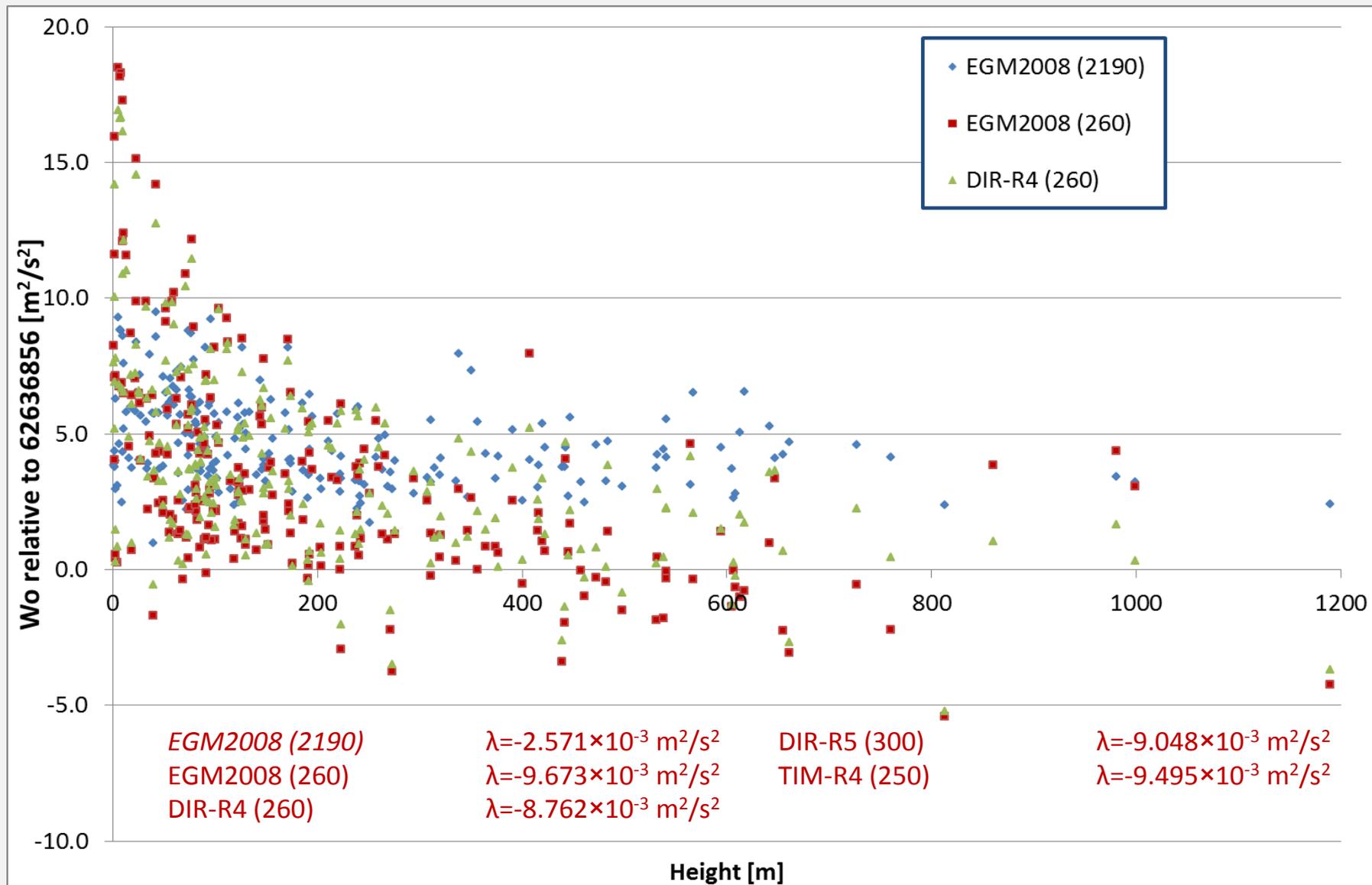
*Thank you for your  
attention*

## E.LE.V.A.T.I.ON. Project

co-financed by the E.U. (European Social Fund) and national funds under the Operational Program “Education and Lifelong Learning 2007 – 2013” in the frame of the action “Archimedes III – Funding of research groups in T.E.I.”

