A geodetic perspective to the estimation of the Quasi-Stationary sea surface topography and current velocities in a closed sea area

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Abstract

Accurate marine geoid determination from satellite altimetry is restrained from, among other things, the need of a well-determined and accurate model of the quasi-stationary sea surface topography. The latter relates the mean sea surface and the geoid, and is necessary for the downward continuation of the sea surface heights from the sea level to the surface of the geoid. Our approach in this paper was to try and estimate, from a geodetic point of view, a QSST model for a closed sea area located in the Eastern Mediterranean. This geodetically oriented QSST determination stands upon the simple principle that the quantity under determination can be derived as the difference between a purely gravimetric and a purely altimetric geoid model. The deviation of these equipotential surfaces should define a smooth surface, but this is not the usual case. This happens because

a) the altimetric heights are contaminated by noise due to the sea variability and
b) the gravity data used for the gravimetric geoid determination come from different sources and campaigns and are usually of an unknown quality (accuracy).

To minimize these effects a twofold procedure was followed. First, in order to reduce the effects of blunders in the gravity data, as well as any datum inconsistencies between the latter and altimetric SSHs, an editing and gross error detection procedure was employed, using least squares collocation, resulting to unifying the gravity data to a consistent reference system. Then, and in order to reduce the noise coming from the altimetric SSHs, a Wiener-type of filter was tested at various cut-off frequencies. The performance of each was based on a minimum signal loss versus maximum noise reduction principle. The QSST model developed was compared against an oceanographic regional model. As a final step, the geostrophic velocities and the direction of the sea currents were estimated to define whether some known features of the Mediterranean Sea circulation could be recovered. From the analysis of the results it was concluded that even with this purely geodetic determination of the QSST we can recover the main features of the sea circulation.

1. Introduction

Since the early missions of GEOS-3 and SeaSat, altimeters onboard satellites have offered a tremendous amount of measurements of the sea surface resulting in the improved knowledge of the Earth’s gravity field over oceanic regions. A direct consequence of that is the continuous development of Mean Sea Surface (MSS) models of the oceans, which are usually combined with satellite-only Earth Gravity Models (EGMs) to estimate models of the Quasi-Stationary Sea Surface Topography (QSST). The QSST is defined as the semi-constant over large periods of time deviation of the mean sea surface from the geoid. It reaches a maximum of +2.2 m and in closed sea areas has very small variations over large regions. This is why most QSST models developed during the last two decades are usually provided in terms of a spherical harmonics expansion of the QSST to low degrees, e.g., 20 (which corresponds to about 2000 km full wavelength). It can be easily concluded that when the area under study is rather small or is characterised as closed, e.g., the Mediterranean Sea, then such global models are insufficient.

From a geodetic point of view, the QSST is needed for the reduction of the altimetric measurements from the sea surface to the geoid. This is so because the basic measurements of satellite altimeters, the sea surface heights (SSHs), refer to the sea surface and not the geoid itself. Therefore, the reduction of these observations to the geoid is necessary to determine a geoid and not a MSS model. Additionally, shipborne gravity measurements refer to the sea surface as well and need to be free-air reduced to the geoid to be used for the determination of a gravimetric geoid in the well-known Helmert scheme (Vergos 2006). The quantity needed for this reduction is the QSST, which is the “marine” counterpart of orthometric heights. It can be easily concluded that the QSST is significant for the precise and accurate determination of gravity-field related quantities, while local models are highly necessary as well to serve local to regional geoid modelling.

These form the basis for the present work, i.e., to investigate whether a determination of the QSST from a geodetic point of view, i.e., using traditional geodetic methods and quantities, is possible. Studies on a *geodetic determination* of the QSST have begun since the work by Engelis (1983) who presented in a very elegant way their feasibility (OSU83 QSST model). Consequently, there have been more works on a global determination of the QSST in terms of surface spherical harmonics (SH) (Engelis, 1984; 1985; 1987), while Knudsen (1992) presented a local model for the North Sea. Finally, Lemoine et al. (1997) estimated a QSST model complete to degree and order 20 during the development of the EGM96, while Pavlis et al. (1998) used Proudman functions and data from the POCM-4 model to estimate the QSST to degree and order 20 and Andritsanos (2000) estimated QSST models and current velocities from an analysis of altimetric exact repeat mission data.

