GOCE GGM Analysis through wavelet decomposition and synthesis and validation with GPS/Leveling data

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Introduction

GRACE and GOCE have contributed to the representation of the Earth's static and time-variable gravity field with increasing accuracy to the medium band of the spectrum.

This work focuses mainly on the evaluation of GGMs that come from GOCE and GRACE data, both for gravity anomalies and geoid heights.

Gravity anomaly evaluation is carried out through the WGM2012 gravity field covering the entire European area, while geoid heights are evaluated over an extensive network of collocated GPS/Level BMs which covers the Greek mainland.

GOCE/GRACE GGM processing is carried out with a wavelet multi-resolution analysis. Although wavelet multiresolution analysis is too young compared to FFT, it has been developed in order to overcome FFT deficiencies.

The advent of wavelets transformations in geosciences brought a flexibility in the analysis process for over a decade.

Open Problems and Objectives

In this work wavelet transform (WT) is used to analyze both gravity anomalies and geoid heights in approximation and detail coefficients for various levels of decomposition, which correspond to different spatial scales.

To improve the GGM performance, as to their spectral content in the higher bands of the spectrum, a combination scheme has been followed through wavelet decomposition, filtering and reconstruction.

The aim of this work is to generate new GGMs, where both GOCE, GRACE and EGM2008 are used, and to evaluate these models in order to conclude on the improvements they bring to gravity field and geoid modeling.

Gravity anomalies and Geoid heights data used

The data used are gravity anomalies (Δg) and geoid heights (N) coming from five GGM's. **EGM2008** (Earth Gravitational Model 2008) presents a spherical harmonics expansion of the Earth's potential to a maximum degree n_{max}=2159, consisting of both satellite (Grace, Champ, SLR) and local data.

GOCO03S presents a spherical harmonics expansion of the Earth's potential to a maximum degree n_{max}=250 employing (a) 7.5-years ITG-Grace2010s data (d/o 180), (b) 18-months of GOCE Satellite Gravity Gradiometry (SGG) observations, (d/o 250), (c) 12-months of GOCE satellite-to-satellite tracking in high-low mode (SST-hl), (d/o 110), (d) 8-years of CHAMP data, and (d/o 120) and (f) 5-years of SLR data from 5 satellites (d/o 5).

GO_CONS_GCF_2_TIM_R4 presents a spherical harmonics expansion of the Earth's potential to a maximum degree n_{max}=250, employing 26.5 months of GOCE data.

GO_CONS_GCF_2_DIR_R4 presents a spherical harmonics expansion of the Earth's potential to a maximum degree n_{max}=260. It is based on data from GOCE (27.5 months), GRACE (9 years) and LAGEOS .

EIGEN-6C2 is a combined GGM to a maximum degree n_{max}=1949 employing, 7.5-years GRACE data (GPS-SST), 1 year GOCE SGG data, 25-years LAGEOS data, local gravity and altimetry data (from EGM2008).

Data cover the entire European continental (30° ≤ ϕ ≤ 60° and -10° ≤ λ ≤ 30°) .