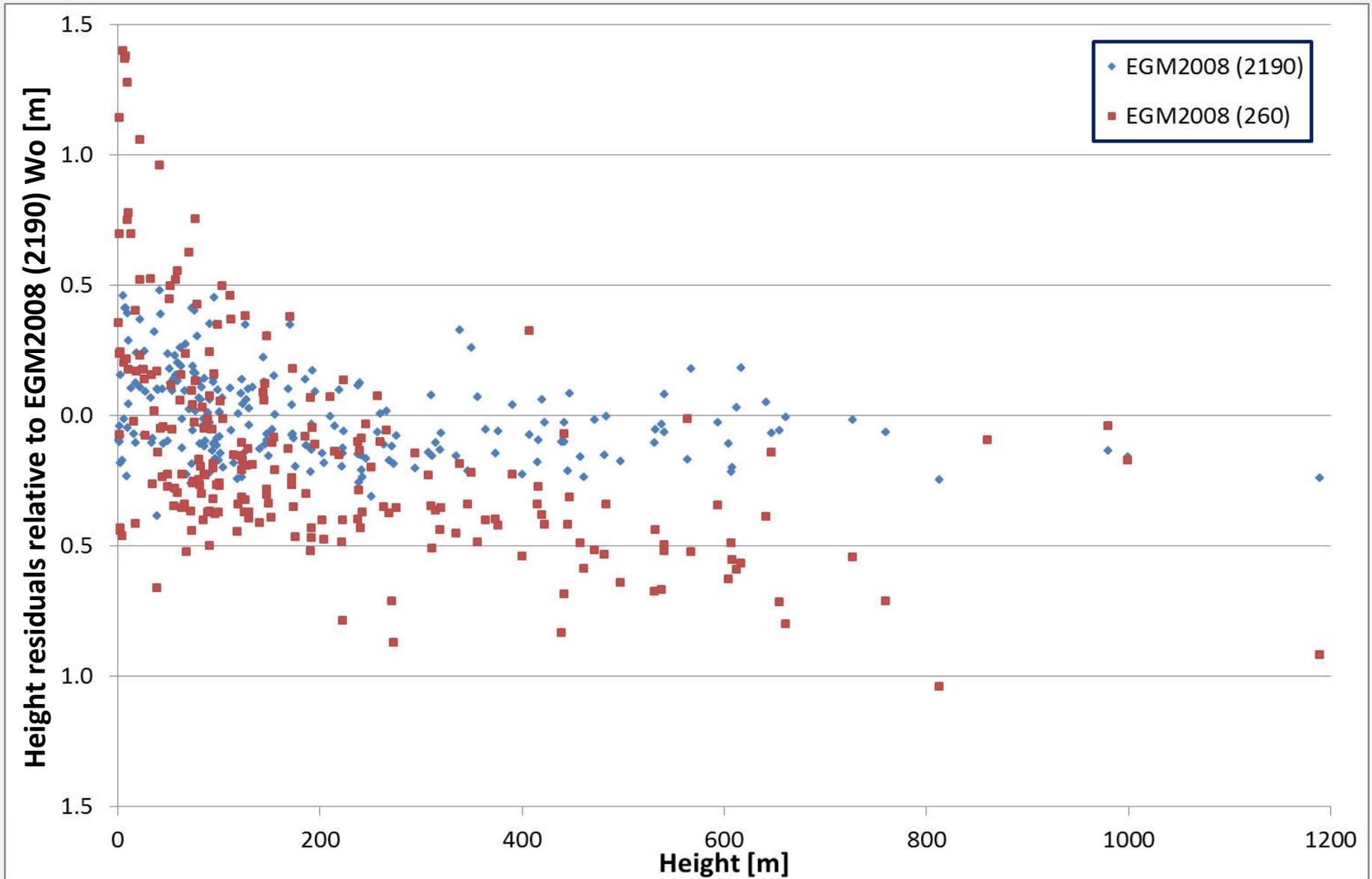
# BACK-UP SLIDES

# Numerical Results I