The area of the present study is the Eastern part of the Mediterranean Sea bounded between $33^o \le \varphi \le 38^o$ and $20^o \le \lambda \le 28^o$. This region was selected due to

a) the fact that it is a closed sea, thus global models are insufficient and
b) some well-known currents are present so they can provide a reasonable validation of the proposed method.

The determination is based on well-known geodetic algorithms and uses purely "geodetic" data, i.e. satellite altimetry geoid heights and shipborne gravity anomalies. For the estimation of the QSST, the simple formula connecting altimetric and gravimetric geoid heights, i.e., that their difference gives the QSST, was employed. With this as a starting point, the use of low pass filtering (LPF) with a Wiener-type of filter and a blunder detection test is proposed to filter the resulting QSST field and lead to a better approximation of the SST. This filtering operation is necessary to reduce high-frequency oceanic effects contaminating geodetic mission (GM) altimetry, while the blunder removal is needed to smooth the differences between the altimetric and shipborne gravity data, due to blunders in the latter.

2. Sea Surface Topography Modeling

For the determination of the QSST an altimetric and a gravimetric geoid model for the area were used. These models were developed by the authors (Vergos et al. 2004) in the frame of previous studies, aiming at the determination of high-resolution and high-accuracy marine geoid and mean sea surface models and the monitoring of sea level variations in the Mediterranean. The development of these models will be briefly discussed since the models themselves and the methodology followed are well documented in Vergos and Sideris (2003) and Vergos et al. (2003; 2004; 2007).

The altimetric geoid was estimated from a combination of ERS1 and GEOSAT GM data for the area under study. The well-known remove-compute-restore method was employed, while the EGM96 (Lemoine et al., 1998) global geopotential model was used as a reference surface. Finally, an altimetric geoid of 1′×1′ resolution in both latitude and longitude was determined for the area under study.

For the determination of the gravimetric geoid model, an effort was made to collect all available marine, land and airborne gravity data for the area under study. Then, an editing and blunder detection and removal process, using least squares collocation, took place to construct a homogeneous and accurate gravity database. Finally, a gravimetric geoid model was estimated using EGM96 as a reference surface and the 1D FFT spherical Stokes convolution to evaluate Stokes' function (Vergos et al., 2004). The statistics of the altimetric and gravimetric geoid models are summarized in Table 1.

***Table 1**. Statistics of the altimetric and gravimetric geoid models. Unit: [m].*

Model	**max**	**min**	**mean**	**σ**
$N^{gravimetric}$	39.813	0.780	21.185	±10.352
$N^{altimetric}$	40.206	1.057	21.376	±10.484

Employing the so-derived geoid models for the area under study, a preliminary quasi-stationary sea surface topography model for the area was estimated as

$$QSST = N^{alt} - N^{grav} \tag{1}$$

where, we suppose that the gravity anomalies used to determine the gravimetric geoid are free-air reduced, i.e., reduced from the sea surface to the geoid. The statistical characteristics of this preliminary QSST are given in Table 2, while the model itself is depicted in Figure 1. From Figure 1 (see solid circle) and Table 1 it is evident that the QSST estimated presents some unreasonably large variations within the area (3.3 m) and reaches a maximum of 2.2 m. Therefore it is clear that blunders are present in the estimated field. Finally, from Figure 1 some noisy features are evident (see the dotted circle), thus low-pass filtering (LPF) is needed to reduce these effects.

