Preliminary estimation of the geopotential value W_0 for the Local Vertical Datum of Argentina using EGM2008 and GPS/Levelling data

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Objective

Estimate the zero-height geopotential value W_0^{LVD} for the local vertical datum of Argentina. Two methodologies are employed:

- Method 1: Estimation of W_0^{LVD} using a Least-Squares adjustment scheme, Helmert orthometric heights and surface gravity and geopotential values computed from a Global Geopotential Model (GGM).
- Method 2: Estimation of W_0^{LVD} using surface gravity and geoid heights from GGM and GPS/Levelling data.

This will allow the connection with other regional or global height systems. One of the foremost objectives of the SIRGAS (South American Geocentric Reference System) Project is to define and establish a unified vertical datum within a global definition.

We will provide a link between the W_0 IAG conventional value (62636856.00 m² s⁻²) with the zero-height geopotential level for the local vertical datum determined for Argentina.

Methodology

Consider a terrestrial network of *{i= 1, 2,……, m}* Levelling Benchmarks (BMs), with:

- Orthometric heights H_i from traditional spirit Levelling
- GPS derived ellipsoidal heights h_i
- g_i and W_i are computed from a GGM

W_0^{LVD}(estimated value)

Method 1

Basic physical model Helmert orthometric heights:

$$H_i^{Helmert} = \frac{C_i^{LVD}}{\bar{g}_i^{Helmert}} = \frac{W_0^{LVD} - W_i}{\gamma_i^{Helmert}}, \quad (1)$$

where W_i is the actual gravity potential or geopotential at the BM (computed by a GGM) and $\bar{g}_i^{Helmert}$ is the mean value of gravity related to the observed gravity/surface gravity (g_i) computed by a GGM at each station given by:

$$\bar{g}_i^{Helmert} = g_i + 0.024\, H_i,$$

From GGM

Equation (1) has only one unknown,

$$W_0^{LVD} = H_i^{Helmert}\bar{g}_i^{Helmert} + W_i,$$

which can be computed through a Least Squares adjustment of the zero height geopotential value over the BMs network as:

$$\hat{W}_0^{LVD} = \frac{\sum_{i=1}^{m} p_i(\bar{g}_i^{Helmert} H_i^{Helmert} + W_i)}{\sum_{i=1}^{m} p_i},$$

where p_i *are positive observation weights, so that the residuals after the adjustment are minimized as* $\sum_{i=1}^{m} p_i \delta W_i^2 = min$.

Method 2

$$C^{CVD} = W_0^{CVD} - W_i, \quad C_i^{LVD} = W_0^{LVD} - W_i,$$

The differences between the local and global vertical datum can be described by the next formula:

$$\Delta C_i^{CVD/LVD} = W_0^{CVD} - W_0^{LVD},$$

$$W_0^{LVD} = \frac{\sum_{i=1}^{m} W_0^{LVD}}{m} = W_0^{CVD} - \frac{\sum_{i=1}^{m} \Delta C_i^{CVD/LVD}}{m}.$$

From GGM

where $\Delta C_i^{CVD/LVD}$ is given by: $\Delta C_i^{CVD/LVD} = (h_i - H_i^{Helmert} - N_i - N_0)\bar{g}_i^{Helmert}$,

where N_0 represents the contribution of the zero-degree harmonic to the GGM geoid undulations with respect to a specific reference ellipsoid. It has been computed according to the formula (Heiskanen and Moritz 1967, Eq. 2.182):

$$N_0 = \frac{GM - GM_0}{R\gamma} - \frac{W_0 - U_0}{\gamma}.$$

In both methods, the computation of surface gravity g_i at each BM ws based on EGM2008 as:

$$g_i = \gamma_{i(BM)} - \frac{\partial T}{\partial r}$$

All SHs computations (by *harm_synth*, Pavlis et al. 2012) were carried out in the Tide-Free system using GRS80 as the reference ellipsoid.