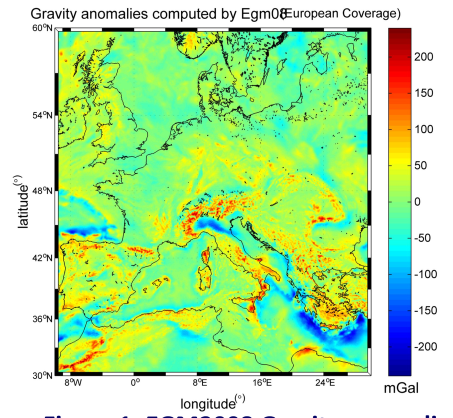


Figure 1: EGM2008 Gravity anomalies

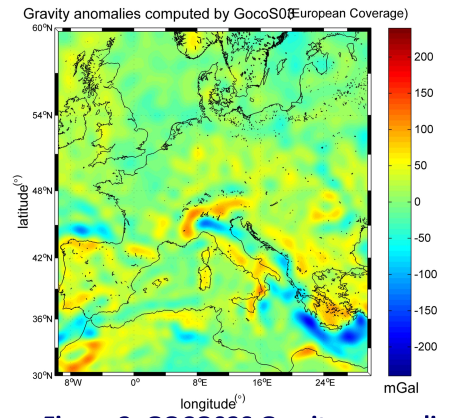


Figure 2: GOCO03S Gravity anomalies

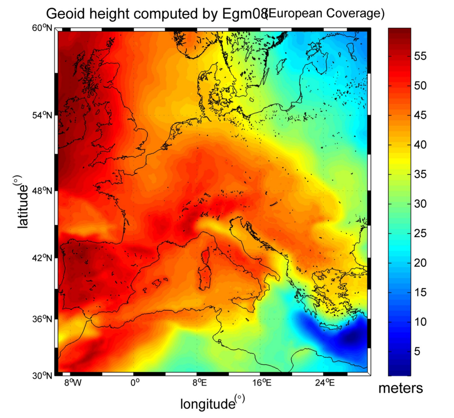


Figure 3: EGM2008 Geoid heights

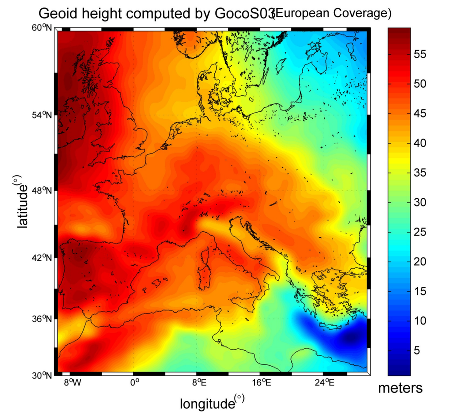


Figure 4: GOCO03S Geoid heights

For the evaluation of GGMs gravity anomalies from the BGI global model were used, while GPS/Leveling geoid heights over BMs were used for the validation of geoid heights. The BGI database was used over the whole European continent (30° ≤ ϕ ≤ 60° and -10° ≤ λ ≤ 30°) and Geoid heights from the GPS/Leveling network of Greece was employed (34° ≤ ϕ ≤ 42° and 19° ≤ λ ≤ 30°).

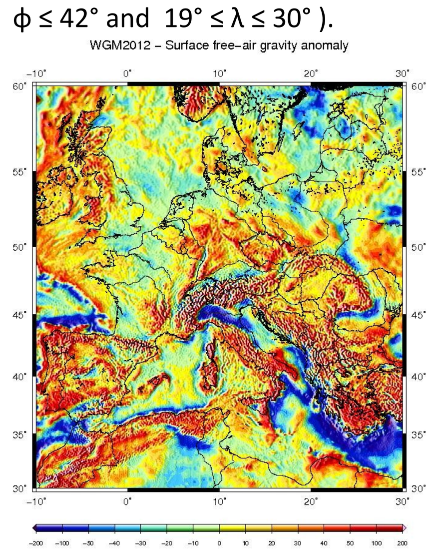


Figure 5: WGM2012 Gravity anomalies

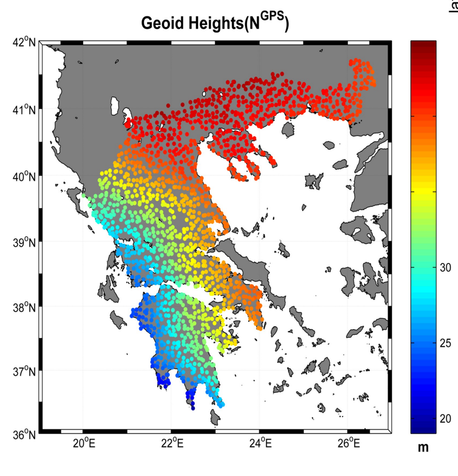


Figure 6: Geoid heights over the network of collocated GPS/leveling BMs.

Initial Evaluation of GOCE/GRACE GGMs

With the inclusion of more GOCE data in the GOCE/GRACE GGMs (R1, R2, R3, R4 and the coming R5) their representation of the Earth's gravity field achieved higher accuracies to smaller wavelength of the spectrum.

To evaluate their performance ,external data for both gravity anomalies and geoid heights are used referring to GPS/Leveling geoid heights over Greece and the WGM2012 model computed by BGI.

Table 1 presents the statistics of gravity anomalies differences between the available GGM and those from WGM2012, while Table 2 presents the corresponding statistics with the GPS/Leveling geoid heights.