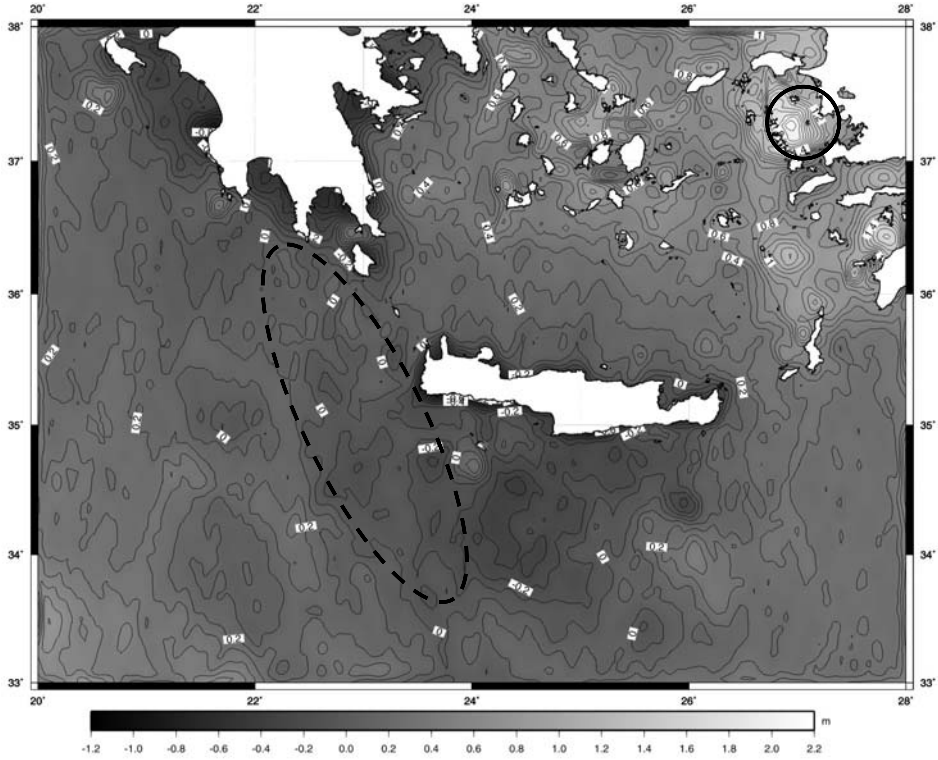

Figure 1: *The preliminary QSST model.*

For the detection and removal of blunders, a simple 3 rms test was performed, i.e., points with a QSST value larger than 3 times the rms of the preliminary field were removed. The statistics of the QSST model after this test are given in Table 2 as well. To low-pass filter the preliminary QSST model, a collocation-type of filter

(Wiener filtering) was used, assuming the presence of white noise in the QSST field that needs to be filtered (Schwarz et al., 1990). Furthermore, we assume that Kaula's rule for the decay of the geoid power spectrum holds, i.e., that the geoid heights PSD decays like k^{-4} where k is the radial wavenumber. Finally, we arrive at the filtering function shown in (2).

$$F(\omega)=\frac{\omega_c^4}{\omega^4+\omega_c^4} \tag{2}$$

***Table 2**. Statistics of the preliminary QSST model before and after the 3rms test. Unit: [m].*

	max	**min**	**mean**	**σ**
before	2.177	–1.112	0.224	±0.326
after	0.977	–0.958	0.190	±0.269

To filter the wanted field, the desired cut-off frequency needs to be selected. The latter relates to the final resolution of the filtered field and the reduction of the noise in the data. Thus, a trade-off is necessary, since higher resolution means more noise will pass the filter, while higher noise reduction means lower resolution of the final model. A high resolution is vital in the determination of regional to local QSST models, since if a high value cannot be achieved then a so-derived local model has little to offer compared to a global solution. It can be clearly seen, that the disadvantage of Wiener filtering is that the selection of the cut-off frequency in based on the spectral characteristics of the field only, while its spatial characteristics are not taken into account. Furthermore, the selection of the cut-off frequency is based on solely objective criteria (noise reduction). Thus, a trial and error process, based on maximum noise reduction with minimum signal loss, is needed to determine the desired cut-off frequency.

