Wavelet multi-resolution analysis of recent GOCE/GRACE GGMs

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Introduction and Problem

Monitoring and understanding of the Earth's gravity field parameters, at various spatial scales, has been the focus of many studies during the past decades. The missions of GRACE and GOCE offer new opportunities for gravity field approximation with higher accuracy at the medium wavebands, while wavelets (WL) provide powerful gravity field analysis tools in the frequency domain.

This work focuses on the spectral analysis of GOCE, GOCE/GRACE and combined Global Geopotential Models (GGMs) through wavelet decomposition, filtering and reconstruction in order to improve their performance as to their spectral content in the higher bands of the spectrum. Moreover an investigation of the coherence and the correlation between GGMs and topography is carried out.

Gravity anomaly evaluation is carried out through the WGM2012 gravity field covering the entire European area, while geoid heights are evaluated over an extensive network of collocated GPS/Leveling BMs which cover the Greek mainland. Coherence and correlation between Topography and gravity information are computed for the Amazon area.

Table 1: GGMs used

Models	n max	Data	Reference
EGM2008	2190	S(GRACE), G, A	Pavlis et al., 2008
GOCO03S	250	S(GOCE, GRACE, CHAMP, SLR)	Mayer-Gürr, et al. 2012
DIR_R4	260	S(GOCE, GRACE, LAGEOS)	Bruinsma et al, 2013
TIM_R4	250	S(GOCE)	Pail et al., 2011
EIGEN6C2	1949	S(Goce,Grace,Lageos),G,A	Förste et al, 2012

(Data: S = Satellite Tracking Data, G = Gravity Data, A = Altimetry Data
GRACE (Gravity Recovery And Climate Experiment)
CHAMP (CHAllenging Mini-satellite Payload)
GOCE (Gravity field and steady state Ocean Circulation Explorer)
LAGEOS (Laser GEOdynamics Satellite)
SLR (Satellite Laser Ranking)

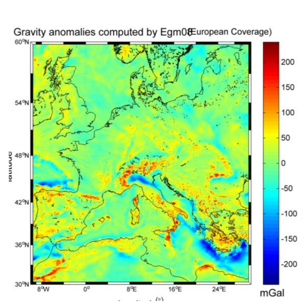

Figure 1: EGM08 Gravity Anomalies

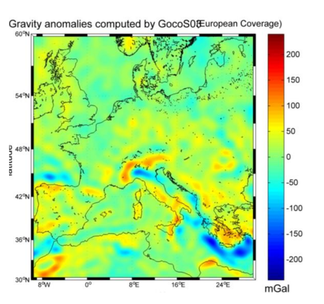

Figure 2: GOCO03s Gravity Anomalies

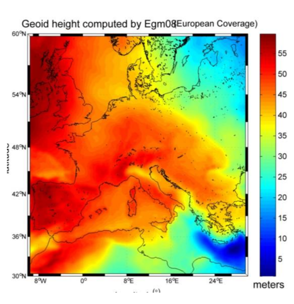

Figure 4: EGM08 Geoid Heights

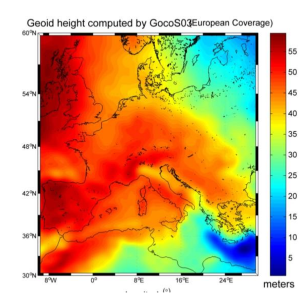

Figure 5: GOCO03s Geoid Heights

Initial Evaluation of GOCE/GRACE GGMs

With the inclusion of more GOCE data in the GOCE/GRACE GGMs (R1, R2, R3, R4 and the coming R5) their representation of the Earth's gravity field achieved higher accuracies to smaller wavelength of the spectrum.

To evaluate their performance, external data for both gravity anomalies and geoid heights are used referring to GPS/Levelling geoid heights over Greece and the WGM2012 model computed by BGI.

