

# Estimation of the time-varying dynamic ocean topography from the combined adjustment of geodetic and oceanographic data

**G.S. Vergos**

*Department of Geodesy and Surveying, Aristotle University of Thessaloniki,  
University Box 440, 541 24, Thessaloniki, Greece, Fax: +30 31 0995948,  
E-mail: vergos@topo.auth.gr*

## Abstract

The determination of the time-varying part of the sea surface topography is vital for both oceanographic and geodetic applications, since in the former it allows the determination of sea level anomalies, as deviations from a static mean sea level, while it is also fundamental for geoid determination when employing altimetric observations. The varying part of the SST has been given little attention, due to its small magnitude and random nature, since it can be treated as noise in altimetric SSHs and subsequently removed through a wiener type of low-pass filter. Of course, modeling the varying SST in such a way does not lead to rigorous results, since simple low-pass filtering can cause signal deterioration and loss, therefore, even though the influence of the time-varying SST will be removed, part of the SSH signal will be lost as well. On the other hand, the present day availability of both geodetic- and oceanographic-oriented models of the SST allow for their combined use in order to determine the varying part of the SST. This is the scope of the present work, where two SST models have been combined in a hybrid deterministic and stochastic adjustment in order to determine the time-varying part of the SST. The deterministic part represents models the systematic differences between the available mean dynamic ocean topography models, while the stochastic signal modeled provides a first outlook of the time-varying SST. Various parametric models are validated in order to model the deterministic differences between the geodetic and oceanographic SST, while an analytic covariance function for the varying part is determined for rigorous error propagation.

## 1. Introduction

From the early missions of GEOS-3 and SeaSat in the mid 70s to the recent ones of Jason-2 and ENVISAT, altimeters onboard satellites offer an unprecedented database of instantaneous measurements of the sea surface. The basic altimetric measurement refers to the satellite height above the non-static sea surface, determined as the two-way travel time needed for the radar pulse emitted from the satellite to reach the sea surface and received back by the altimeter. The difference between that height and the altitude of the satellite above a reference ellipsoid leads

to the determination of the instantaneous sea surface height (SSH) which successfully represents the geometric height of the non-static sea surface. This abundance of measurements for the Earth's oceans lead to an improved knowledge of the Earth's gravity field over oceanic regions and the monitoring of sea level variations over large time and spatial scales. Repeated satellite altimetry data span nowadays over a period of about 35 years, if one considers the exact repeat mission (ERM) of GEOSAT as a landmark and the latest missions of JASON-2 and ENVISAT. This record of measurements about the variations and mean level of the Earth's oceans, manage to provide reliable monitoring tools for time periods as short as ten days, useful for sea level anomaly determination, and long enough in order to provide a more-or-less reliable estimate of trends in mean sea level (MSL) rise.

One of the most vital quantities needed to determine marine geoid models from altimetric measurements in the sea surface topography, which is defined as the separation between the instantaneous sea level and the geoid. The SST can be decomposed in a quasi-stationary part known as the quasi-stationary SST (QSST) or mean dynamic ocean topography (DOT) and a time-varying part, which will be denoted in the sequel as time-varying SST (TSST). The DOT is defined as the semi-constant over large periods of time deviation between the time-averaged mean sea surface and the geoid. It reaches a maximum of +2.2 m and in closed sea areas has very small variations over large regions. For instance in the eastern part of the Mediterranean Sea the DOT has a variation of ~12-15 cm (Rio, 2004). The DOT is influenced mainly by ocean circulation and the salinity, temperature and pressure of the ocean water. Given its definition the DOT measures the long-term-averaged strength of ocean currents, i.e., the "steady-state" ocean circulation. On the other hand, its time-varying counterpart is defined as the deviation between the instantaneous and the time-averaged mean sea surface. The time-varying DOT is mainly attributed to atmospheric forcing, storm surges, un-modeled tidal effects and other varying-with-time influences to the marine environment (Pond and Pickard, 2000). It has a small magnitude and a random nature, so that in most cases until now it has been modeled by applying geophysical corrections to altimetric data such as the inverse barometric effect and tidal corrections, followed by a Wiener type of low-pass filtering of the available altimetric sea level anomalies in order to remove the high-frequency time-varying SST. Note, that in all physical geodesy related studies it is assumed that the geoid does not change with time, i.e., that it is a stationary signal at least for the duration of the study. Due to that assumption, we are only interested in the mean dynamic part of the SST and not the time-varying one. The later can be determined in a straight forward way from the analysis of the repeated tracks of altimetric satellites, which are on an exact repeat missions, so that it will be then removed from the observations. Another way to remove the influence of the time-varying SST is to consider it as noise of the altimetric observations and remove it during the crossover adjustment of the altimetric observations along with the bias and tilt parameters of the radial orbit error.