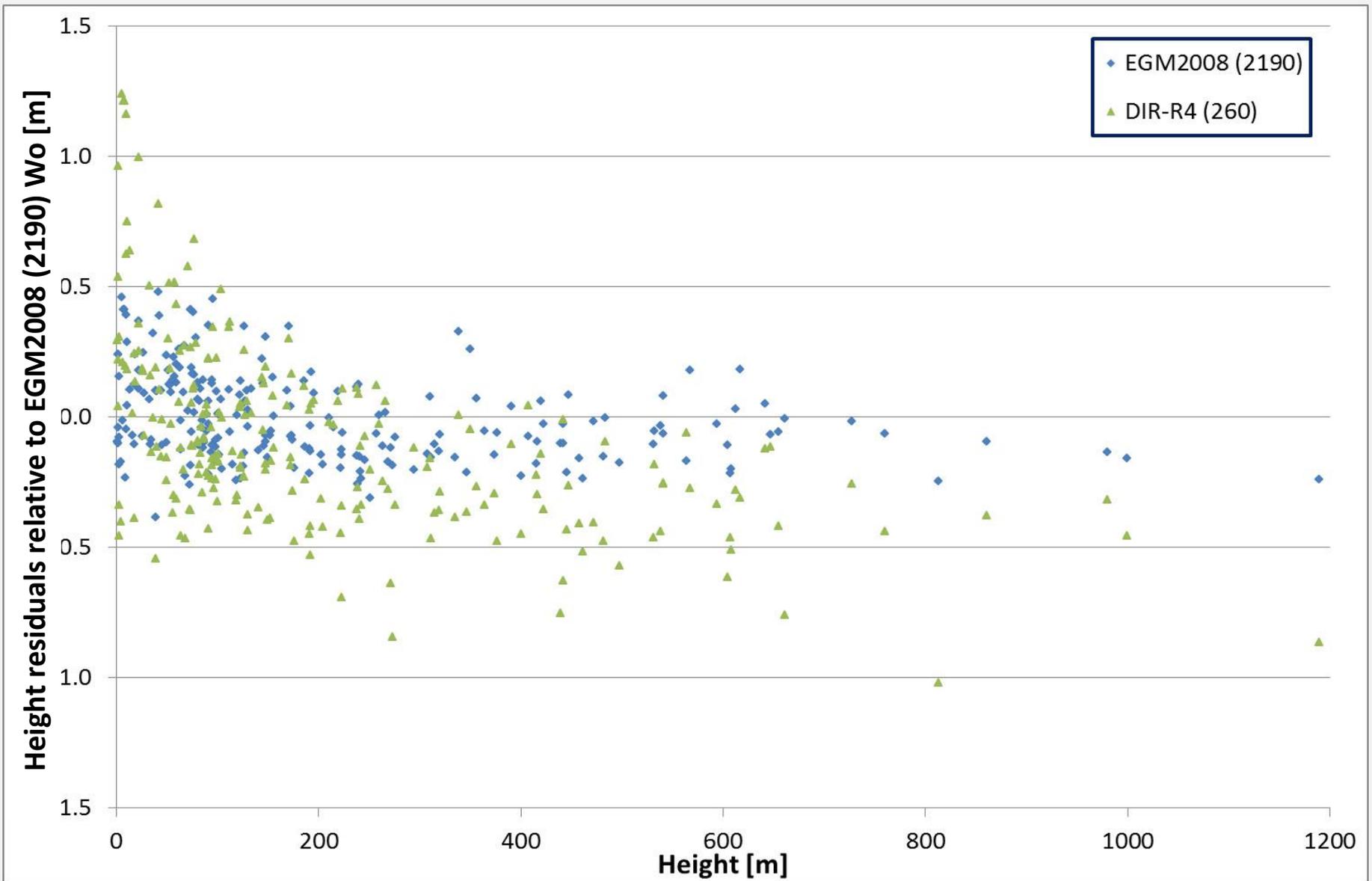


Wo residuals relative to  $\hat{W}_0^{LVD}$  from EGM2008 (2190)