Table 2: Δg differences between local gravity anomalies and GGMs' [Units: mGal]

	min	max	mean	std
DG_BGI-EGM08	-49.66	128.50	0.31	±3.25
DG_BGI-GOCO03S	-204.97	272.23	0.11	±22.49
DG_BGI-TIM_R4	-206.98	269.35	0.11	±22.14
DG_BGI-DIR_R4	-201.93	271.43	0.11	±21.93
DG_BGI-EIGEN6C2	-68.89	140.35	0.30	±4.84

Table 3: Geoid height differences between GPS levelling and GGMs [Units: m]

	min	max	mean	std
NGPS-No-N_EGM08	-0.853	0.104	-0.372	±0.134
NGPS-No-N_GOCO03S	-1.735	1.110	-0.359	±0.464
NGPS-No-N_TIM_R4	-1.597	1.155	-0.358	±0.450
NGPS-No-N_DIR_R4	-1.540	1.105	-0.366	±0.442
NGPS-No-N_EIGEN6C2	-0.915	0.095	-0.386	±0.131

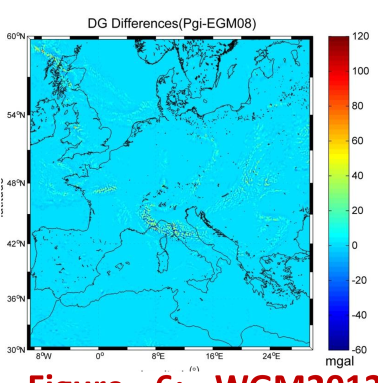

Figure 6: WGM2012 and EGM08 Gravity anomalies differences

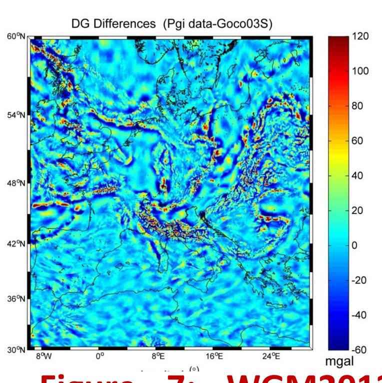

Figure 7: WGM2012 and GOCO03s Gravity anomalies differences

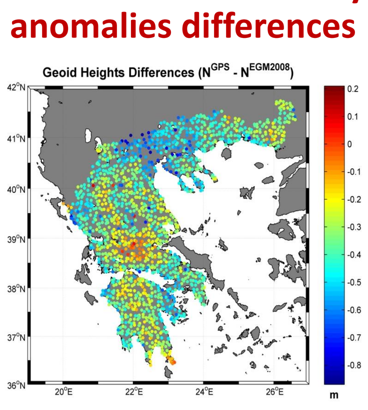

Figure 8: GPS and EGM2008 Geoid height Differences

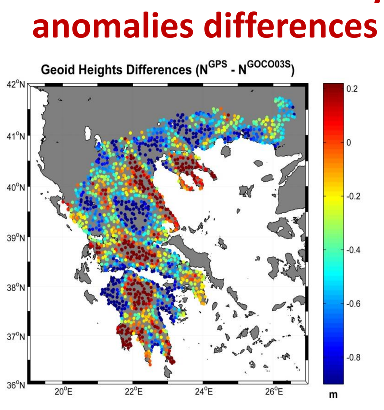

Figure 9: GPS and Goco03s Geoid height Differences

The maximum degree of each GGM affects the quality of both gravity anomalies and geoid heights. As a result EGM2008 followed by EIGEN-6c2, which have the maximum degree, present gravity field parameters with considerably higher accuracy than the other GGMs which maximum degree range between 250 and 260. The higher the maximum degree the better the GGM performance.

Wavelets and Multiresolution Analysis

WT is based on wavelets ψκ(x) as basis function in order to represent other functions. The wavelet function (ψ) carries valuable information about the signal, while the scaling function (φ), reveals the functional approximation. Since wavelets are base functions with localization properties in both space (time) and frequency (scale) domains, there can be a multiresolution analysis (MRA) at various levels of decomposition.

The 2D WT provides coefficients that correspond to different spatial resolutions, related to the signal frequencies. According to the wavelet decomposition algorithm, each scale analysis (level) of the signal, is analyzed in an approximation coefficient that carries the main information of the signal, and three detail coefficients (horizontal, vertical and diagonal).