The determination of the DOT has been significant attention especially during the last two decades, so that purely geodetic, oceanographic and combined models have been determined. The geodetic approach refers to the combination of an altimetry derived MSS with a marine geoid model so that their separation is modeled as the DOT, while the oceanographic approach combines ocean water salinity, temperature and pressure data (Andersen and Knudsen, 2009; Barzaghi et al., 2008; Engelis, 1985, 1987; Knudsen, 1992; Knudsen et al., 2004, Rio, 2004; Rio and Hernandez, 2004; Vergos, 2006; Vergos et al., 2007). Contrary to its quasi-stationary counterpart, the time-varying SST has been given little attention in geodetic work, since it is a signal that needs to be removed in the utilization of altimetric SSHs for marine geoid and gravity field modeling. On the other hand, the availability of a marine geoid model from gravimetry and the abundance of satellite altimetry measurements of the instantaneous sea surface, allow the application of purely geodetic algorithms such as parametric least squares collocation in order to determine both the DOT and the time-varying SST. This is the scope of the present work, i.e., the utilization of a gravimetric geoid model and altimetric SSHs in a hybrid deterministic and stochastic combination scheme in order to determine a model of the time-varying SST in the area under study.

## 2. Observation equations for time-varying DOT modeling

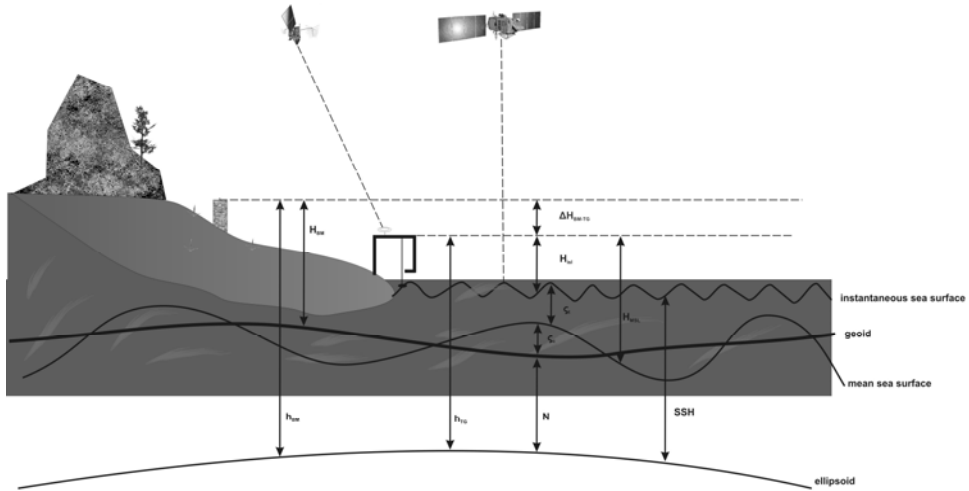
In order to determine a model of the time-varying DOT and layout the necessary observation equations we should consider the case presented in Figure 1. Within this scenario we have available a gravimetric geoid model  $N$  and altimetric SSHs from single- or multi-satellite missions and we need to determine the time-varying DOT which is denoted with  $\zeta_t$  in Figure 1. Note that  $\zeta_c$  in the same Figure represents the mean dynamic ocean topography, i.e., the stationary separation between the MSS and the geoid. From Figure 1 we can readily write the equation that connects geoid heights, SSHs and the sea surface topography as:

$$SSH = N + \zeta, \quad (1)$$

where  $SSH$  denotes the instantaneous geometric height of the sea and  $SST$  the sea surface topography, which can be decomposed in its mean dynamic  $\zeta_c$  and time-varying constituents  $\zeta_t$  as:

$$\zeta = \zeta_c + \zeta_t. \quad (2)$$

Eq. (1) resembles the one connecting geometric and geoid heights with Helmert orthometric heights on land. Note that for Eq. (1) to hold, all geophysical and instrumental corrections need to be applied to the altimetric SSHs. In such cases the SST in marine areas is analogous to the orthometric height in continental regions and due to that similarity it is called topography of the sea.



