Estimation of the reference geopotential value for the local vertical datum of continental Greece using EGM08 and GPS/leveling data

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Abstract

Estimation of the zero-height geopotential level represented by W_o^{LVD} in a local vertical datum (LVD) is a problem of main importance for a wide range of geodetic applications related to different height frames and plays a fundamental role in the connection of traditional height reference systems into a global height system or even a modern geoid-based vertical datum. This paper aims primarily at the estimation of W_o^{LVD} for the continental part of Greece, with the use of surface gravity data and geopotential values computed from EGM08 in conjunction with GPS and orthometric heights over an extensive network which covers sufficiently the test area. The method used focuses on the estimation of W_o^{LVD} from a least squares adjustment scheme that is applied on the Helmert model for orthometric heights, using surface geopotential and gravity values (as obtained from EGM08 and the known 3D geocentric coordinates of each benchmark) along with the local Helmert heights over all network stations. Moreover, an attempt is made towards the modeling and removal of any height correlated errors in the available data according to this adjustment procedure. Different weighting schemes are tested, and, finally, some conclusions are drawn considering the accuracy of the obtained results.

Keywords

Local Vertical Datum, Hellenic Vertical Datum, Helmert orthometric heights, zero-height geopotential level

1 Introduction

The use of heights is of main importance for a wide range of geodetic, surveying and engineering applications. In the case of orthometric heights, i.e., heights above the geoid, differences are determined nationwide by conventional spirit leveling accompanied by gravity measurements along dedicated traverses. The orthometric heights of all established benchmarks are then obtained, through a Least-Squares (LS) adjustment of the entire vertical network, as height differences with respect to (w.r.t.) a selected benchmark (BM) that serves as the origin point of the country's vertical reference system. It has been customary for the origin point to coincide with a tide gauge (TG) station at which the local mean sea level (MSL) has been determined over a long period of time – the latter provides the primary realization of a zero-height reference surface w.r.t. which all orthometric heights are referred and measured thereafter. In this way, the orthometric height of a point P on the Earth's surface w.r.t. a local vertical datum (LVD) is obtained, in principle, through the geopotential difference between the Earth's gravity potential W_{P} at that point and the reference geopotential value $W_{\mathrm{o}}^{\mathrm{LVD}}$ on the associated zero-height surface of the LVD divided by the mean gravity along the corresponding plumbline segment (Heiskanen and Moritz 1967, sect. 4-4).

Many studies have been carried out in the past decades towards the unification of independent realizations of height systems under a common vertical datum through a global reference value W_{o}. Moreover, vertical shifts between national vertical datums have been determined by various approaches in terms of height or geopotential differences of their corresponding zero-height reference surfaces. For example, Burša et al. (2001) computed geopotential differences among several LVDs and unified them into a World Height System (WHS) while Burša et al. (2002) demonstrated the practical realization of a WHS by determining geopotential values at TG stations and the geopotential differences between the LVDs and a global vertical datum. Ardalan et al. (2002) used GPS data, orthometric heights and a Global Geopotential Model (GGM) for deriving a zero-height geopotential value for TG stations located in the region of the Baltic Sea. Burša et al. (2004) determined a global vertical reference frame through the unification of the North American, Australian, French and Brazilian height datums. Sánchez (2007), after determining a global W_0 value, attempted the unification of the South American height datums into a WHS. Ardalan et al. (2010) used potential and gravity differences for height datum unification within a test area in Southwest Finland.

The determination of the zero-height geopotential level of an existing vertical datum is considered of main importance for the connection of traditional height reference systems into a global height system or even a modern geoid-based vertical datum. Various methods have been used in practice for estimating the fundamental parameter W_0^{LVD} which can be broadly classified into two basic categories as described in Kotsakis et al. (2012). The first one is based on the combined adjustment of GGM and GPS/leveling data while the second one employs the formulation of a geodetic boundary value problem (GBVP) with the use of gravity anomaly data over different LVD zones. Both approaches have been extensively utilized by the scientific community but the present paper will focus only on the first approach.