Each Level of decomposition corresponds to a spatial resolution. To determine the number of levels the initial grid step of the data used (1o~110km). The first level extends from 5.5km~11km, the second from 11~22km etc., until the last levels' spatial analysis reaches the grid's perimeter. As a result when the grid step is 3 arcmin, there are 12 Levels of decomposition.

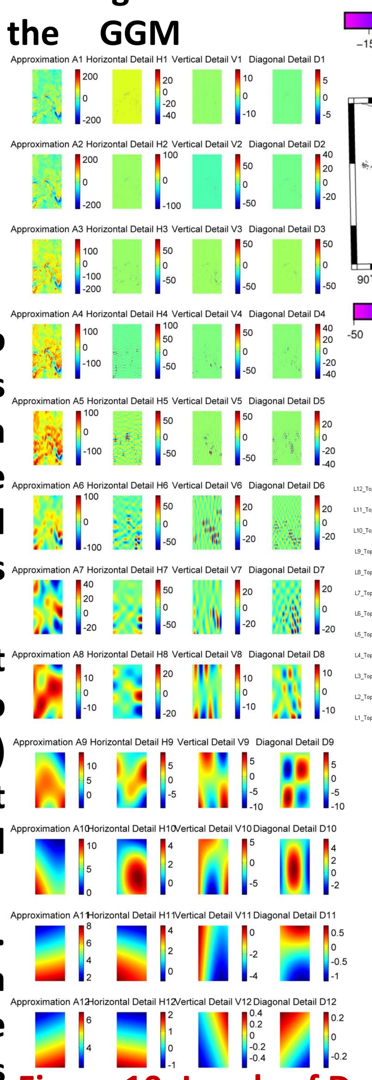

Figure 10: Levels of Decomposition and Detail Coeff. of Gravity anomalies of EGM2008 GGM

$$\varphi_{(j,m,n)}(x)=2^{(j/2)}\varphi\left(2^j x-m,2^j x-n\right)$$

$$\psi^i_{(j,m,n)}(x)=2^{(j/2)}\psi\left(2^j x-m,2^j x-n\right), i=H,V,D$$

GGMs Synthesis through WT MRA

Through the synthesis process various GGMs can be combined, since each Level can be composed by a different GGM given each spatial resolution and performance at each specific Level of analysis. Synthesis is defined as the algebraic sum of the detail coefficients of each Level used and the approximation coefficient of the last Level. The spectral content at each level is analyzed in order to conclude on the gravity field signal power that each GOCE/GRACE GGM represents compared to EGM2008 . The choice of the GGM that will be used at each level depends on its resolution and the gravity field content w.r.t. EGM2008.

$$Synthesis=A12+(H,V,D)_{12}+(H,V,D)_{11}+ +...(H,V,D)_2+(H,V,D)_1$$

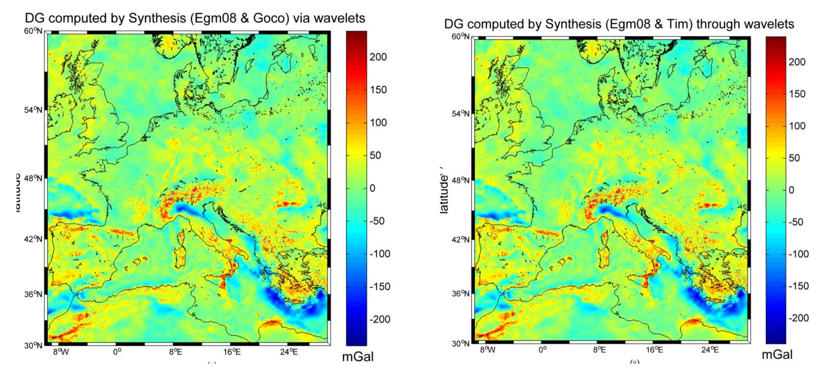

Figure 11: Synthesized GGMs' (Gravity Anomalies)