**Figure 1:** Definition of the fundamental constituents for the determination of the time-varying DOT.

From Eqs. (1) and (2), it becomes evident that given the availability of a gravimetric geoid model, altimetric SSHs and an oceanographic model of the mean DOT, one can estimate the time-varying SST through the combined adjustment of the available observations. In the present study, a gravimetric geoid model for the southern Aegean Sea was available (Vergos et al., 2005) along with altimetric observations from the GEOSAT, ERS1/2, TOPEX/Poseidon and JASON-1 satellites (Vergos et al., 2007). These form the basis of a so-called geodetic model of the mean DOT, which is combined with an oceanographic model of the DOT available from Rio and Hernandez (2004). Based on Eqs. (1) and (2) we can write the observation equation for the combined adjustment of the gravimetric geoid model and the altimetric SSHs as:

$$\mathbf{b}_i = (SSH_i - N_i) - \zeta_i^c \text{ ocean} = \zeta_i^c \text{ geod} - \zeta_i^c \text{ ocean} = (\mathbf{a}_i^T \mathbf{x} + s_i) + v_i = \mathbf{a}_i^T \mathbf{x} + \mathbf{e}_i, \quad (4)$$

where  $\mathbf{b}$  is the observation vector, i.e., the deviation between the dynamic sea surface and the geoid,  $\mathbf{a}_i^T \mathbf{x}$  denotes the deterministic parameters to be estimated,  $s_i$  is the stochastic signal to be determined and  $v_i$  stand for the observation errors. Note that Eq. (4) outlines a hybrid deterministic and stochastic combination scheme where it is possible to estimate both the mean DOT as well as the time-varying SST. This mixed adjustment allows the estimation of parameters  $x_i$ , depending on the parametric model chosen, in order to remove any datum-like differences from the observations as well as the estimation of some stochastic signal  $\mathbf{s}$  that still remains in the reduced observations  $SSH_i - N_i - \zeta_i^c \text{ ocean}$ , which in our case is the time-varying SST. According to Kotsakis and Sideris (1999) the stochastic part of the signal is incorporated from the beginning in the observation equations so that it

takes the following form in matrix notation:

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{s} + \mathbf{K}\mathbf{v}. \quad (5)$$

In Eq. (5) matrix  $\mathbf{K}$  is an identity matrix which in the present problem of combining altimetric SSHs and gravimetric geoid heights ( $\text{SSH}_i - N_i - \zeta_i^{\text{c ocean}} - \zeta_i^{\text{t}} = 0$ ) takes the form  $\mathbf{K} = [\mathbf{I}_n \quad -\mathbf{I}_n \quad -\mathbf{I}_n \quad -\mathbf{I}_n]$  where  $n$  is the number of observations. Within this scheme, the minimization principle becomes:

$$\mathbf{s}^T \mathbf{Q}_s^{-1} \mathbf{s} + \mathbf{v}_{\text{SSH}}^T \mathbf{Q}_{\text{SSH}}^{-1} \mathbf{v}_{\text{SSH}} + \mathbf{v}_N^T \mathbf{Q}_N^{-1} \mathbf{v}_N + \mathbf{v}_{\zeta^{\text{c ocean}}}^T \mathbf{Q}_{\zeta^{\text{c ocean}}}^{-1} \mathbf{v}_{\zeta^{\text{c ocean}}} = \min, \quad (6)$$

where  $\mathbf{Q}_s^{-1}$  is an appropriate weight matrix for the unknown signal. The solution of the system of normal equations is the same as in the case of the well-known least square adjustment with observation equations, so that:

$$\mathbf{P} = \left( \mathbf{Q}_{\text{SSH}} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^{\text{c ocean}}} + \mathbf{Q}_s \right)^{-1}, \quad (7)$$

$$\hat{\mathbf{x}} = \left( \mathbf{A}^T \mathbf{P} \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{P} \mathbf{b}, \quad (8)$$