In practice, the primary realization of most vertical datums is not accompanied by the specification or the determination of its associated W_0^{LVD} value. This is also the case for Greece, with the main problem being the existence of hundreds of islands, where no hydrostatic leveling has been applied to connect the orthometric heights of the reference BMs at the islands with the official origin of the Hellenic Vertical Datum (HVD), i.e. the TG reference station at Piraeus harbour. Apart from this difficulty, the HVD suffers also from local biases that can be attributed to the lack of a common adjustment of the whole vertical network (Tziavos et al. 2012). Under these conditions, estimates of the zero-height reference levels W_0^{LVD} for the independent vertical datums within the 16 largest Greek islands were derived by Kotsakis et al. (2012) while for the continental part the only available estimate has been given by Vatrt et al. (2009) from a combined processing of geoid heights with a limited number of GPS/leveling data.

The main objective of this study is to obtain a representative estimate W_0^{LVD} for the HVD zero-height level over the entire continental part of Greece using Helmert orthometric heights, GPS measurements and a high-resolution GGM. The used data provide an extremely dense coverage of the test area, and the accuracy of our final estimate is limited only by the GGM commission error over the spatial wavelengths that overly exceed the size of the country. In Europe there has been a significant amount of work towards the determination and the modernization of the European Vertical Reference System (EVRS) by combining several geodetic datasets from many different countries (Ihde et al. 2002). The absence of the Greek leveling (and other types of) data from such an effort and the practical exclusion of the country from the EVRS efforts give good evidence on the necessity of the present study.

2 Method for the determination of W_o^{LVD}

The simplest approach for determining the unknown zero-height geopotential value W_o^{LVD} in a local vertical datum based on heterogeneous height data over a terrestrial network of m leveling BMs relies on the following LS estimator

$$\hat{W}_o^{\mathrm{LVD}} = W_o - \frac{\sum_1^m \left(h_i - H_i - N_i\right)\gamma_i}{m}, \qquad (1)$$

where h_i is the ellipsoidal height obtained from space geodetic techniques, N_i is the geoid height from a GGM or a regional geoid model, and H_i is the known orthometric height of each benchmark w.r.t. the underlying LVD. The term W_o specifies the equipotential surface that is realized by the GGM or the geoid model (i.e., as deduced by the zero-degree term in the known geoid heights N_i) while γ_i is the normal gravity on the reference ellipsoid computed through Somigliana’s formula; for more details on the use of Eq. (1) see Jekeli (2000) and Kotsakis et al. (2012).

In our current study we devise an alternative estimator $\hat{W}_o^{\mathrm{LVD}}$ to be used in conjunction with orthometric heights and a high-resolution GGM, without the need to compute geoid heights at the leveling BMs. This is advantageous since it abolishes the effect of terrain modeling errors that are inherently present whenever geoid heights are determined through a spherical harmonic series expansion of Earth's disturbing gravity field (e.g., Rapp 1997, Smith 1998). Moreover, the approximate character of the orthometric heights used in geodetic practice (Helmert orthometric heights) is taken into account in our alternative estimator, thus avoiding any errors due to their theoretical inconsistency with GPS/GGM derived orthometric heights (note that such errors are always present in the result of Eq. (1)).

Based on the definition of Helmert orthometric heights (Heiskanen and Moritz 1967, ch. 4) we have the following equation:

$$H_i^{\text{helm}} = \frac{W_{\text{o}}^{\text{LVD}} - W_i}{\bar{g}_i}, \qquad (2)$$

where H_i^{helm} is the known Helmert orthometric height w.r.t. the underlying LVD, $W_{\text{o}}^{\text{LVD}}$ is the (unknown) reference geopotential level of the LVD, W_i is the gravity potential at the leveling benchmark, and $\bar{g}_i$ is the mean gravity value along the plumb line between the LVD's zero-height equipotential reference surface and the Earth's surface according to the Poincare-Prey reduction scheme (ibid., Eq. 4-24)

$$\bar{g}_i = g_i + 0.0424 H_i^{\text{helm}} \ \ (g_i \text{ and } \bar{g}_i \text{ in mgal, } H_i^{\text{helm}} \text{ in m}). \qquad (3)$$

Both terms W_i and $\bar{g}_i$ in Eq. (2) can be determined on the basis of a GGM and the knowledge of the spatial position of the leveling benchmark from space geodetic measurements (e.g., GPS). Specifically, the gravity potential W_i can be synthesized from a gravitational part obtained by the GGM spherical harmonic coefficients and a centrifugal part using the benchmark's known spatial position and the Earth's conventional rotational velocity (Petit and Luzum 2010), whereas the surface gravity value g_i in Eq. (3) can be simply reconstructed through the GGM-derived gravity disturbance as described in Filmer et al. (2010).