Table 4: Gravity anomaly differences between WGM2012 and the WL MRA synthesis (Lev. 5, 6, 7). [Units: mGal]

	min	max	mean	std
DG_BGI-DG_Egm08-GOCO03S	-89.09	129.87	0.32	±9.38
DG_BGI-DG_Egm08-TIM_R4	-90.52	134.80	0.31	±8.85
DG_BGI-DG_Egm08-DIR_R4	-87.10	129.69	0.29	±8.47
DG_BGI-DG_Egm08-Eigen6c2	-51.01	128.15	-0.33	±3.46

Table 6: Gravity anomaly differences between WGM2012 and the WL MRA synthesis (Lev. 6, 7). [Units: mGal]

	min	max	std
DG_BGI-DG_Egm08-GOCO03S	-89.09	129.87	±9.38
DG_BGI-DG_Egm08-TIM_R4	-50.17	123.54	±3.49
DG_BGI-DG_Egm08-DIR_R4	-51.12	123.89	±3.44

Table 5: Geoid height differences between GPS/Levelling and the WL MRA synthesis (Lev. 5, 6, 7). [Unit: m]

	min	max	mean	std
NGPS-No-N_Egm08-GOCO03S	-1.083	0.453	-0.387	±0.259
NGPS-No-N_Egm08-TIM_R4	-1.151	0.399	-0.381	±0.239
NGPS-No-N_Egm08-DIR_R4	-1.048	0.401	-0.392	±0.223
NGPS-No-N_Egm08-Eigen6c2	-0.409	0.638	0.129	±0.155

Table 7: Geoid height differences between GPS/Levelling and the WL MRA synthesis (Lev. 6, 7). [Unit: m]

	min	max	std
NGPS-No-N_Egm08-GOCO03S	-0.855	0.093	±0.124
NGPS-No-N_Egm08-TIM_R4	-0.838	0.053	±0.122
NGPS-No-N_Egm08-TIM_R4	-0.842	0.072	±0.123

Filtering

During the process that different GGM's levels are synthesized (Synthesis), there is a significant noise leakage and undesirable frequencies in specific levels, due to the resolution that each GGM has. Increasing the SNR (signal/noise) demand a digital or spatial filter implementation. A spatial filter is an image operation where each pixel value *I(u,v)* is changed by a function of the intensities of pixels in a neighborhood of *(u,v)*. Two types of isotropic filters, i.e. a boxcar and a Gaussian one have been tested in order to investigate whether they improve the results for the synthesized GGMs.

Boxcar filter

$$h(x,y)=2\lambda_c\, sinc\left(2\lambda_c\left(x^2+y^2\right)\right) \qquad H(u,v)=\prod\left(\frac{\omega}{2\omega_c}\right)$$

Gaussian filter

$$h(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad H(u,v)=2\pi\sigma^2 e^{-2\pi^2\sigma^2\left(u^2+v^2\right)}$$

Table 8: Differences between WGM2012 and filtered GGM Synthesis [Unit: mGal]

	min	max	mean	std
DG_BGI-Filtered_gauss_goco03s	-72.77	127.91	0.31	±6.48
DG_BGI-Fiiltered_gauss_timr4	-74.88	132.14	0.29	±6.36
DG_BGI-Fiiltered_boxcar_timr4	-75.29	135.47	0.29	±6.81
DG_BGI-Fiiltered_gauss_dir4	-73.01	130.56	0.29	±6.25
DG_BGI-Fiiltered_boxcar_dir4	-73.96	134.30	0.29	±6.72

Table 9: Differences between GPS/Levelling and filtered GGM Synthesis [Unit: m]

	min	max	mean	std
N_GPS-No-N_Goco03S_Gauss	-0.870	0.206	-0.377	±0.176
N_GPS-No-N_Goco03S_Boxcar	-0.898	0.244	-0.373	±0.192
N_GPS-Fiiltered_TimR4_Gauss	-1.011	0.314	-0.373	±0.213
N_GPS-Fiiltered_TimR4_Boxcar	-0.927	0.279	-0.377	±0.190
N_GPS-Fiiltered_DirR4_Gauss	-0.925	0.279	-0.378	±0.187
N_GPS-Fiiltered_DirR4_Boxcar	-1.027	0.316	-0.373	±0.212