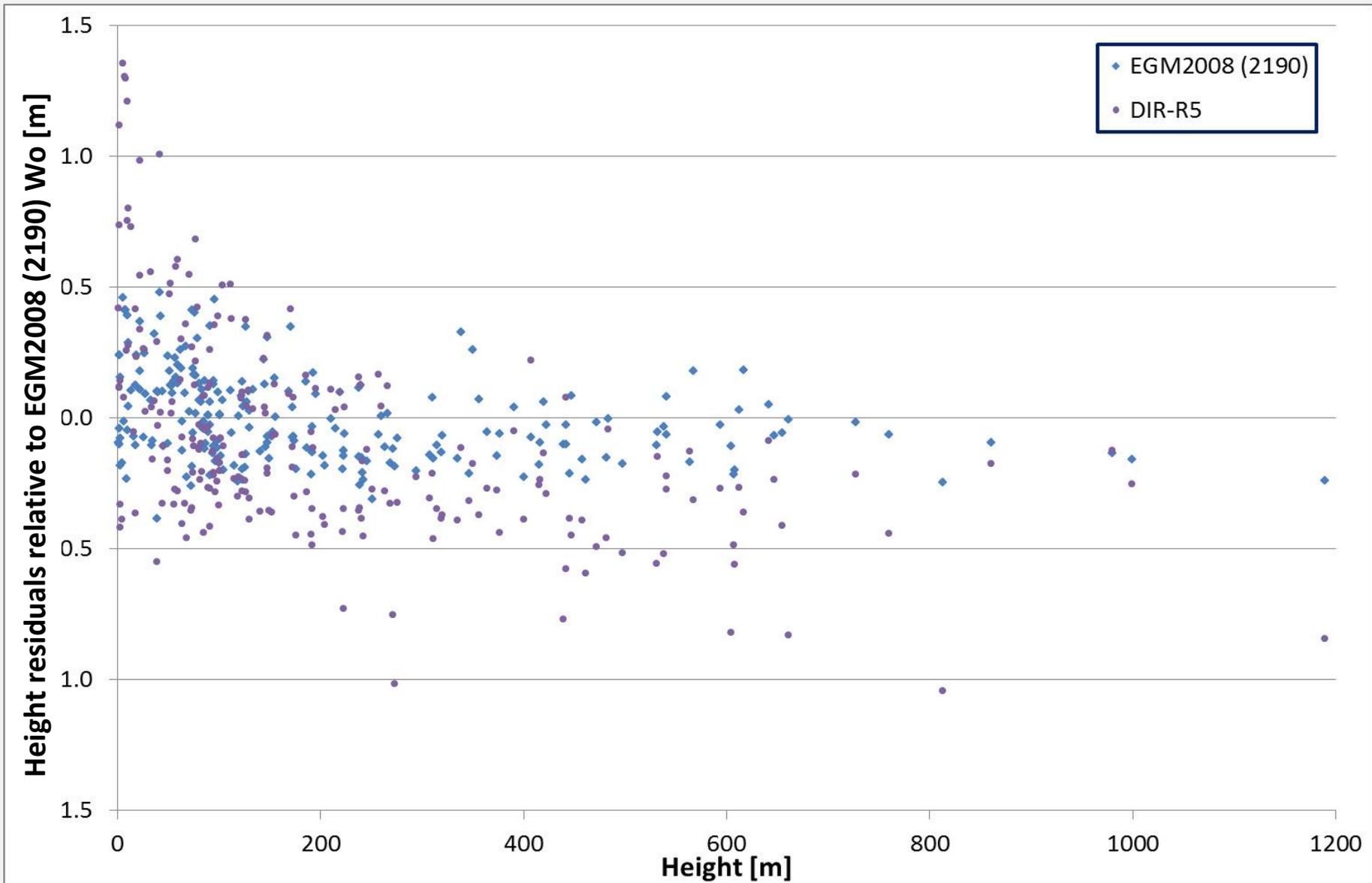
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