Hence, the implementation of Eq. (2) over a network of leveling BMs yields a system of "observation equations" with a single unknown parameter ($W_{\text{o}}^{\text{LVD}}$) which can be resolved in terms of the general weighted LS estimator

$$\hat{W}_{\text{o}}^{\text{LVD}} = \frac{\sum_i p_i y_i}{\sum_i p_i} = \frac{\sum_i p_i \left(W_i + H_i^{\text{helm}} \bar{g}_i \right)}{\sum_i p_i}, \qquad (4)$$

where p_i is a positive weight factor associated with each BM such that

$$\sum_i p_i \delta W_i^2 = \min . \qquad (5)$$

(see Fig.1 for a graphical description of the above estimation scheme). Note that the estimate $\hat{W}_\mathrm{o}^\mathrm{LVD}$ according to Eq. (4) is rather insensitive to the uncertainty of the GGM-derived surface gravity values that are used for computing the terms $\bar{g}_i$ at the leveling BMs. Indeed, if we apply a straightforward error propagation to Eq. (4) and assuming for simplicity that $p_i = 1$, we have

$$\sigma_{\hat{W}_\mathrm{o}^\mathrm{LVD}} = \frac{\sqrt{\sum_i^m \left(H_i^\mathrm{helm}\right)^2}}{m} \sigma_g , \qquad (6)$$

which implies an uncertainty less than 0.1 m^{2}s^{-2} for the zero-height geopotential value even in mountainous test networks and surface gravity accuracy σ_g reaching up to 20 mGal.

FIGURE 1

3 Available data

The Hellenic Vertical Datum (HVD) was established by the Hellenic Geographic Military Service within the period 1963-1986. In principle, the physical heights in the HVD were modeled as Helmert orthometric heights in the mean-tide system. They refer to the TG station at the Piraeus harbor, where local MSL was computed from sea level measurements over the period 1933-1978 (Takos 1989). The true accuracy of the HVD's leveling network is largely unknown. Additionally, a zero-height geopotential value was not originally associated with the HVD and hence is also considered as unknown.

In the present study, Helmert orthometric heights referring to the HVD along with GPS derived ellipsoidal heights at 1629 control stations of the Hellenic geodetic network are used for estimating the unknown parameter $W_\mathrm{o}^\mathrm{LVD}$ of the HVD according to the methodology presented in Sect. 2. The ellipsoidal heights refer to the ITRF2000 frame (epoch t=2007.236, tide-free system) and they were obtained from a number of GPS measurement campaigns which were performed over Greece for the establishment of the Hellenic Positioning (HEPOS) system (Gianniou 2008). All control stations used in our study are located in the mainland part of the country (see Fig. 2) and the horizontal and vertical accuracy of their GPS derived spatial positions is 1-4 cm (1σ) and 2-5 cm (1σ), respectively, whereas their orthometric heights had been computed by the Hellenic Geographic Military Service through spirit and/or trigonometric leveling ties with the primary national leveling network (Takos 1989). A total of 94 stations out of the 1629 were rejected from our analysis (see next section) having failed to pass the standard outlier detection (3σ) test during the preliminary LS adjustment of Eq. (2) over our test network, most probably due to existing blunders in their HVD orthometric heights.

FIGURE 2

All required computations for our analysis have been carried out in the tide-free system. The conversion, given in meters, from mean-tide H_{MT} to tide-free H_{TF} orthometric heights was implemented according to the following formula (Ekman 1989, combination of Eqs. 1, 6 and 7):

$$H_{\mathrm{TF}} - H_{\mathrm{MT}} = \left(29.6\sin^2\varphi - 9.9\right)\gamma \cdot 10^{-2}, \qquad (7)$$

where φ is the geodetic latitude and γ is the elasticity factor of the Earth which was set equal to 0.68 (Ekman 1989).

The computation of the gravity potential values (W_i) at all control stations has been performed with the Earth Gravitational Model 2008 (EGM08) (Pavlis et al. 2012), complete to degree and order 2159, in the tide-free system using (for the gravitational part) the harmonic synthesis program of Holmes and Pavlis (2006) and (for the centrifugal part) the GPS derived spatial coordinates of each station. The surface gravity at each station, which is required for the computation of the mean gravity term $\bar{g}_i$ as per Eq. (3), was also determined through the EGM08-based gravity disturbances according to the following formula

$$g_{\mathrm{BM}} = \gamma_{\mathrm{BM}} - \frac{\partial T}{\partial r}, \qquad (8)$$

where γ_{BM} is the normal gravity on the Earth's surface and $\partial T / \partial r$ is the radial derivative of the disturbing potential. Note that the GRS80 reference ellipsoid has been used for all related computations described in the previous paragraphs.