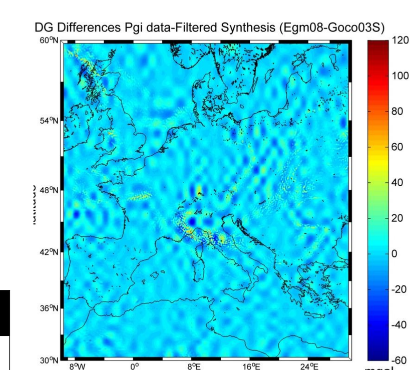

Figure 12: Gravity anomaly differences between WGM2012 and GOCO03S filtered WL Synthesis.

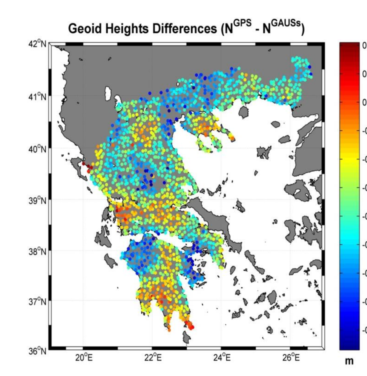

Figure 13: Geoid height differences between GPS/Leveling and GOCO03S filtered WL Synthesis.

GOCO03S L5 extends from 88-176 km, when the maximum resolution of GOCO03S is 80km and GOCE useful wavelength are >100 km. As a result the high frequencies of L5 (wavelengths shorter than 120 km) in GOCO03S carry significant noise, demanding filtering. L5 for the TIM-R4 and DIR-R4 GGMs exhibit the same behavior. The 120 km cut-off frequency was the one providing the most rigorous results.

GGMs and Topography Correlation and Coherence

The spectral coherence examines the relation between two signals. It is commonly used to estimate the power transfer between input and output of a linear system.

Spectral Correlation is a technique that can show whether and how strongly two signals are related.

Coherence and Correlation between Topography/Bathymetry and GGM-derived Gravity Anomalies & Trr for the Amazon area are estimated.