Table 1: Δg differences between local gravity anomalies and GGMs' [Units: mGal]

	min	max	mean	stdev
DG_PGI-EGM08	-49.66	128.50	0.31	3.25
DG_PGI-GOCO03S	-204.97	272.23	0.11	22.49
DG_PGI-TIM_R4	-206.98	269.35	0.11	22.14
DG_PGI-DIR_R4	-201.93	271.43	0.11	21.93
DG_PGI-EIGEN6C2	-68.89	140.35	0.30	4.84

Table 2: Geoid height differences between GPS leveling and GGMs [Units: m]

	min	max	mean	stdev
NGPS-No-N_EGM08	-0.853	0.104	-0.372	0.134
NGPS-No-N_GOCO03S	-1.735	1.110	-0.359	0.464
NGPS-No-N_TIM_R4	-1.597	1.155	-0.358	0.450
NGPS-No-N_DIR_R4	-1.540	1.105	-0.366	0.442
NGPS-No-N_EIGEN6C2	-0.915	0.095	-0.386	0.131

It can be seen that the maximum degree of each GGM affects the quality of both gravity anomalies and geoid heights. As a result EGM2008 followed by EIGEN-6c2, which have the maximum degree, present gravity field parameters with considerably higher accuracy than the other GGMs which maximum degree range between 250 and 260. The higher the maximum degree the better the GGM performance.

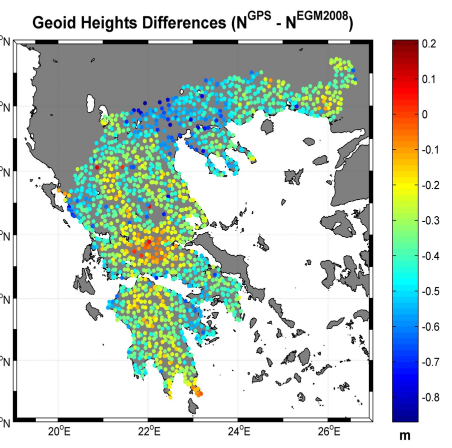


Figure 7: GPS and Egm2008 Geoid heights differences

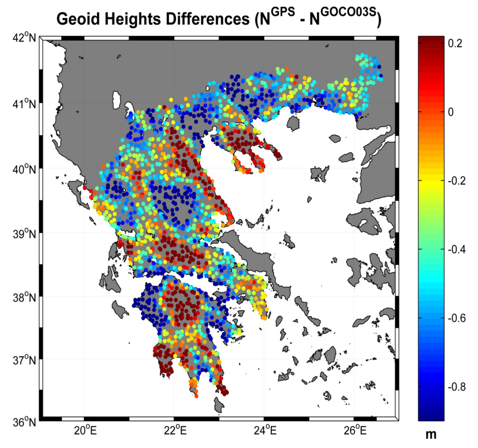


Figure 8: GPS and Goco03S Geoid heights differences

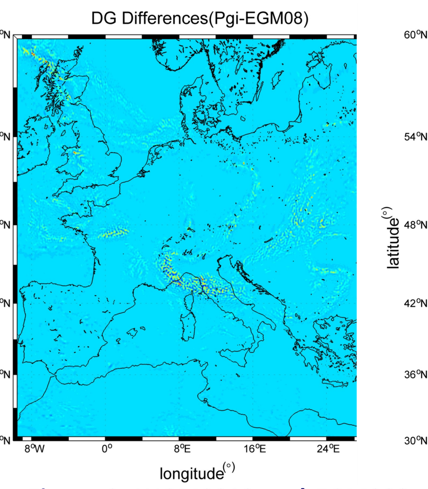


Figure 9: WGM2012 and EGM2008 Gravity anomalies differences

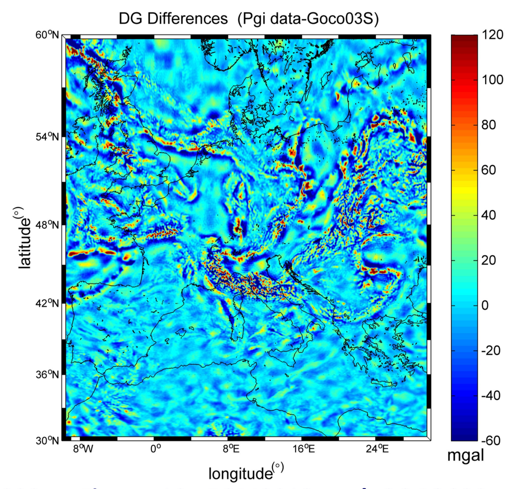


Figure 10: EGM2012 and GOCO03S Gravity anomalies differences

Wavelets and Multiresolution Analysis

It is known, that wavelets can localize both in the spatial an frequency domain. Wavelet Transformation (WT) is considered more effective in signal processing than Short-time Fourier Transformation.