4 Numerical results

The results for the estimation of $W_{\mathrm{o}}^{\mathrm{LVD}}$ in the HVD using the method and datasets described in Sections 2 and 3 are presented in Table 1. Both an un-weighted and a weighted LS adjustment of Eq. (2) were performed with empirically assigned weights p_i taken as the inverse Helmert orthometric heights of each station. The accuracy level of our results shown in Table 1 corresponds to the standard error (1σ) obtained from the LS estimation algorithm taking into account the a posteriori variance factor that was obtained in each case.

Table 1. Estimates of the zero-height geopotential value for continental Greece.

	$\hat{W}_{\mathbf{o}}^{\mathbf{LVD}}$ **[m^{2}s^{-2}]**
Un-weighted LS	62636859.37 ± 0.04
Weighted LS $\left(p_i = 1 / H_i^{\mathrm{helm}}\right)$	62636860.16 ± 0.03

Looking at the results of Table 1 we notice a significant difference between the un-weighted and the weighted estimate $\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ at the level of 0.79 m^{2}s^{-2} (approximately 8cm). The magnitude of this discrepancy, in conjunction with the high statistical precision of the two estimates, manifests the existence of a bias into our results due to height-dependent systematic errors within the available data. This effect was further investigated by performing additional LS adjustment tests using different subsets of the control stations according to a certain height threshold. The results of these tests are presented in Table 2 for the un-weighted and weighted solutions, respectively. It is evident that, as more stations with higher altitude are included in the LS adjustment, the un-weighted estimate of the zero-height level systematically increases by several cm while the corresponding weighted estimate remains practically unaffected (within a few mm). This implies the presence of a height-correlated bias in our data that originates mostly from the HVD physical heights and partially from EGM08 related systematic errors in the computed W_i values at the test stations. The empirical weighting scheme that was used during the estimation process managed to control the effect of these systematic errors and provided more robust estimates for the HVD's zero-height level $\hat{W}_o^{\mathrm{LVD}}$. The existence of a height-correlated bias in our data is clearly visible in the scatter plot of the adjusted height residuals

$$e_i = H_i^{\mathrm{helm}} - \frac{\hat{W}_o^{\mathrm{LVD}} - W_i}{\bar{g}_i}, \qquad (9)$$

which were obtained from the un-weighted LS solution over the entire test network (1535 BMs), see Fig. 3. The statistics of these residuals are presented in Table 3 indicating an average agreement among the GPS heights, the HVD Helmert orthometric heights and the EGM08 model at the level of 15 cm over the Hellenic mainland.

Table 2. Estimates of the zero-height geopotential value for continental Greece by setting a height threshold to the initial data.

Height threshold for used BMs	**$\hat{W}_o^{\mathrm{LVD}}$ [m²s⁻²]**		**Difference [cm]**
	Un-weighted LS	**Weighted LS $\left(p_i = 1/H_i^{\mathrm{helm}}\right)$**	
< 200m (514 pts)	62636860.04	62636860.20	~ 1.6
< 500m (866 pts)	62636859.90	62636860.19	~ 3.0
< 1000m (1308 pts)	62636859.65	62636860.17	~ 5.3
< 1500m (1487 pts)	62636859.45	62636860.17	~ 7.3
< 2000m (1535 pts)	62636859.37	62636860.16	~ 8.1

Table 3. Statistics of the height residuals of the un-weighted LS adjustment solution. Unit: [m]

	mean	**min**	**max**	**std**
e_i	0.000	-0.456	0.429	0.150

FIGURE 3

For the treatment of the data biases that were identified in our previous tests, we next employed an extended parametric model towards the LS estimation of the HVD's geopotential value. Specifically, an additional parameter describing the linear part of the height-dependent systematic errors was introduced into the data adjustment procedure according to the general equation

$$H_i^{\text{helm}} \;=\; \frac{W_{\text{o}}^{\text{LVD}} - W_i}{\bar{g}_i} \;+\; \lambda H_i^{\text{helm}} \,. \qquad (10)$$