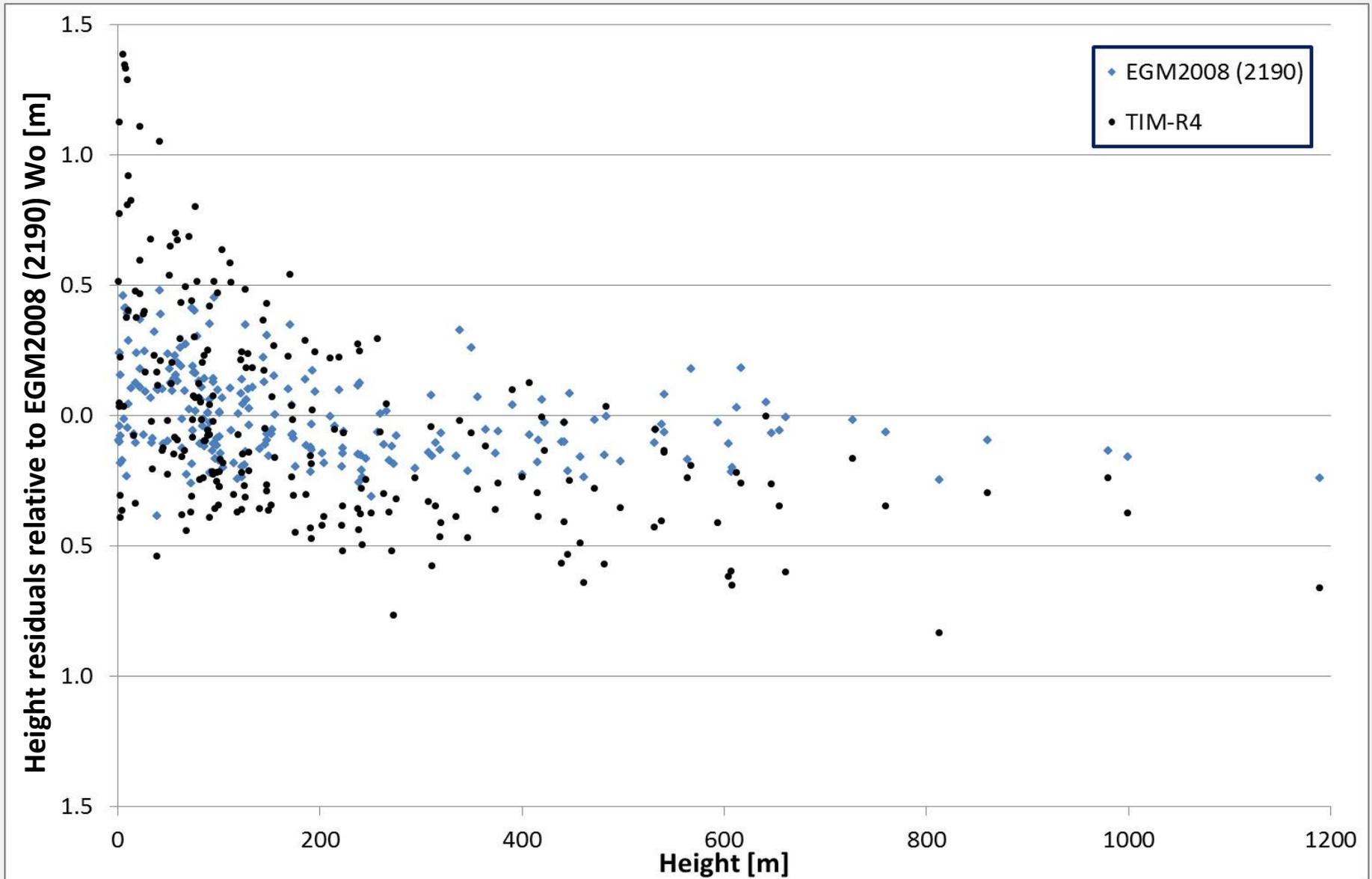
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