Coherence

$$C_{xy}=\frac{\left|G_{xy}\right|^2}{G_{xx}G_{yy}}$$

Correlation

$$R(x,y)=\frac{C(x,y)}{\sqrt{C(x,x)C(y,y)}}$$

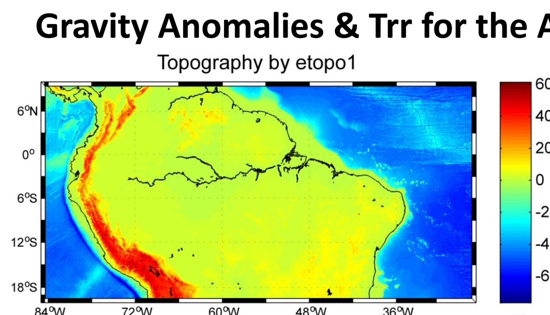

Figure 14:etopo1 Topography

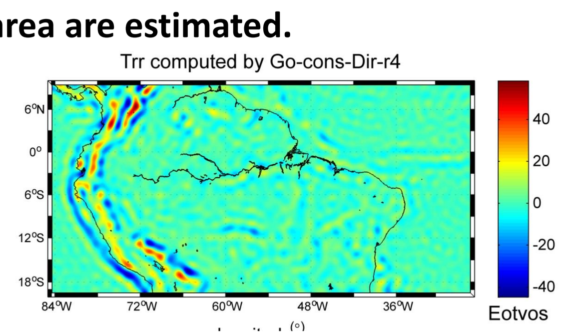

Figure 15: Dir_R4 Trr

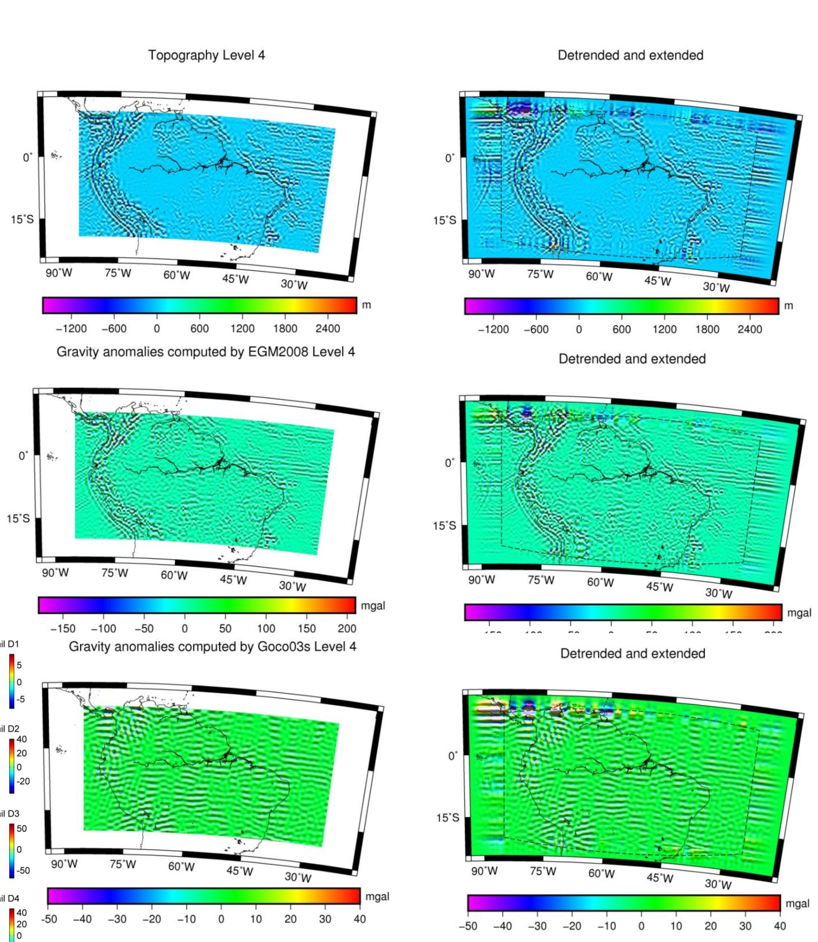

Figure 16: Topography, EGM08 and GOCO03S Level 4

Coherency between Topography and EGM08 Gravity Anomalies is~60% for Level 4 waveband (44~88km), while GOCE/GRACE GGMs' display a small coherence for wavelengths smaller than 50km.

Trr Coherency reveals a bigger connection between EGM08 and Topography, while there is a descending coherence all over Levels' 4 spectrum for GOCE/GRACE GGMs'.