The estimated values of the fundamental parameter $W_{\text{o}}^{\text{LVD}}$ and the nuisance parameter λ from the LS adjustment of the above equation over the entire test network are provided in Table 4. Four different solutions were computed depending on the data weighting scheme that was adopted in the estimation algorithm. It should be noticed that two additional scenarios are now included in our experiments which correspond to the weight choices $p_i = \left(1/H_i^{\text{helm}}\right)^{1/2}$ and $p_i = \left(1/H_i^{\text{helm}}\right)^{2}$. The statistics of the adjusted height residuals that were obtained from these tests, i.e.,

$$e_i \;=\; H_i^{\text{helm}} - \frac{\hat{W}_{\text{o}}^{\text{LVD}} - W_i}{\bar{g}_i} - \hat{\lambda} H_i^{\text{helm}} \qquad (11)$$

are summarized in Table 5.

From the results contained in Tables 4 and 5 it is noticed that the LS solution based on the weight choice $p_i = \left(1/H_i^{\text{helm}}\right)^{2}$ becomes unstable with respect to the nuisance parameter λ while the corresponding mean value of the adjusted height residuals in Table 5 is unacceptable. This is not a surprising result since the particular weight factor is rather harsh and significantly down-weights most of the available BMs, thus blocking them from the adjustment procedure. Therefore, this weight factor makes the separation of $\hat{\lambda}$ and $\hat{W}_{\text{o}}^{\text{LVD}}$ practically impossible.

Table 4. Estimates of the zero-height geopotential value for continental Greece from the revised model of Eq. (10).

LSA Schemes	$\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ **[m^{2}s^{-2}]**	$\hat{\lambda}$ **[× 10^{-4}]**
Un-weighted	62636860.30 ± 0.05	-1.882 ± 0.073
Weighted $p_i = \left(1/H_i^{\mathrm{helm}}\right)^{1/2}$	62636860.28 ±0.04	-1.832 ± 0.095
Weighted $p_i = \left(1/H_i^{\mathrm{helm}}\right)$	62636860.23 ± 0.03	-1.725 ± 0.221
Weighted $p_i = \left(1/H_i^{\mathrm{helm}}\right)^2$	62636860.12 ± 0.01	1.339 ± 3.660

Table 5. Statistics of the height residuals from the LS adjustment of Eq. (10). Unit: [m]

	Mean	**Min**	**Max**	**Std**
Un-weighted	0.000	-0.481	0.415	0.125
Weighted $p_i = \left(1/H_i^{\mathrm{helm}}\right)^{1/2}$	0.000	-0.479	0.412	0.125
Weighted $p_i = \left(1/H_i^{\mathrm{helm}}\right)$	0.000	-0.474	0.405	0.126
Weighted $p_i = \left(1/H_i^{\mathrm{helm}}\right)^2$	-0.144	-0.821	0.340	0.188

On the other hand, the un-weighted solution and the weighted solutions with the weight choices $p_i = \left(1/H_i^{\mathrm{helm}}\right)^{1/2}$ and $p_i = \left(1/H_i^{\mathrm{helm}}\right)$ provide similar results that are statistically equivalent within their precision level. Any of the three corresponding estimates $\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ can be actually used as a representative estimate for the zero-height reference value of the HVD in continental Greece. In order to further evaluate these results, an alternative estimate was also computed using Eq. (1). For this purpose, the required "global" parameter W_{o} was set equal to the International Earth Rotation and Reference System Service's (IERS) conventional value 62636856.00 m^{2}s^{-2} (Petit and Luzum 2010) while the surface gravity data and the geoid heights were extracted from EGM08. An estimated value $\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ = 62636859.66

$\mathrm{m^2s^{-2}}$ was obtained in this case. This result has a small difference (0.29 $\mathrm{m^2s^{-2}}$) with the one given by the un-weighted LS solution of Table 1 while it is smaller by 0.64 $\mathrm{m^2s^{-2}}$ compared to the LS estimates of the revised model of Eq. (10). These differences signify that the result provided by Eq. (1) is also affected by a height-correlated bias. The same conclusion may be also drawn from the comparison that was made with the value provided by Vatrt et al. (2009) for the HVD, i.e., $\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ = 62636859.44 $\mathrm{m^2s^{-2}}$. The difference found is 0.86 $\mathrm{m^2s^{-2}}$ which corresponds approximately to 9 cm.