WT is based on wavelets $\psi_r(x)$ as basis function in order to represent other functions. The wavelet function (ψ) carries valuable information about the signal, while the scaling function (φ), reveals the functional approximation.

The orthogonal Daubechies 10 mother wavelet (db10) was employed for the analysis of the gravity field functionals.

2-D Wavelet Transformation functions

$$\psi^{i}_{(j,m,n)}(x)=2^{(j/2)}\psi\left(2^{j}x-m,2^{j}x-n\right), i=H,V,D$$

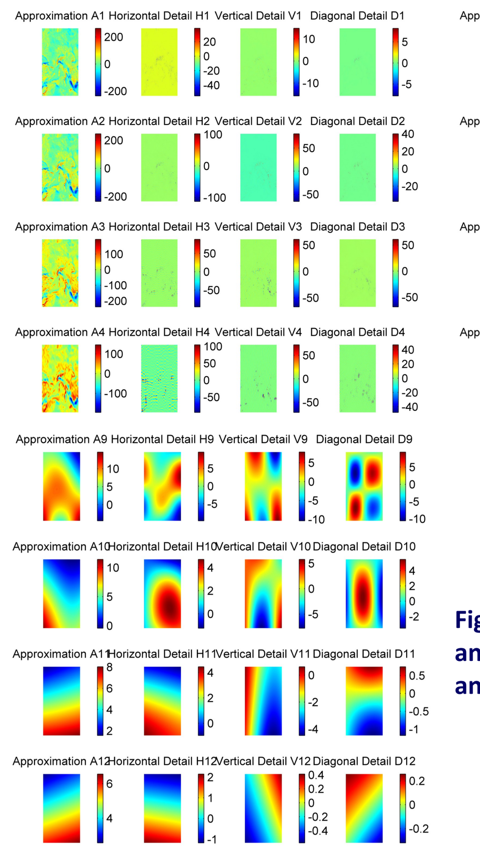
$$\varphi_{(j,m,n)}(x)=2^{(j/2)}\varphi\left(2^{j}x-m,2^{j}x-n\right)$$

Signal Decomposition

Since wavelets are base functions with localization properties in both space (time) and frequency (scale) domains, there can be a multiresolution analysis (MRA) at various levels of decomposition.

The 2D WT provides coefficients that correspond to different spatial resolutions, related to the signal frequencies. According to the wavelet decomposition algorithm , each scale analysis (level) of the signal, is analyzed in an approximation coefficient that carries the main information of the signal, and three detail coefficients (horizontal, vertical and diagonal).

Each Level of decomposition corresponds to a spatial resolution. To determine the number of levels the initial grid step of the data I used (1°~110km). The first level extends from 5.5km~11km, the second from 11~22km etc., until the last levels' spatial analysis reaches the earth's perimeter. As a result when the grid step is 3 arcmin, there are 12 Levels of decomposition.



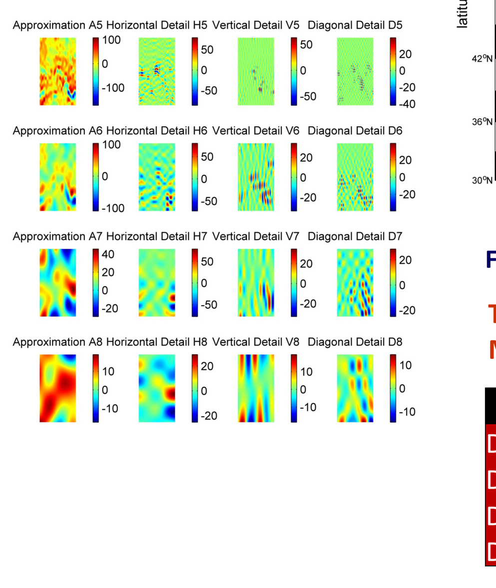


Figure 11: Levels of Decomposition and Detail Coefficients of Gravity anomalies of EGM2008 GGM

GGM synthesis through WT multi-resolution analysis

Through the synthesis process various GGMs can be combined, since each Level can be composed by a different GGM given each spatial resolution and performance at each specific Level of analysis. Synthesis is defined as the algebraic sum of the detail coefficients of each Level used and the approximation coefficient of the last Level.

$$Synthesis = A12+(H,V,D)_{12}+(H,V,D)_{11}+ +\ldots(H,V,D)_{2}+(H,V,D)_{1}$$

The spectral content at each level is analyzed in order to conclude on the gravity field signal power that each GOCE/GRACE GGM represents compared to EGM2008 The choice of the GGM that will be used at each level depends on its resolution and the gravity field content w.r.t. EGM2008.