Various cut-off frequencies have been tested corresponding to wavelengths of 5, 10, 20, 40, 60, 100 and 120 km and finally we selected a wavelength of 100 km (about 1° or harmonic degree 180) since it offered the minimum signal loss with maximum noise reduction. Wavelengths shorter than 100 km left too much noise in the field, while those larger than 100 km were reducing not only the noise but the characteristics of the field as well. If we would select a longer wavelength, then, and if the area was significantly larger (e.g. the entire Mediterranean Sea) it would have been possible to identify larger in scale QSST features and distinguish them from smaller ones. The problem in this case is that shipborne gravity data in such high resolutions are not available for large regions.

The final QSST field after the filtering is shown in Figure 2, while the statistics are given in Table 3. From the aforementioned figure it can be seen that the noise

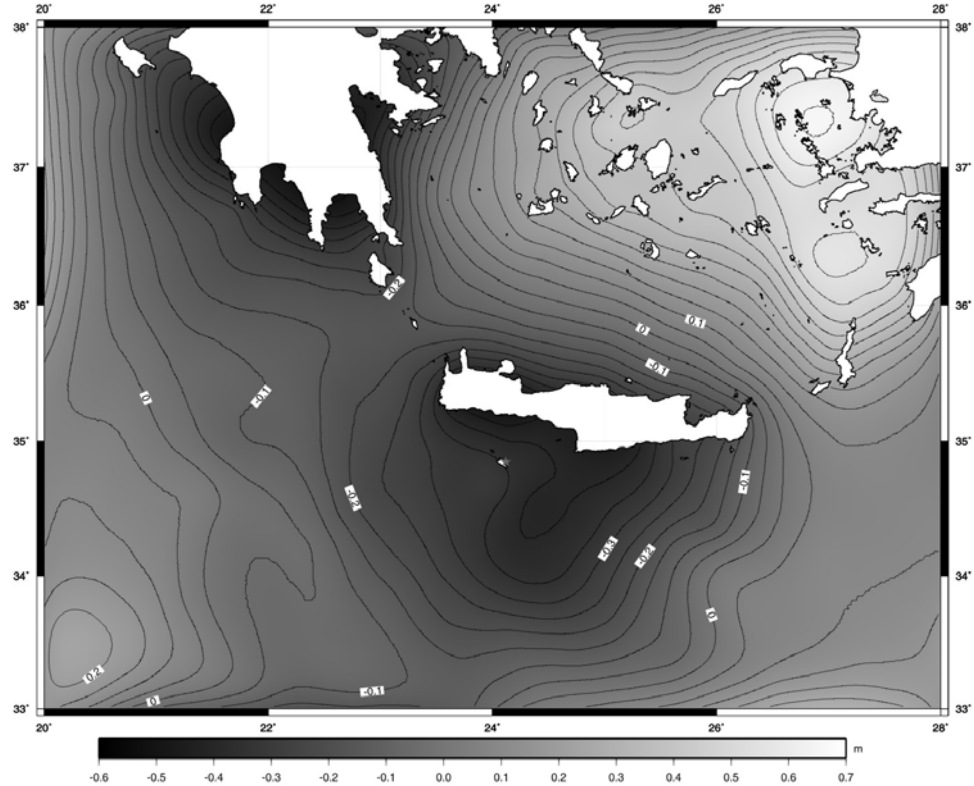

Figure 2: *The final quasi-stationary sea surface topography model.*

Table 3. *Statistics of the final QSST model. Unit: [m].*

	max	min	mean	σ
QSST	0.675	–0.510	0.014	±0.238
MDT-QSST	0.481	–0.293	0.000	±0.090

present in the preliminary model is reduced significantly, while blunders cannot be identified. For validation purposes, the estimated QSST model has been compared with a Mean Dynamic Topography model estimated for the entire Mediterranean Sea from an analysis of satellite altimetry and oceanographic data (Rio, 2004). The latter is given as a grid of mean QSST values of 3.75′×3.75′ resolution in both latitude and longitude. The statistics of the differences between the MDT and the estimated QSST models are given in Table 3 (last row). From the comparison it can be concluded that the two models agree very well to each other (standard deviation at the ±9 cm level only). The maximum and minimum values of the differences are found close to land areas only, where both models are inadequate. This comparison gives evidence that the estimated QSST model is at least in good agreement with existing regional oceanographic MDT models. Furthermore, it is a welcoming fact, which supports the appropriateness of the proposed methodology for the determination of a geodetic QSST model. This is important, since geodesy and oceanography are in need of the, more or less, same quantities, i.e., the geoid and the SST to support their studies, so the interaction between the two is vital especially in the frame of the recent gravity field dedicated mission of GOCE (Barzaghi et al., 2007. The methodology followed for the determination of the QSST, is given schematically in Figure 3.