For a more realistic assessment of the accuracy level in our results given in Table 4, we have to consider the fact that EGM08 contributes an apparent bias into the LS estimate $\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ due to its commission error over spatial wavelengths that overly exceed the extent of our test network. Obviously, this error component cannot be reduced through the (un-weighted or weighted) data averaging that takes place within the adjustment procedure and it is fully propagated into the final results. Its magnitude can be evaluated, in a statistical sense, from the following formula

$$\delta\hat{W}_{\mathrm{o}}^{\mathrm{LVD}} = \sqrt{\sum\nolimits_{n=2}^{n*} \sigma_e^2\left(V_n\right)}, \qquad (12)$$

where $n*$ depends on the extent ΔA of the test area ($n* << 180/\Delta A$) and $\sigma_e^2\left(V_n\right)$ are the gravitational potential error degree variances given by

$$\sigma_e^2\left(V_n\right) = \left(\frac{GM}{\alpha}\right)^2 \sum\nolimits_{m=0}^{n} \left(\sigma_{\bar{C}_{nm}}^2 + \sigma_{\bar{S}_{nm}}^2\right) \qquad (13)$$

for

$$V = \frac{GM}{r} + \sum\nolimits_{n=2}^{\infty} V_n . \qquad (14)$$

In the above equations, V is the gravitational potential, $\sigma_{\bar{C}_{nm}}$ and $\sigma_{\bar{S}_{nm}}$ are the error standard deviations of the GGM spherical harmonic coefficients, GM is the Earth gravitational constant, α is the semi-major axis of the ellipsoid and r is the geocentric radius. For our study area that covers a region of 5.5°×6°, the maximum degree $n*$ was selected equal to

15 (~1300 km, full-wavelength) leading to an apparent bias into the various estimates $\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ which is about 3.5 cm or, equivalently, 0.35 $\mathrm{m}^2\mathrm{s}^{-2}$. Therefore, the total accuracy of our results should be represented by the final error estimate

$$\hat{\sigma} = \sqrt{\sigma_{\mathrm{LS}}^2 + \left(\delta \hat{W}_{\mathrm{o}}^{\mathrm{LVD}}\right)^2} \simeq 0.35 \ \mathrm{m}^2\mathrm{s}^{-2} \, . \qquad (15)$$

where $\delta \hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ is obtained from Eq. (12) and σ_{LS} corresponds to the statistical accuracy of the LS adjusted solutions given in Table 4. Note that the EGM08 *omission* error for *n* > 2159 should not be separately considered as its propagated effect on $\hat{W}_{\mathrm{o}}^{\mathrm{LVD}}$ is well averaged out due to the large number of test points used in our study. It is evident that the EGM08 commission error over spatial wavelengths > 1300 km dominates the estimation accuracy of the zero-height geopotential value for the HVD. Therefore, further accuracy improvements could be achieved with the use of an improved GGM, like for example a GOCE-based model (see, e.g., Gatti et al. 2013; Gerlach and Rummel 2013).

5 Conclusions and future work

In this study a method based on a weighted LS adjustment has been presented for the determination of the fundamental parameter $W_{\mathrm{o}}^{\mathrm{LVD}}$ of the HVD for the continental part of Greece utilizing Helmert orthometric heights, GPS data and EGM08 derived gravity and gravity potential values in a terrestrial network of 1629 control stations. A series of LS adjustment tests using empirical height-dependent weighting and an augmented parametric model (see Eq. (10)) were carried out in order to account for the systematic part of the data errors which has been attributed to a detected height-correlated bias. As discussed in the previous section, any of the first three geopotential estimates given in Table 4 can be selected as representative values for the zero-height reference level in the HVD, since the differences among them are statistically insignificant within their actual accuracy level.

In terms of future work, geoid heights from local high-resolution geoid solutions could be used along with GOCE-based GGMs for an improved determination of $W_{\mathrm{o}}^{\mathrm{LVD}}$ through the proposed or other combination methods. Moreover, further investigation regarding the weighting schemes of the LS adjustment procedure could be carried out.

Acknowledgement

Dr. Grigoriadis, Dr. Vergos and Prof. Tziavos wish to acknowledge the funding provided for this work by the European Space Agency in the frame of the ESA-PRODEX GOCESeaComb project (C4000106380). The GPS data were kindly provided in the frame of a cooperation agreement between the Department of Geodesy and Surveying of the Aristotle University of Thessaloniki and KTIMATOLOGIO SA.