Table 3: GGMs' Synthesis at various levels

	Resolution from (km)	Resolution to (km)	Synthesis Egm08-Goco03S	Synthesis Egm08-Tim_R4	Synthesis Egm08-Dirr_R4	Synthesis Egm08-Eigen6c2
Level1	5.5	11	Egm08	Egm08	Egm08	Egm08
Level2	11	22	Egm08	Egm08	Egm08	Egm08
Level3	22	44	Egm08	Egm08	Egm08	Egm08
Level4	44	88	Egm08	Egm08	Egm08	Egm08
Level5	88	176	**Goco03S**	**Tim_R4**	**Dir_R4**	**Eigen6c2**
Level6	176	352	**Goco03S**	**Tim_R4**	**Dir_R4**	**Eigen6c2**
Level7	352	704	**Goco03S**	**Tim_R4**	**Dir_R4**	**Eigen6c2**
Level8	704	1408	Egm08	Egm08	Egm08	**Eigen6c2**
Level9	1408	2816	Egm08	Egm08	Egm08	**Eigen6c2**
Level10	2816	5632	Egm08	Egm08	Egm08	**Eigen6c2**
Level11	5632	11264	Egm08	Egm08	Egm08	**Eigen6c2**
Level12	11264	22528	Egm08	Egm08	Egm08	Egm08

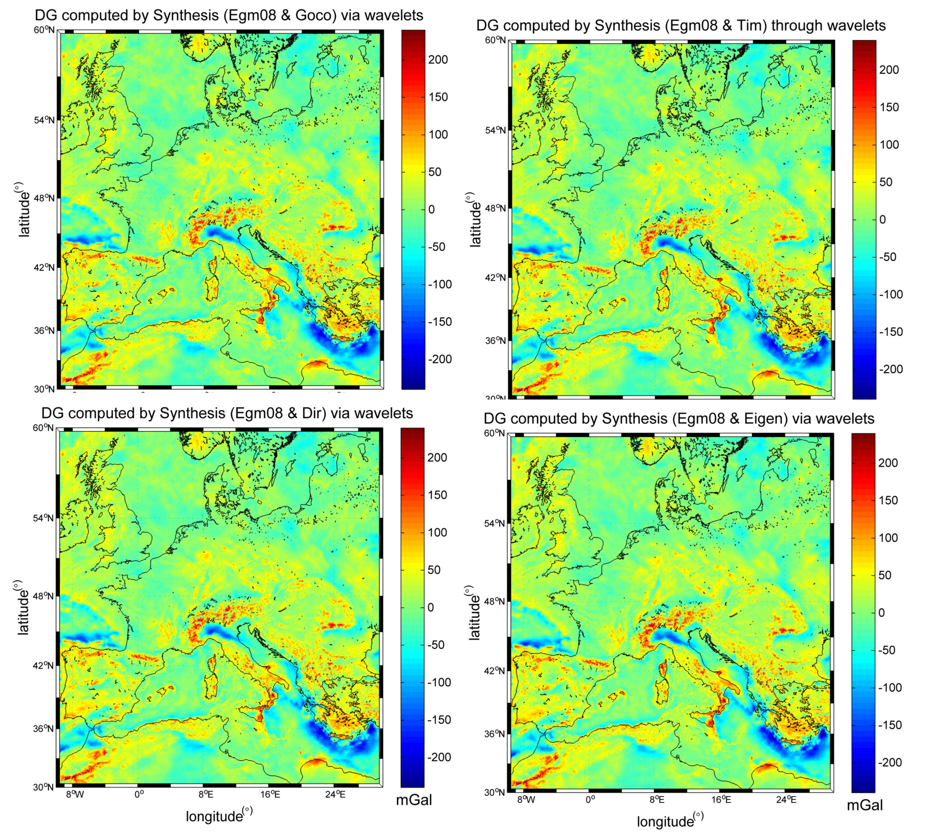


Figure 12: Synthesized GGMs (Gravity anomalies)

Table 4: Gravity anomaly differences between WGM2012 and the WL MRA synthesis [Units: mGal]

	min	max	mean	stdev
DG_PGI-DG_Egm08-Goco03S	-89.09	129.87	0.32	9.38
DG_PGI-DG_Egm08-Tim_R4	-90.52	134.80	0.31	8.85
DG_PGI-DG_Egm08-Dir_R4	-87.10	129.69	0.29	8.47
DG_PGI-DG_Egm08-Eigen6c2	-51.01	128.15	-0.33	3.46

There is a significant improvement when the WL MRA Synthesis is implemented, since the std of the differences drops by about 13-15mGal.