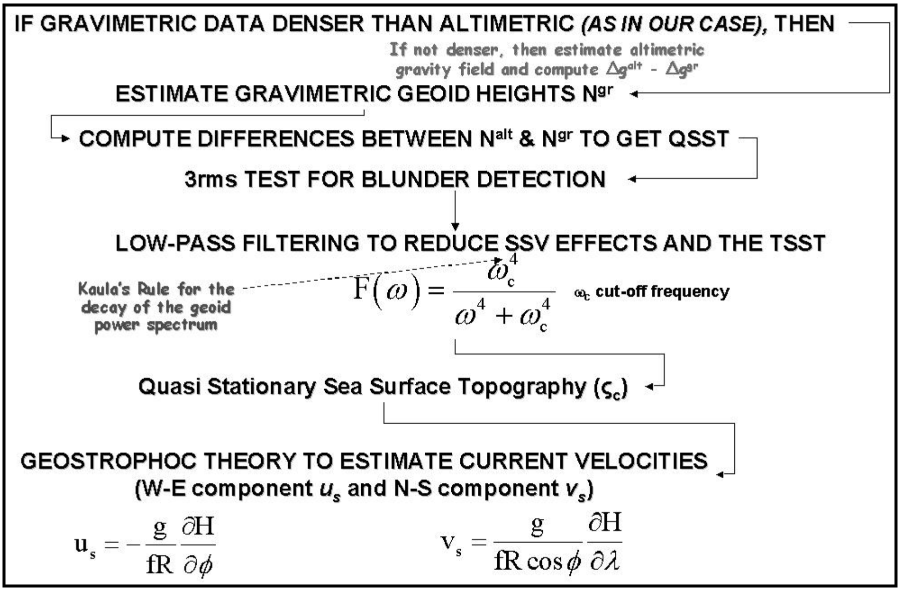

***Figure 3:** Sea surface topography modeling.*

3. Geostrophic Velocity Estimation

Following the determination of the QSST model for the area under study, the direction and velocities of the ocean currents can be estimated. That was achieved by following the theory of geostrophic flow, i.e., that the Coriolis force and the pressure gradient acting on the currents are in balance. This method is more related to oceanographic studies and products, but its main advantage is that it quickly gives velocity estimates and takes into account the properties of the ocean as a fluid. One of its disadvantages is that it diverges close to coastal areas, thus making the current estimates in such regions unreliable. The equations of geostrophic flow in spherical approximation, are given as (Pond and Pickard, 2000)

$$u_s = -\frac{g}{fR}\frac{\partial H}{\partial \phi} \tag{3}$$

$$v_s = \frac{g}{fR\cos\phi}\frac{\partial H}{\partial \lambda} \tag{4}$$

where u_s and v_s are the horizontal constituents of geostrophic flow, R is a mean earth radius (6371 km), φ and λ denote geographic latitude and longitude respectively, f is the Coriolis force and H the QSST previously estimated. Using (3) and (4), the north-south (u_s) and west-east (v_s) components of the currents geostrophic velocities have been estimated for the area under study. Table 4 summarizes the statistics of the estimated velocities and the total velocity field (last row), while Figure 4 depicts the direction and magnitude of the current velocities. From Fig. 4 we can clearly distinguish some well-known jets in the area like the Mid-Ionian (MIJ) and Mid-Mediterranean ones (Mid-MED Jet), the Cretan Cyclone (CC) and the Ierapetra Anticyclone (AC). Furthermore, South of the island of Crete we can identify a small (in terms of magnitude) jet (dotted lines), which can be either a branch of the Mid-Mediterranean one or a jet by its own. Finally, there is a clear flow from the Aegean Sea (jets J1, J2, and J3) which merge into the Cretan Cyclone and probable "feed" the MIJ. These give good evidence that the proposed methodology for the "geodetic" estimation of the QSST can provide good estimates of the current velocities as well, since most known features are identified.