$$\hat{\mathbf{v}} = \mathbf{b} - \mathbf{A} \hat{\mathbf{x}}. \quad (9)$$

Note, that we assume that either there are no errors in the estimates of the signal  $\hat{\mathbf{s}}$  or that they do exist but their magnitude is negligible. Moreover, it is assumed that the observation errors are contained entirely in the estimation errors. In the aforementioned Eqs. the error matrix  $\mathbf{Q}_s$  of the signal to be predicted is unknown so in the first step of the adjustment the identity matrix is selected. According to Kotsakis and Sideris (1999) this initial selection for the signal error matrix can be considered as the smoother one which fits best to the available observations  $\mathbf{b}$ , the selected parametric model  $\mathbf{a}_i^T \mathbf{x}$  and the stochastic model that has been selected for the random error of the observations ( $\mathbf{Q}_{\text{SSH}}, \mathbf{Q}_N, \mathbf{Q}_{\zeta^{\text{c ocean}}}$ ). The first estimate for the unknown signal  $\mathbf{s}$  can then be determined as:

$$\mathbf{W} = \mathbf{I}_n - \mathbf{A} \left( \mathbf{A}^T \left( \mathbf{Q}_{\text{SSH}} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^{\text{c ocean}}} + \mathbf{I}_n \right)^{-1} \mathbf{A} \right)^{-1} \times \quad (10)$$

$$\times \mathbf{A}^T \left( \mathbf{Q}_{\text{SSH}} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^{\text{c ocean}}} + \mathbf{I}_n \right)^{-1}$$

and

$$\hat{\mathbf{s}}_{\text{init}} = \left( \mathbf{Q}_{\text{SSH}} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^{\text{c ocean}}} + \mathbf{I}_n \right)^{-1} \mathbf{W} \mathbf{b}. \quad (11)$$

Note that in this initial solution of our adjustment problem, the unknown parameters of the deterministic model are estimated according to Eq. 8 together with

a first estimate of the unknown signal  $\hat{\mathbf{s}}_{init}$  according to Eq. 11. In the next step we need to estimate the general trend in the unknown signal  $\mathbf{s}$  through the adjustment of a smooth corrector surface in the initial signal estimates  $\hat{\mathbf{s}}_{init}$ . After the estimation of the differences  $\hat{\mathbf{m}}_s$ , which result from the fit of the corrector surface on the observations, we can construct the reduced vector of observations and the reduced vector of the unknown signal as:

$$\mathbf{b}_r = \mathbf{b} - \hat{\mathbf{m}}_s, \quad (12)$$

and

$$\mathbf{s}_r = \mathbf{s} - \hat{\mathbf{m}}_s. \quad (13)$$

It should be noted that this reduction of both the observations and the signals with a smooth corrector surface is needed in order to remove any biases between the observations so that we can safely assume that the reduced signals to be predicted have a zero mean value ( $E\{\mathbf{s}_r\} = 0$ ). In this way, we can also estimate and empirical covariance function for the reduced signals, which will describe their statistical characteristics and will be used for the estimation of the signal error matrix  $\mathbf{Q}_{s_r}$ . Then, using this updated and improved description of the stochastic model, more rigorous predictions for the signal estimates can be performed. The final solution of the system is now given as:

$$\mathbf{W} = \mathbf{I}_n - \mathbf{A} \left( \mathbf{A}^T \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1} \mathbf{A} \right)^{-1} \times \\ \times \mathbf{A}^T \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1}, \quad (14)$$

$$\hat{\mathbf{x}} = \left( \mathbf{A}^T \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1} \mathbf{A} \right)^{-1} \times \\ \times \mathbf{A}^T \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1} \mathbf{b}_r, \quad (15)$$

$$\hat{\mathbf{s}}_r = \mathbf{Q}_{s_r} \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1} \mathbf{W} \mathbf{b}_r, \quad (16)$$

while we can also estimate the individual errors of the observations:

$$\hat{\mathbf{v}}_{SSH} = -\mathbf{Q}_{SSH} \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1} \mathbf{W} \mathbf{b}_r, \quad (17)$$

$$\hat{\mathbf{v}}_N = -\mathbf{Q}_N \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1} \mathbf{W} \mathbf{b}_r, \quad (18)$$

$$\hat{\mathbf{v}}_{\zeta^c \text{ ocean}} = -\mathbf{Q}_{\zeta^c \text{ ocean}} \left( \mathbf{Q}_{SSH} + \mathbf{Q}_N + \mathbf{Q}_{\zeta^c \text{ ocean}} + \mathbf{Q}_{s_r} \right)^{-1} \mathbf{W} \mathbf{b}_r. \quad (19)$$