Estimation of the zero-height geopotential value for Argentina

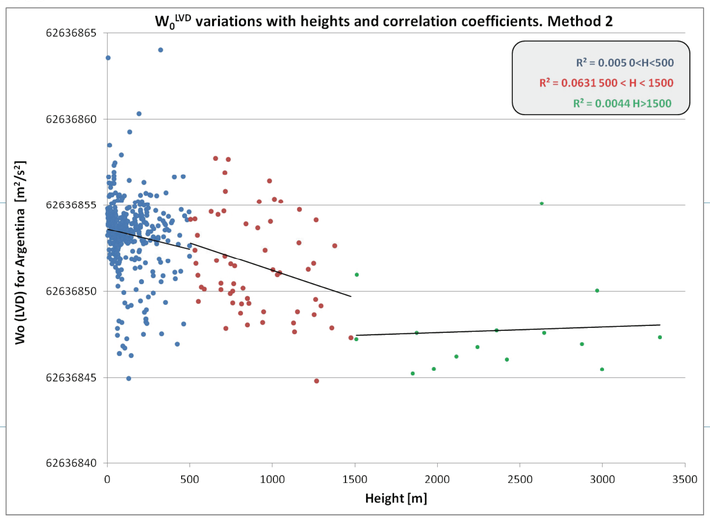
	$\hat{W}_0^{LVD}$ [m² s⁻²]		
	Un-weighted	Weighted	Difference [cm]
Method 1	*H < 500 m* *62636853.26 ± 0.042* *H < 1500 m* *62636853.00 ± 0.040* *H < 3500 m* *62636852.79 ± 0.039* Height dependency 62636852.79 ± 0.039	62636853.98 ± 0.015 $p_i=(1/H_i^{Helmert})$	11.9
		62636854.39 ± 0.008 $p_i=(1/H_i^{Helmert})^2$	16.1
		62636853.46 ± 0.025 $p_i=(1/H_i^{Helmert})^{1/2}$	6.8
Method 2	62636852.96 Arithmetic average		
Difference Method 2 minus Method 1 (un-weighted)			1.7

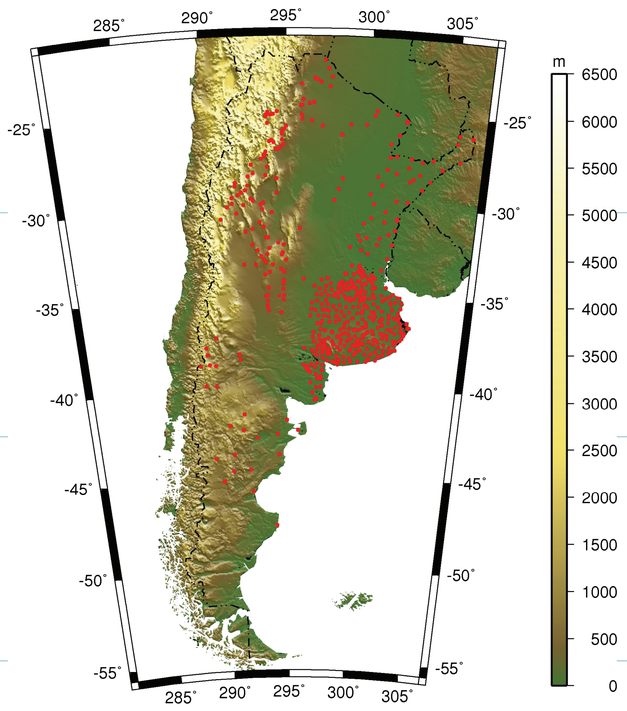
Statistics of the height residuals of the un-weighted LS adjustment solution. Method 1.