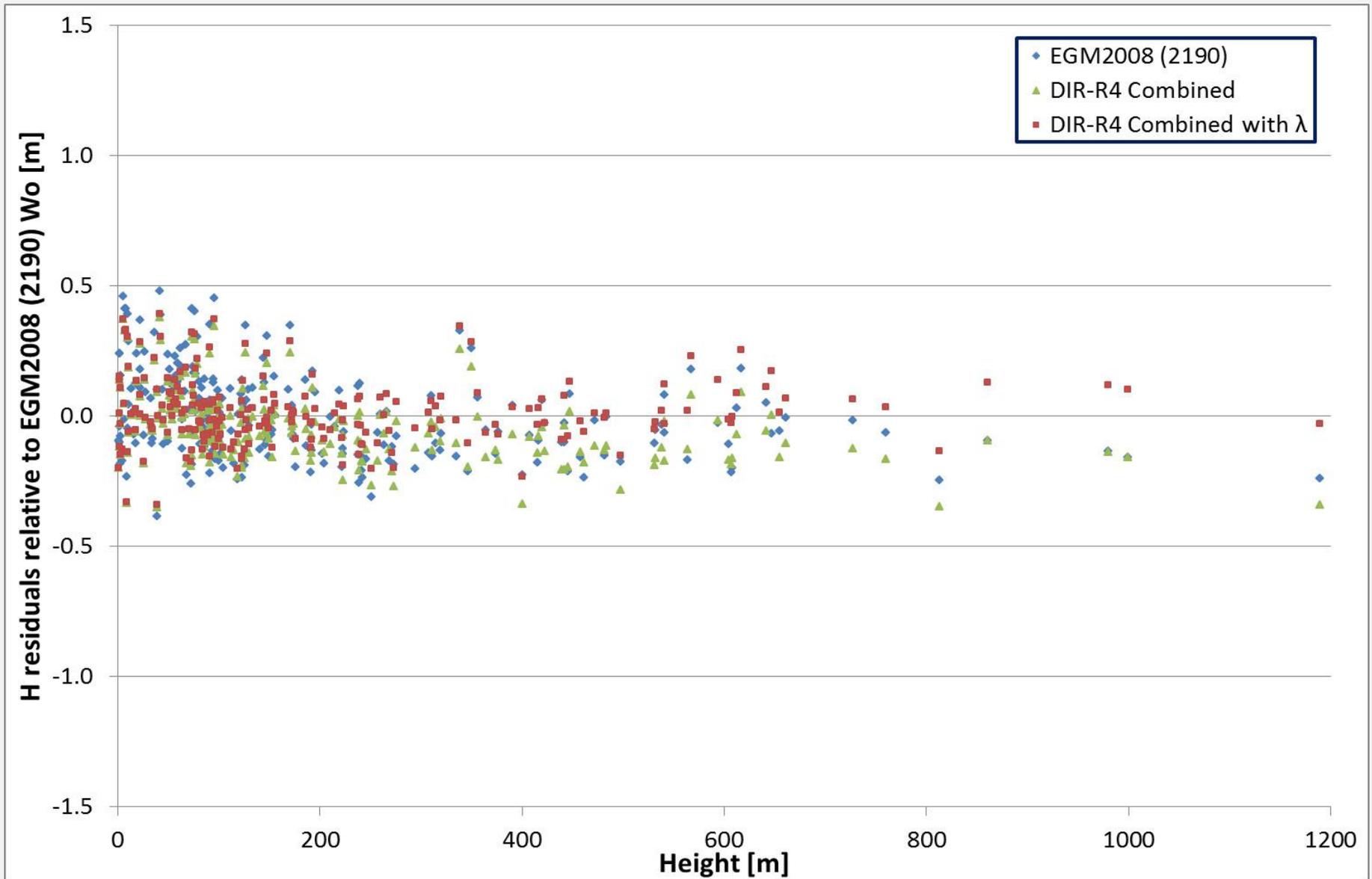
# Numerical Results I



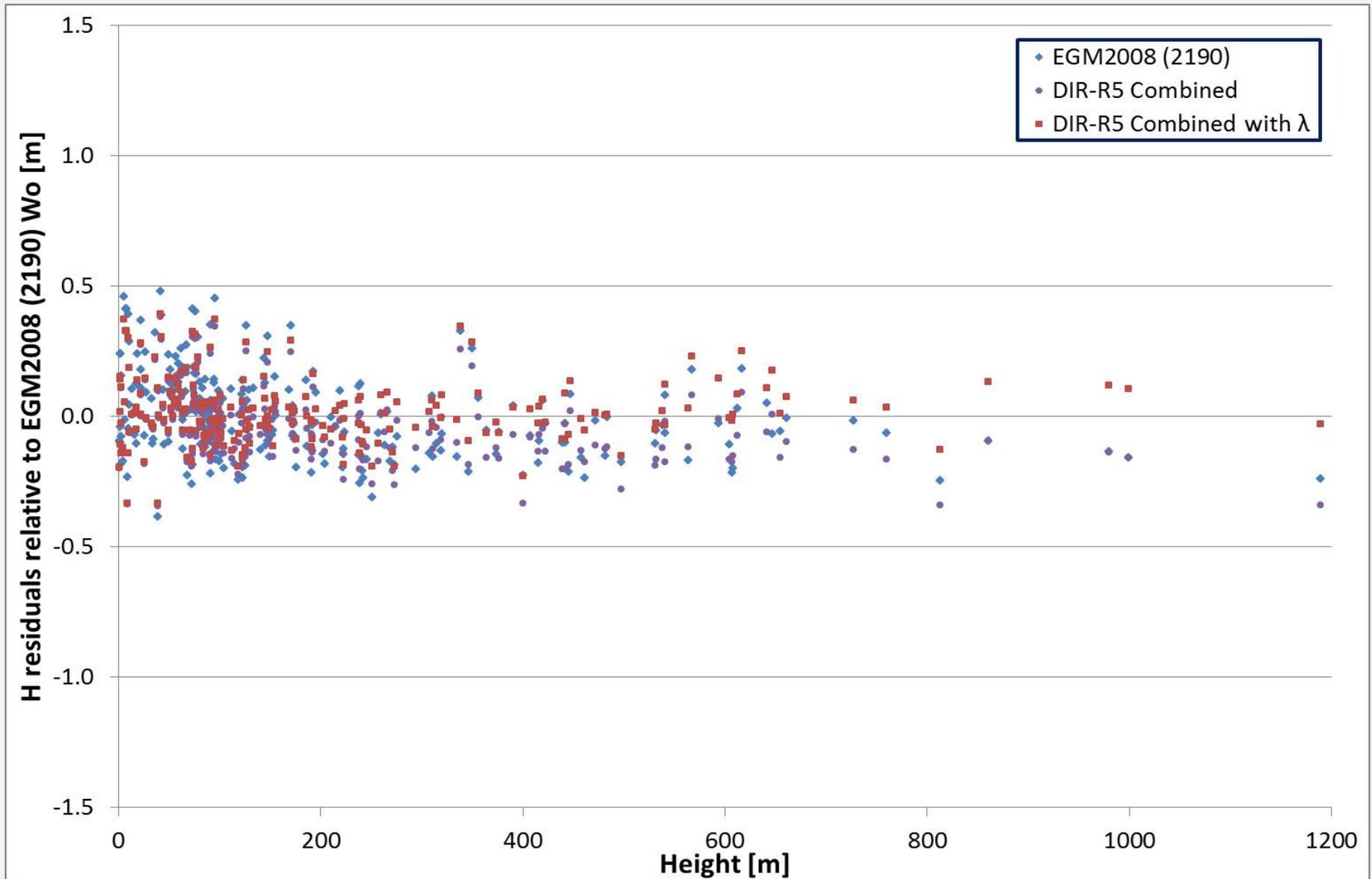
# Numerical Results I



# Numerical Results III



# Numerical Results II



# Numerical Results III