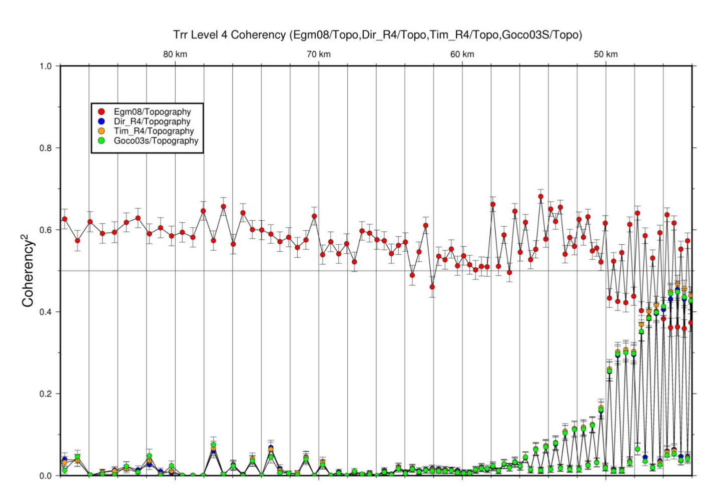

Figure 17: Topography and GGMs' Gravity anomalies Coherency Level 4

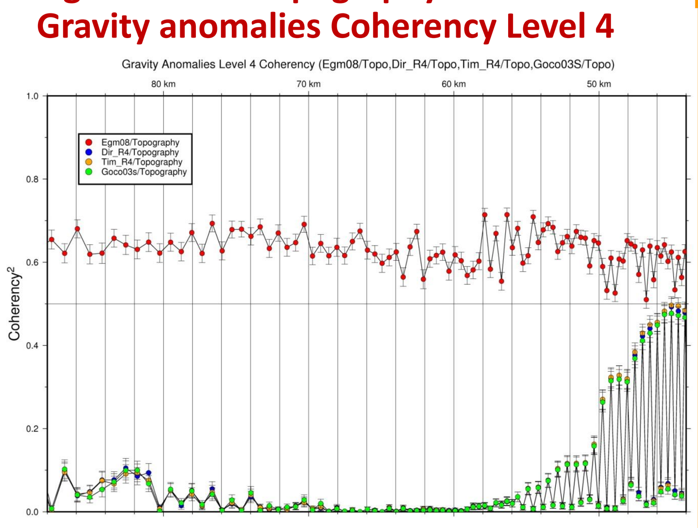

Figure 18: Topography and GGMs' Trr Coherency Level 4

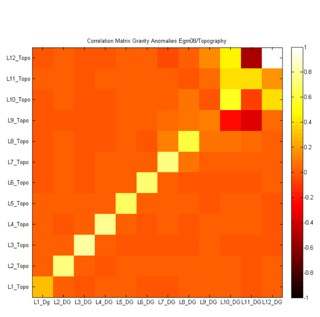

Figure 19: Topography and EGM08 Gravity Anomalies Correlation

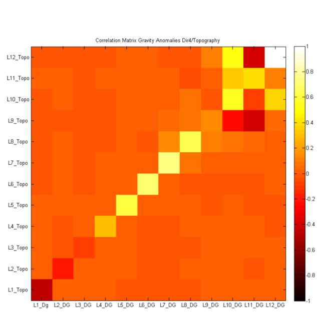

Figure 20: Topography and DIR_R4 Gravity Anomalies Correlation

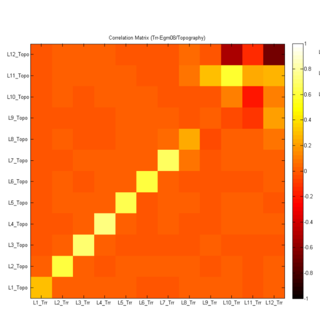

Figure 21: Topography and EGM08 Trr Correlation

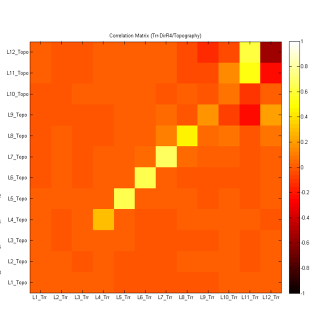

Figure 22: Topography and DIR_R4 Trr Correlation

Table 10: Correlation between Topography and Trr

	EGM08-Topo	DIR-Topo
Cor_Level1	28.80%	0.30%
Cor_Level2	59.80%	0.20%
Cor_Level3	71.90%	2.50%
Cor_Level4	74.20%	30.10%
Cor_Level5	65.50%	63.00%
Cor_Level6	62.40%	62.70%
Cor_Level7	67.70%	66.10%
Cor_Level8	22.10%	45.10%
Cor_Level9	3.00%	16.50%
Cor_Level10	10.10%	8.00%
Cor_Level11	24.80%	52.30%
Cor_Level12	-66.90%	-53.90%

Both EGM2008 and DIR-R4 gravity anomalies are completely correlated for Level 12 with topography. GOCE/GRACE GGMs' 5.5-44 km wavebands have a lower correlation than EGM2008, while they reach the same of higher correlation between 88-704 km. Wavebands 44~1408km are high correlated for all the GGMs.

Topography and EGM2008 Trr correlation, for the entire spectrum, is 8% while GOCE/GRACE GGMs' reach 18%. The wavebands 44~704km are high correlated for all the GGMs, while EGM08 displays a higher cross-correlation compared to the GPCE/GRACE GGMs.

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