Table 4. *The final geostrophic velocities for the area under study. Unit: [m/s].*

	max	**min**	**mean**	**σ**
u_s	1.298	–0.999	0.016	±0.292
v_s	1.292	–1.211	–0.005	±0.284
total field	1.307	0.000	0.347	±0.214

4. Conclusions

A method to determine the quasi-stationary sea surface topography from a purely geodetic point of view and using geodetic data has been presented. It is based on the simple relationship connecting altimetric and gravimetric geoid heights and a blunder removal and noise filtering procedure. From the results obtained we can conclude that the proposed methodology provides reasonably good results, since it gives small differences w.r.t. a MDT model derived from altimetry and oceanographic data. Furthermore, from the current velocities estimated, it was possible to identify known features of the circulation in the area under study like the Mid-Ionian and Mid-Mediterranean Jets, the Cretan Cyclone and the Ierapetra Anticyclone, which were clearly depicted.

Such a local QSST model is invaluable to geodetic studies for the reduction of altimetric sea surface heights from the sea surface to the geoid. Furthermore, it

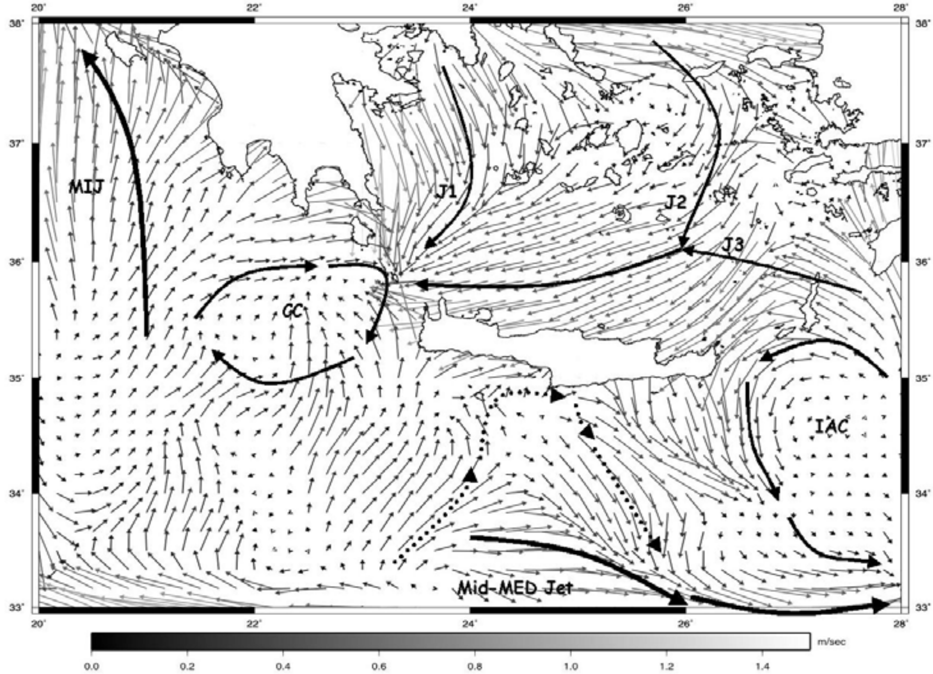

Figure 4: *Direction and magnitude of the geostrophic currents.*

provides a reference surface for oceanographic studies, where other measurements can be referred. The results of the present study offer an encouraging prospect for the synergy between geodesy and oceanography with respect to sea level monitoring, sea surface topography determination and marine geoid modeling.

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