References

Ardalan A, Grafarend E, Kakkuri J (2002) National Height Datum, the Gauss-Listing Geoid Level Value W(o) and its Time Variation (Baltic Sea Level Project : epochs 1990.8, 1993.8, 1997.4. J of Geodesy 76: 1-28.

Ardalan A, Karimi R, Poutanen M (2010) A bias-free geodetic boundary value problem approach to height datum unification. J Geodesy 84: 123–134.

Burša M, Kenyon S, Kouba J, Raděj K, Vatrt V, Vojtíšková M and Šimek J (2002) World height system specified by Geopotential at tide gauge stations. IAG Symp, 124, Springer-Verlag Berlin Heidelberg, 291–296.

Burša M, Kenyon S, Kouba J, Sima Z, Vatrt V and Vojtísková M (2004) A Global Vertical Reference Frame Based on Four Regional Vertical Datums. Stud Geophys Geod (48): 493-502.

Burša M, Kouba J, Müller A, Raděj K, True SA, Vatrt V and Vojtísková M (2001) Determination of geopotential differences between local vertical datums and realization of a world height system. Stud Geoph Geod (45): 127-132.

Ekman M (1989) Impacts of geodynamic phenomena on systems for height and gravity. B Geod (63): 281-296.

Filmer MS, Featherstone WE, Kuhn M (2010) The effect of EGM2008-based normal, normal-orthometric and Helmert orthometric height systems on the Australian levelling network. J Geodesy 84(8): 501-513.

Gatti A, Reguzzoni M, Venuti G (2013) The height datum problem and the role of satellite gravity models. J Geodesy 87: 15-22.

Gerlach C, Rummel R (2013) Global height system unification with GOCE: a simulation study on the indirect bias term in the GBVP approach. J Geodesy 87: 57-67.

Gianniou M (2008) HEPOS: Designing and implementing an RTK network. Geoinformatics 11(1): 10-13.

Heiskanen W, Moritz H (1967) Physical geodesy. WH Freeman, San Francisco.

Holmes SA, Pavlis NK (2006) A Fortran program for very-high-degree harmonic synthesis (version 05/01/2006). Program manual and software code available at *http://earth-info.nima.mil/GandG/wgs84/gravitymod/egm2008/*.

Ihde J, Augath W, Sacher M (2002) The vertical reference system for Europe. IAG Symp, 124, Springer-Verlag Berlin Heidelberg, 345–350.

Jekeli C (2000) Heights, the Geopotential and Vertical Datums. Department of Civil, Environmental Engineering and Geodetic Science, The Ohio State University, OSU Report No. 459, Columbus, Ohio

Kotsakis C, Katsambalos K, Ampatzidis D (2012) Estimation of the zero-height geopotential level W_o^{LVD} in a local vertical datum from inversion of co-located GPS, leveling and geoid heights: a case study in the Hellenic islands. J Geodesy 86(6): 423-439.

Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2012) The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). J Geophys Res 117(B4), B04406.

Petit G, Luzum B (eds) (2010) IERS Conventions 2010. International Earth Rotation and Reference Systems Service, IERS Technical Note No. 36, Verlag des Bundesamtes für Kartographie und Geodäsie, Frankfurt am Main.

Rapp RH (1997) Use of potential coefficient models for geoid undulation determinations using a spherical harmonic representation of the height anomaly/geoid undulation difference. J Geod, 71: 282-289.

Sánchez L (2007). Definition and Realization of the SIRGAS Vertical Reference System within a Globally Unified Height System. IAG Symp, 130, Springer-Verlag Berlin Heidelberg, 638-645.

Smith DA (1998) There is no such thing as ‘the’ EGM96 geoid: subtle points on the use of a global geopotential model. International Geoid Service Bullettin, 8: 17-27.

Takos I (1989) New adjustment of the national geodetic networks in Greece (in Greek). Bulletin of the Hellenic Military Geographic Service, 49(136): 19-93.

Tziavos IN, Vergos GS, Grigoriadis VN, Andritsanos VD (2012) Adjustment of collocated GPS, geoid and orthometric height observations in Greece. Geoid or orthometric height improvement? IAG Symp, 136, Springer-Verlag Berlin Heidelberg, 481-488.