$e_i = H_i^{Helmert} - \frac{\hat{W}_0^{LVD} - W_i}{\bar{g}_i}$	Unit: [m]			
e_i	min	max	mean	σ
Un-weighted	-1.142	1.142	0.000	± 0.265

Data available

- The physical heights H correspond to the National Altimetric Network
- Argentina, like most countries does not make any luni-solar correction for precise levelling and ended with the Mean Tide system (MT). Orthometric Heights need to be converted from the MT system to the TF system with the expression: $H_{TF} = H_{MT} - 0.68(0.099 - 0.296\, sin^2\varphi)$
- The ellipsoidal heights h are referred to POSgAR 07
- Total number of 542 GPS/Levelling BMs stations
- EGM2008





Geographical Distribution of GPS/Levelling BMs in Argentina

Estimation of the zero-height geopotential value for Argentina using a height threshold for both un-weighted and weighted solutions. Method 1.

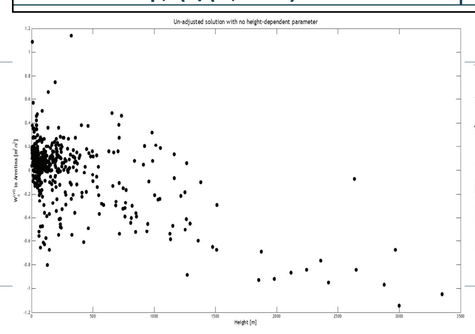
	$\hat{W}_0^{LVD}$ [m² s⁻²]						
Height threshold	Un-weighted	Weighted $p_i=(1/H_i^{Helmert})$	Difference [cm]	Weighted $p_i=(1/H_i^{Helmert})^2$	Difference [cm]	Weighted $p_i=(1/H_i^{Helmert})^{1/2}$	Difference [cm]
H < 500 m (464 points)	62636853.56	62636854.32	-7.7	62636854.51	-9.5	62636853.95	-3.9
H < 1500 m (527 points)	62636853.60	62636854.22	-6.2	62636854.49	-8.9	62636853.86	-2.6
H < 3500 m (542 points)	62636853.68	62636854.20	-5.2	62636854.49	-8.1	62636853.86	-1.8

Estimation of the zero-height geopotential value for Argentina including a parameter into the data adjustment using the following formula:

	$H_i^{Helmert} = \frac{W_0^{LVD} - W_i}{\bar{g}_i} + \lambda H_i^{Helmert}$	
	$\hat{W}_0^{LVD}$ [m² s⁻²]	λ
Un-weighted	62636853.68 ±0.05	-3.343E-04 ± 9.600E-06
Weighted $p_i=(1/(H_i^{Helmert})$	62636854.20 ± 0.31	-5.300E-04 ± 2.897E-04
Weighted $p_i=(1/(H_i^{Helmert})^2$	62636854.49 ± 0.91	1.218E-03 ± 4.886E-03
Weighted $p_i=(1/(H_i^{Helmert})^{1/2}$	62636853.86 ±0.14	-3.654E-04 ± 5.491E-05

Statistics of the height residuals from the LS adjustment of the following formula:

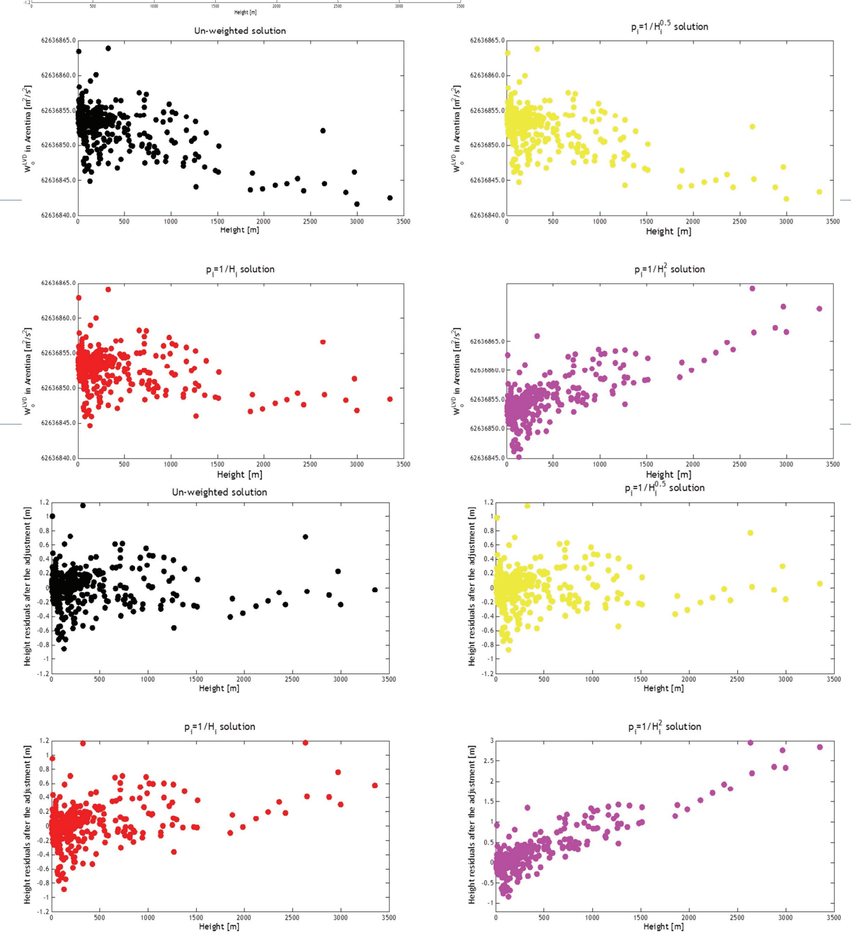
$e_i = H_i^{Helmert} - \frac{\hat{W}_0^{LVD} - W_i}{\bar{g}_i} + \hat{\lambda} H_i^{Helmert}$	Unit: [m]			
e_i	min	max	mean	σ
Un-weighted	-0.849	1.152	0.000	± 0.214
Weighted $p_i=(1/(H_i^{Helmert})$	-0.874	1.184	0.000	± 0.230
Weighted $p_i=(1/(H_i^{Helmert})^2$	-0.816	1.457	0.114	± 0.345
Weighted $p_i=(1/(H_i^{Helmert})^{1/2}$	-0.865	1.149	-0.010	± 0.215



The most robust estimate is the one with $p_i=1/H_i$ and since $p_i=1/H_i^{0.5}$ since the mean value with $p_i=1/H_i^2$ is unacceptable. The original h-H-$N^{EGM2008}$ σ is at the 24.4 cm, so the height residuals after the LS are improved (21.4 and 21.5 cm for the robust estimates).

Estimation of the zero-height geopotential value for Argentina using a height threshold. Method 2 (Arithmetic average).

Height threshold	$\hat{W}_0^{LVD}$ [m² s⁻²]	W_0^{CVD} [m² s⁻²] IAG conventional value	ΔW[m² s⁻²]
All points (542 points)	62636852.96	62636856	-3.04
H < 500 m (464 points)	62636853.32	62636856	-2.68
500 m < H < 1500 m (63 points)	62636851.57	62636856	-4.43
Average H > 1500 m (15 points)	62636847.72	62636856	-8.28



Conclusions

- A preliminary determination of W_0 for Argentina was carried out considering a terrestrial network of BMs with collocated physical heights (H) and ellipsoidal heights (h).
- A strong correlation with height is evident for BMs of higher elevation, as much 12 cm) so a height dependant parameter is introduced. The more robust estimates are those with $p_i=1/H_i$ and $p_i=1/H_i^{0.5}$ along with the height dependant parameter. Their difference is 3.4 cm, while the latter has a difference of just 1.8 cm with the un-weighted solution.
- A realistic estimate of $\hat{W}_0^{LVD}$ for Argentina is that of 62636853.86 m²/s², i.e., 21.4 cm below the conventional IAG W_0.
- Future work will involve new GGMs from GOCE mission.