### 3. Determination of the time-varying DOT

As it has already been mentioned, the determination of the time-varying DOT was based on available altimetric SSHs, a gravimetric geoid model and an oceanographic mean DOT model for the south Aegean Sea. Table 1 summarizes the statistics of the so-called geodetic DOT model ( $SSH_t - N_t$ ) and those of the oceanographic one, as well their differences that were used as observations in the adjustment. The last row of Table 1 were the input data that have been used, along with an initial signal covariance matrix  $\mathbf{Q}_s$  equal to the identity matrix, so that the initial estimates of the unknown signals  $\hat{s}_{init}$  have been predicted. For the deterministic parameters, various models have been tested in order to investigate which one provides the best fit to the available observations. Therefore, four- and five- parameter similarity transformation models have been tested along with zero, first, second and third-order polynomial ones. The analytic description of the design matrix  $\mathbf{A}$  depends on the selection of the parametric model and is not given here since it is well documented in the literature (Fotopoulos, 2003; Heiskanen and Moritz, 1967; Kotsakis and Sideris, 1999; Vergos, 2006). Given the selection of a parametric model, the reduced signals and observations are determined after removing the differences  $\hat{m}_s$ , which result from the fit of the deterministic model.

**Table 1:** Statistics of the altimetric and gravimetric geoid models. Unit: [m].

	max	min	mean	rms	std
$\zeta^{c \text{ geod}}$	0.675	-0.510	0.014	$\pm 0.238$	$\pm 0.238$
$\zeta^{c \text{ ocean}}$	0.096	-0.176	-0.040	$\pm 0.066$	$\pm 0.053$
$\zeta^{c \text{ geod}} - \zeta^{c \text{ ocean}}$	0.478	-0.635	-0.058	$\pm 0.208$	$\pm 0.200$

Table 2 presents the statistics of the corrector surfaces for some of the parametric models tested, where  $\mathbf{A}$  denotes the 3<sup>rd</sup> order polynomial model and  $\mathbf{B}$ ,  $\mathbf{C}$  denote the four- and five-parameter similarity transformation ones respectively. Note that the lower order polynomial models are not listed, since their results were inferior compared to the aforementioned ones and they provided larger prediction errors. The first part of Table 2 presents the statistics of the corrector surfaces, while the second part gives some statistical measures of the system of normal equations (condition number, adjusted and simple coefficient of determination), which represent the goodness of fit and the stability of the achieved solution (Fotopoulos, 2003; Vergos, 2006). From Table 2 it becomes evident that the 3<sup>rd</sup> order polynomial model (model  $\mathbf{A}$  in that Table) provides the smaller condition number, i.e., a more stable solution, along with the larger adjusted and simple coefficient of determination i.e., a better fit, compared to the similarity transformation models. Note that the condition numbers for the similarity transformation models are four orders

**Table 2:** Statistics of the corrector surface values for the estimation of the time-varying SST and system condition number, adjusted and simple coefficients of determination. Unit: [m].

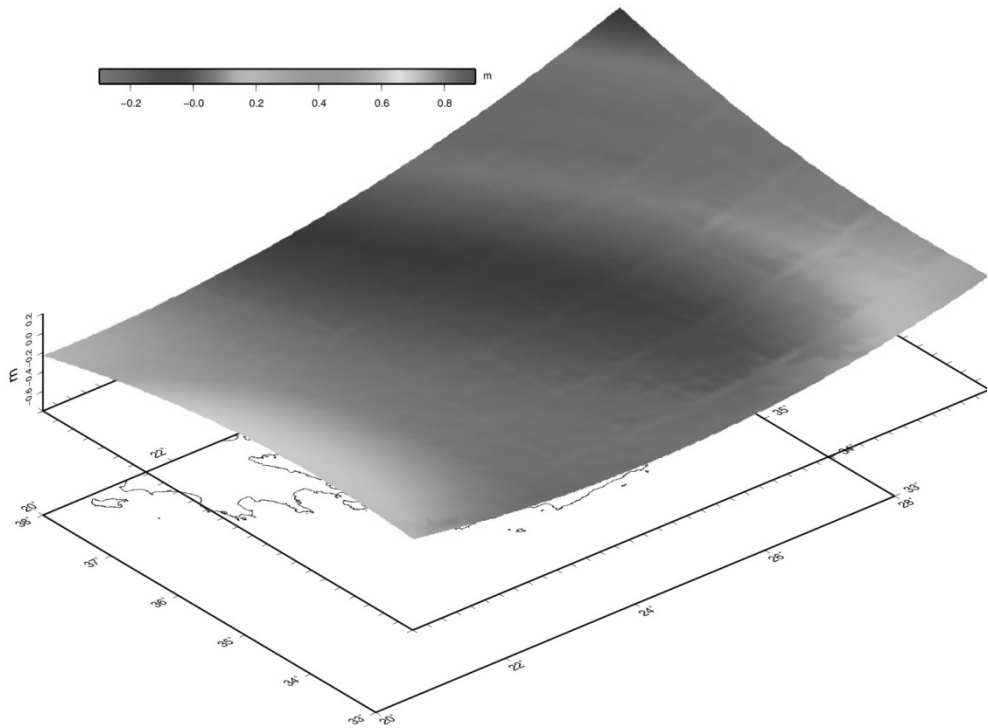
	max	min	mean	rms	std
<b>A (trend)</b>	0.882	-0.232	0.044	±0.201	±0.196
<b>B (trend)</b>	0.607	-0.167	0.044	±0.175	±0.169
<b>C (trend)</b>	0.577	-0.149	0.044	±0.181	±0.175
	A		B		C
<b>R<sup>2</sup></b>	0.66		0.49		0.52
<b>R<sub>a</sub><sup>2</sup></b>	0.65		0.48		0.51
<b>con</b>	1.3111×10 <sup>3</sup>		1.16×10 <sup>7</sup>		3.63×10 <sup>7</sup>