Vatrt V, Sima Z, Vojtiskova M (2009) Vertical shifts of LVDs by EGM08 and Wo - preliminary result. Presented at the annual EUREF 2009 Symposium, Florence, May 27-30, 2009.

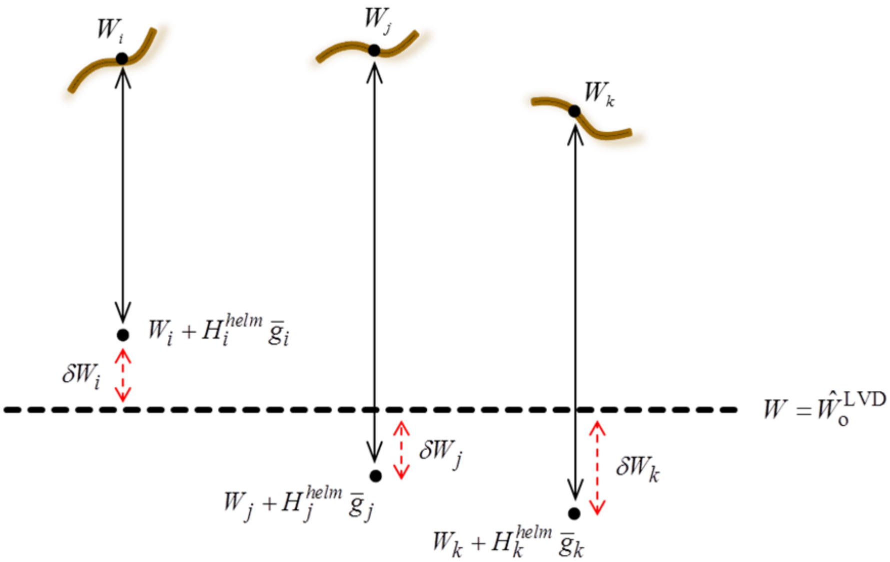

Fig. 1 Zero-height geopotential value determination from a network of stations.

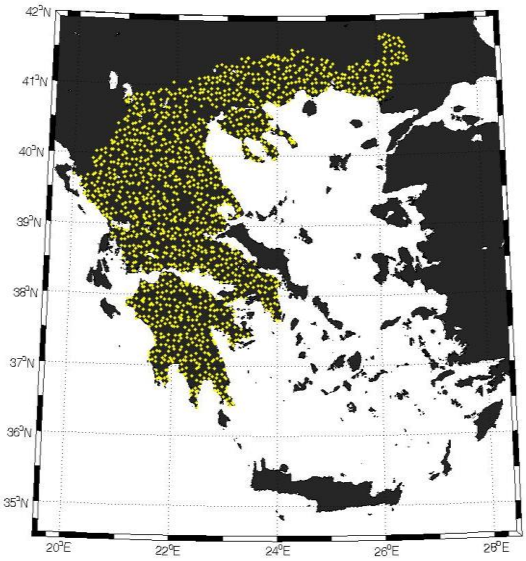

Fig. 2 Spatial distribution of the 1629 GPS/leveling control stations in continental Greece.

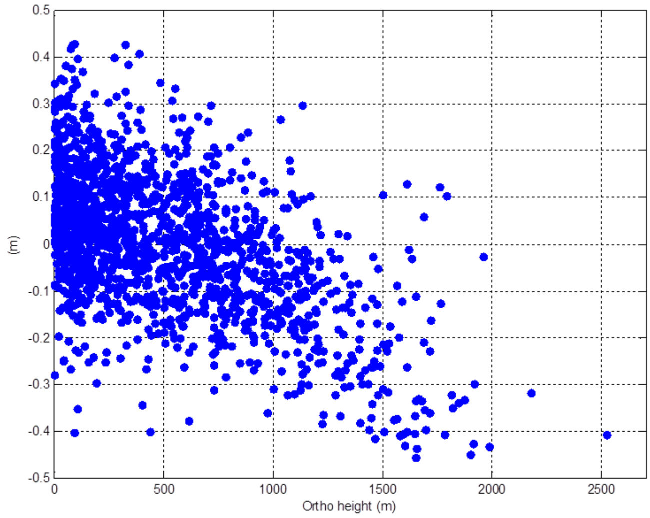

Fig. 3 Residual heights computed from the un-weighted LS adjustment of Eq. (2) over the Hellenic test network.