The synthesis of EIGEN6C2 with EGM2008 shows a slight improvement at the 1 mGal level.

For the low-degree GGMs the range of the differences reduces by more than 250 mGal.

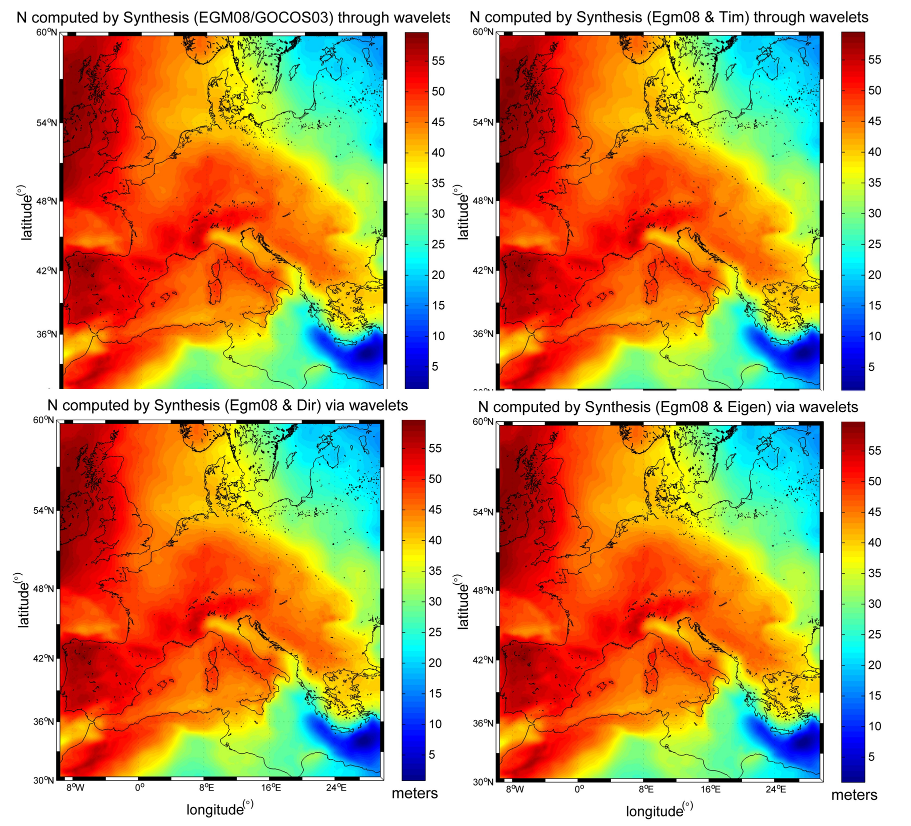


Figure 13: Synthesized GGMs (Geoid heights)

Table 6: Geoid height differences between GPS/Leveling and the WL MRA synthesis. [Unit: m]

	min	max	mean	stdev
NGPS-No-N_Egm08-Goco03S	-1.083	0.453	-0.387	0.259
NGPS-No-N_Egm08-Tim_R4	-1.151	0.399	-0.381	0.239
NGPS-No-N_Egm08-Dir_R4	-1.048	0.401	-0.392	0.223
NGPS-No-N_Egm08-Eigen6c2	-0.409	0.638	0.129	0.155

The std is improved by as much as 20 cm and the range by more than 50 cm, for the low-degree GGMs.

For EIGEN6C2 the std deteriorates by ~2 cm, showing that simple synthesis of the various levels is not enough in order to achieve a performance equal or better than EGm2008.

Synthesis External Validation

Tables 5 and 6 reveal that there is a significant improvement when the WL MRA synthesis process is implemented, where gravity anomalies and Geoid heights standard deviation is improved by ±15mgal and ±20cm respectively.