of magnitude larger than those of the parametric model, which signals that these models are less stable in the solution that they provide. Moreover, from the prediction errors estimated for the three parametric models, the 3<sup>rd</sup> order polynomial one provides a standard deviation of ±4.5 cm compared to ±6.2 cm ±6.9 cm for the five- and four- parameter models respectively. Therefore, it was concluded that the 3<sup>rd</sup> order polynomial model will be the one used to reduce the observations and the signal and model the systematic differences between the altimetric SSHs, the gravimetric geoid heights and the oceanographic DOT model. Figure 2 depicts the corrector surface estimated from the 3<sup>rd</sup> order polynomial model.

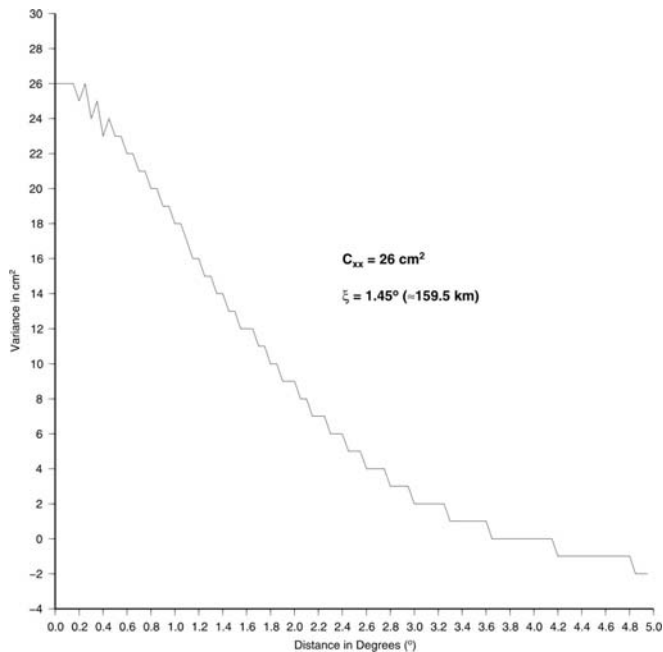
After the reduction of the observations and of the signal has been performed according to Eqs. (12) and (13), the empirical covariance function of the reduced time-varying SST signal has been estimated. Figure 3 presents the time-varying SST empirical covariance function, which was then used to compute the more reliable, compared to the identity matrix, variance-covariance matrix of the reduced signal. From the empirical covariance function it can be seen that the reduced signal has a correlation length of ~160 km with a variance of 26 cm<sup>2</sup> only.

With this information available, the final estimation for the time-varying SST signal has been carried out for the area under study. Table 3 presents the statistics of the estimated time-varying sea surface topography model, which is also depicted in Figure 4. From Figure 4 it becomes evident that the estimated time-varying SST values are realistic, i.e., they do not show extreme maxima and minima, while their mean value is zero. Note that the standard deviation of the predicted signal is at the ±2.9 cm level, showing a significant variation over Cyclades, where the respective anti-cyclone is located, in northwest Crete over the Western Cretan gyre and in southeast Crete over the Cretan anticyclone. Unfortunately, no other model of the time-varying SST for the area is available in order to do some comparisons for the external validation of the model estimated. Such a solution could be available if dedicated oceanographic measurements have been performed from whose analysis





**Figure 2:** The estimated corrector surface from the fit of the 3<sup>rd</sup> order polynomial model.

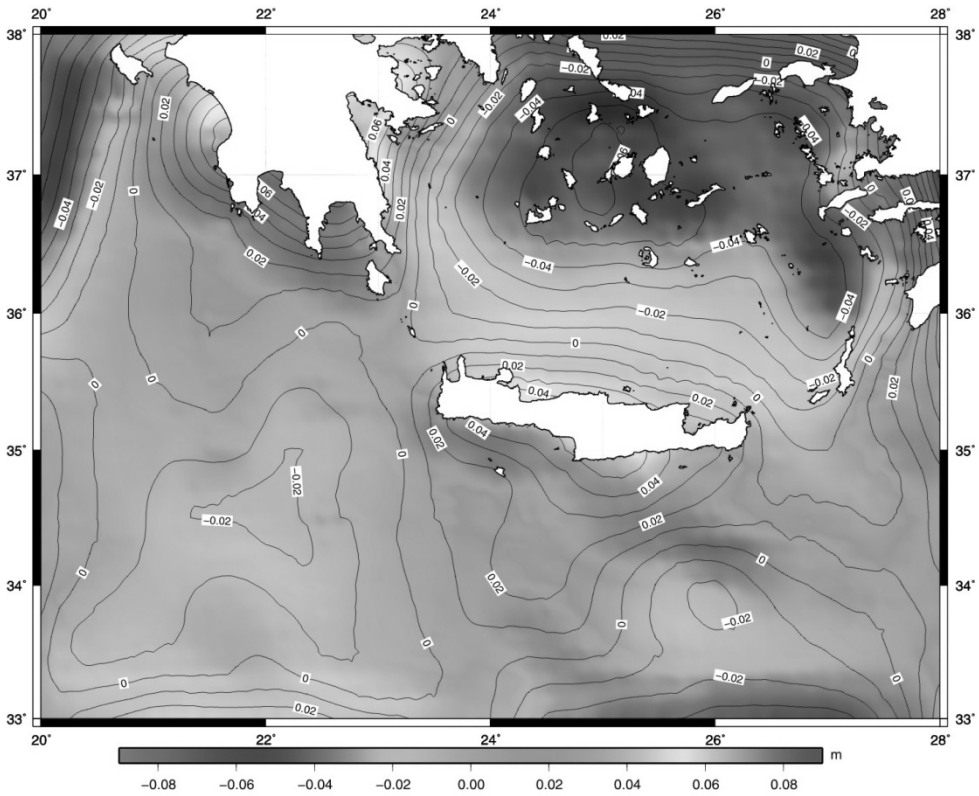


**Figure 3:** The estimated empirical covariance function of the time-varying DOT.

**Table 3:** Statistics of the final time-varying sea surface topography model for the southern Aegean Sea. Unit: [m].

	max	min	mean	rms	std
$\zeta^t \text{ adj}$	0.085	-0.086	0.000	$\pm 0.029$	$\pm 0.029$

a reliable model of the time-varying SST would be derived. The present solution for the time-varying SST could form the basis for respective studies and a validation model for other ones that will be determined either from oceanographic or GRACE-type of data.



**Figure 4:** The final time-varying dynamic ocean topography model for the southern part of the Aegean Sea.

## 4. Conclusions

A detailed scheme for the estimation of the time-varying component of the sea surface topography has been presented, relying on the combined adjustment of altimetric sea surface heights, gravimetric geoid heights and data about the mean dynamic ocean topography. The combination scheme is based on a hybrid deterministic and stochastic approach, where the deterministic part aims at the estimation of a corrector surface which will be used to reduce systematic difference from the observations, so that the signal to be predicted will have the characteristics of a stochastic random variable. Various parametric models have been tested to represent the systematic differences between the available observations, and it was concluded that a 3<sup>rd</sup> order polynomial model manages to provide a stable solution (small condition number), the best fit to the residuals (large simple and adjusted coefficient of determination) and the smaller prediction error, compared to the lower order polynomial and the four- and five-parameter transformation models. The final model estimated gives a first look of the time-varying sea surface topography in the southern Aegean Sea, presenting small variations, a standard deviation of the predicted signal at the  $\pm 2.9$  cm level, and significant spatial variations over the main currents of the area, i.e., over the Cyclades anti-cyclone, the Western Cretan gyre and the Cretan anticyclone. Note that currents with large spatial scale like the Mid-Mediterranean Jet do not appear in the time-varying SST model estimated, since they contribute to its quasi-stationary counterpart. With the availability of data from the GRACE and GOCE satellites, representing with high-accuracy the time-variable and static geoid respectively, and in combination with the available and future satellite altimetry data, better models for the time-varying SST will be estimated with enhanced accuracy and resolution.