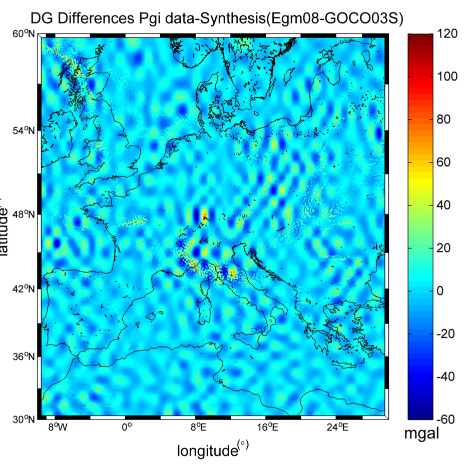


Figure 14: Differences between WGM2012 and Synthesis Gravity anomalies.

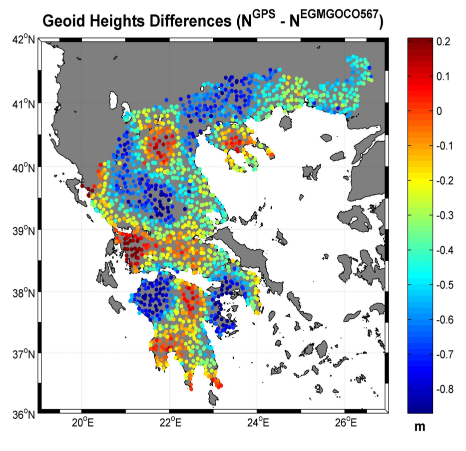


Figure 15: Differences between GPS/ Leveling and Synthesized Geoid heights.

Filtering

During the WL GGM synthesis process, noise, leakage and undesirable frequencies can be removed by filtering. Increasing the SNR, so that the maximum gain from each model will be achieved can be tackled with selective filtering.

Two types of isotropic filters, i.e. a boxcar and a Gaussian one have been tested in order to investigate whether they improve the results for the synthesized GGMs.

Boxcar filter

$$h(x,y)=2\lambda_c\, sinc\left(2\lambda_c\left(x^2+y^2\right)\right)$$

$$H(u,v)=\prod\left(\frac{\omega}{2\omega_c}\right)$$

Gaussian filter

$$h(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$H(u,v)=2\pi\sigma^2 e^{-2\pi^2\sigma^2\left(u^2+v^2\right)}$$

Using different Levels in the WL MRA synthesis process and performing filtering some slight improvement is achieved for the GOCE/GRACE GGMs. When L5 from EGM2008 is used instead of GOCO03S in the EGM08-GOCO03S synthesis, the std is improved by 3 mGal. The std of the geoid height differences is improved from 26 cm to 12 cm. GOCO03S L5 extends from 88-176 km, when the maximum resolution of GOCO03S is 80km and GOCE useful wavelength are >100 km. As a result the high frequencies of L5 (wavelengths shorter than 120 km) in GOCO03S carry significant noise, demanding filtering. L5 for the TIM-T4 and DIR-R4 GGMs exhibit the same behavior. The 120 km cut-off frequency was the one providing the most rigorous results.

Table 7: Differences between WGM2012 and filtered GGM Synthesis [Unit: mGal]

	min	max	mean	stdev
DG_PGI-Filtered_gauss_goco03s	-72.77	127.91	0.31	6.48
DG_PGI-Fiiltered_gauss_timr4	-74.88	132.14	0.29	6.36
DG_PGI-Fiiltered_boxcar_timr4	-75.29	135.47	0.29	6.81
DG_PGI-Fiiltered_gauss_dir4	-73.01	130.56	0.29	6.25
DG_PGI-Fiiltered_boxcar_dir4	-73.96	134.30	0.29	6.72

Table 8: Differences between GPS/Leveling and filtered GGM Synthesis [Unit: m]

	min	max	mean	stdev
N_GPS-No-N_Goco03S_Gauss	-0.870	0.206	-0.377	0.176
N_GPS-No-N_Goco03S_Boxcar	-0.898	0.244	-0.373	0.192
N_GPS-Fiiltered_TimR4_Gauss	-1.011	0.314	-0.373	0.213
N_GPS-Fiiltered_TimR4_Boxcar	-0.927	0.279	-0.377	0.190
N_GPS-Fiiltered_DirR4_Gauss	-0.925	0.279	-0.378	0.187
N_GPS-Fiiltered_DirR4_Boxcar	-1.027	0.316	-0.373	0.212