## References

- Andersen, O. and Knudsen, P., 2009. *DNESC08 mean sea surface and mean dynamic topography models*. J Geophys. Res., 114, C11001, doi:10.1029/2008JC005179.
- Barzaghi, R., Tselfes, N., Tziavos, I.N. and Vergos, G.S., 2008. *Geoid and High Resolution Sea Surface Topography Modelling in the Mediterranean from Gravimetry, Altimetry and GOCE Data: Evaluation by Simulation*. J of Geod., 83(8), doi: 10.1007/s00190-008-0292-z.
- Engelis, T., 1985. *Global circulation from Seasat altimeter data*. Mar. Geod., 9(1): 45-69.
- Engelis, T., 1987. *Spherical harmonics expansion of the Levitus sea surface topography*. Report No. 385, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.
- Fotopoulos G., 2003. *An analysis on the optimal combination of geoid, orthometric and ellipsoidal height data*. UCGE Report Nr. 20185, University of Calgary, Department of Geomatics Engineering, Calgary AB, Canada.

- Heiskanen W.A. and Moritz H., 1967. *Physical Geodesy*. Freeman, San Francisco.
- Knudsen, P., 1992. *Estimation of sea surface topography in the Norwegian sea using gravimetry and Geosat altimetry*. *Bul. Géod.*, 66: 27-40.
- Knudsen, P., Vest, A.L. and Andersen, O., 2004. *Evaluating mean dynamic topography models within the GOCINA project*. In: Proceedings of the 2004 Envisat & ERS Symposium, 6–10 September 2004, Salzburg, Austria (ESA SP– 572, April 2005).
- Kotsakis C. and Sideris M.G., 1999. *On the adjustment of combined GPS/levelling/geoid networks*. *J. Geod.*, **73**(8), 412-421, doi: 10.1007/s00190-005-026-1.
- Pond, S. and Pickard, G., 2000. *Introductory dynamical oceanography*. Butterworth – Heinemann, 2<sup>nd</sup> edition, 329 pp.
- Rio, M.-H., 2004. *A Mean Dynamic Topography of the Mediterranean Sea Estimated from the Combined use of Altimetry, In-Situ Measurements and a General Circulation Model*. *Geoph. Res. Let.*, 6: 03626.
- Rio, M.-H. and Hernandez, F., (2004). *A mean dynamic topography computed over the world ocean from altimetry, in situ measurements, and a geoid model*. *J. Geophys. Res.*, 109, C12032, doi:10.1029/2003JC002226.
- Vergos, G.S., 2006. *Study of the Earth's Gravity Field and Sea Surface Topography in Greece by combining surface data and data from the new satellite missions of CHAMP and GRACE*. PhD Dissertation, School of Rural and Surveying Engineering, Faculty of Engineering, The Aristotle University of Thessaloniki, Department of Geodesy and Surveying, 242 pp.
- Vergos, G.S., Grigoriadis, V., Tziavos, I.N. and Sideris, M.G., 2007. *Combination of multi-satellite altimetry data with CHAMP and GRACE EGMs for geoid and sea surface topography determination*. In: P. Tregoning and C. Rizos (Editors) *Dynamic Planet 2005 - Monitoring and Understanding a Dynamic Planet with Geodetic and Oceanographic Tools*, International Association of Geodesy Symposia, vol.130 Springer – Verlag Berlin Heidelberg, pp. 244-250.
- Vergos G.S., Tziavos I.N. and Andritsanos V.D., 2005. *On the Determination of Marine Geoid Models by Least-Squares Collocation and Spectral Methods Using Heterogeneous Data*. In: F. Sansó (Ed.), *A Window on the Future of Geodesy*, International Association of Geodesy Symposia, **128**, Springer, Berlin Heidelberg, 332-337.