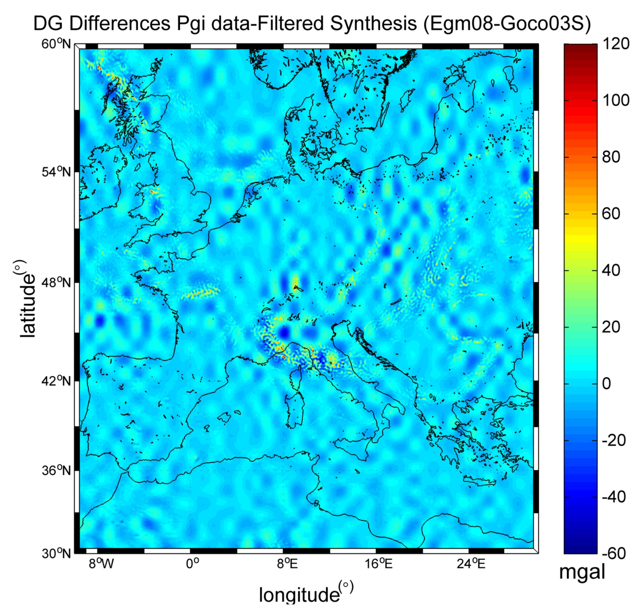


Figure 16: Gravity anomaly differences between WGM2012 and GOCO03S filtered WL Synthesis.

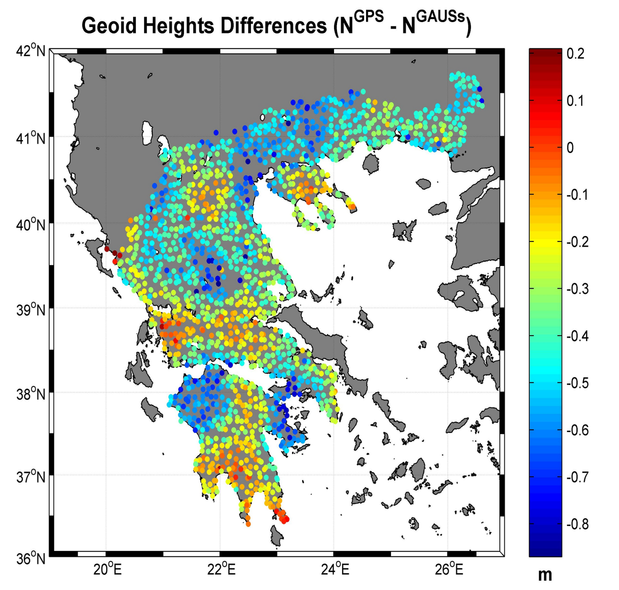


Figure 17: Geoid height differences between GPS/ Leveling and GOCO03S filtered WL Synthesis.

Thresholding

It is known that the smaller the value of coefficient is the more noise they carry, while coefficient with big values have better quality, because of the energy compaction during the wavelet transform. To reduce the effect of the coefficients with high values, soft Thresholding is implemented. In all cases thresholding provides inferior results compared to filtering.

Table 9: Differences between local gravity anomalies and gravity anomalies from Thresholded Synthesis [Unit: mGal]

	min	max	mean	stdev
DG_PGI-GOCO03S_Thresholded	-89.09	129.87	0.32	9.36
DG_PGI-Tim_R4_Thresholded	-91.48	134.45	0.29	8.83
DG_PGI-Dir_R4_Thresholded	-87.10	129.69	0.29	8.46

Table 10 Differences between GPS measurements and Geoid heights from Thresholded Synthesis [Unit: m]

	min	max	mean	stdev
N_GPS-N_GOCO03S_Thresholded	-1.074	0.457	-0.385	0.251
N_GPS-N_Tim_R4_Thresholded	-1.219	0.327	-0.397	0.226
N_GPS-N_Dir_R4_Thresholded	-1.134	0.375	-0.390	0.218

Conclusions

A detailed evaluation has been carried out for the latest GOCE/GRACE GGMs. From the external validation, it can be conducted that gravity anomalies are improved by ±15mgal while geoid heights by ±20m when WL MRA synthesis is implemented.

Moreover, for the low-degree GGMs, filtering L5 that corresponds to a spatial resolution of 88 km-176 km, improved the results. For those GGMs, spatial scales lower than 120 km, carry more noise than signal. Classical filters, Gaussian and Boxcar, implemented to L5 (88km~176km), improved the final synthesized GGM, reducing the std by ~4cm, and gravity anomalies by ~3 mGal.

Thresholding, improved slightly the geoid height differences by 1 cm while its impact on gravity anomalies was minimal. Overall, the GGM performance is improved when they are synthesized with EGM2008, while filtering specific wavebands provides further improvement.

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