On the Determination of Marine Geoid Models by Least-Squares Collocation and Spectral Methods Using Heterogeneous Data

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Abstract. In the frame of the EU-sponsored GAVDOS project the need of a new high-resolution and high-accuracy geoid model for the calibration of altimeters onboard satellites like JASON-1, ENVISAT and EURO-GLOSS and for sea level monitoring purposes has become apparent. That was mainly due to the fact that the already available models have been estimated using outdated datasets and fail to meet the wanted, cm-level, accuracy requirements. To determine the new geoid models multi-satellite (ERS1 and GEOSAT) altimetry and land and marine gravity data have been used. The EGM96 global geopotential model has been employed, while the effect of the bathymetry has been taken into account using recently developed local Digital Depth Models (DDMs). Several solutions have been estimated based on the different datasets used and the two main methodologies followed, i.e., the Fast Fourier Transform (FFT) based Input Output System Theory (IOST) and Least Squares Collocation (LSC). The accuracy of the new models was assessed through comparisons with TOPEX/POSEIDON (T/P) data and the GEOMED geoid solution for the area under study. Finally, the consistency between the estimated solutions has been determined by comparing the geoid height value they provide at the Gavdos Tide Gauge (TG) station on the isle of Gavdos. From the results it was found that the precision of the new geoid models is between ±0.9 and ±3.3 cm, their accuracy ranges between ±5 and ±10 cm and their consistency is at the ±0.5 – 6 cm level.

Keywords. Marine geoid modeling, least-squares collocation, IOST.

1 Introduction

The geoid serves as a reference surface for a wide number of Earth sciences and for various applications. Such are the calibration of altimeters onboard satellites, the determination and monitoring of the mean sea level and its variability and the deformation of the tectonic field, among others. For such applications to be successful a high-accuracy and high-resolution geoid model needs to be determined. Thus, in the frame of the EU-sponsored GAVDOS project, our group is participating with the aim of, among others, determining such a geoid model for the wider area of Gavdos and Crete. These two islands are located in the southern Aegean Sea, Greece, in an area with highly-variable gravity field features, due to the existence of the Hellenic Arc and the submergence of the African plate under the Euro-Asian one. Furthermore, the established calibration facility on the isle of Gavdos is unique since: a) it is under a crossing point of the T/P and JASON-1 tracks, b) it is adjacent to an ERS1/2 and ENVISAT pass, c) it is far from the mainland, and d) the sea circulation and the local tides in the area are relatively small. These facts make the isle of Gavdos ideal for calibrating altimetric satellites, sea level monitoring and tectonics studies, as long as a rigorous in terms of resolution and accuracy geoid model is determined. The first geoid model for the area was developed in the frame of the GEOMED project (1993) and further studies were conducted by Andritsanos et al. (2001b) and Vergos (2002).

In this paper some newly developed geoid models are presented and validated to asses their accuracy and precession. Thus, purely altimetric and gravimetric geoid models are determined, while two combined solutions are estimated as well using both traditional least-squares collocation and the FFT-based IOST method for the optimal combination of heterogeneous data. Furthermore, a rigorous processing methodology is followed, during which the effect of bathymetry and that of the quasi-stationary sea surface topography (QSST) are taken into account in the frame of the well-known remove-compute-restore method. Such effects have a significant influence on the accuracy of marine geoid models and need to be taken into account when cm-level accuracy is sought (Vergos and Sideris 2003a). The effect of the bathymetry needs to be removed for the residual geoid heights or gravity anomalies to be smooth before gridding or prediction. According to Forsberg (1984), when high-quality depths are available, then the smoothing of the data can reach 50%. In marine geoid modeling a residual terrain model (RTM) reduction is used to account for the bathymetric effects (Forsberg 1984).

The effect of the QSST is important in processing altimetry data since the sea surface heights (SSHs) available from the satellites do not refer to the geoid but to the sea surface, thus their processing will determine a very good model of the mean sea surface but not the geoid itself. Thus, it is important to correct the altimetry SSHs due to the presence of the nearly-stationary part of the SST by simply removing its contribution.

The improvement that the newly estimated geoid models offer, compared to the previous ones developed for the Eastern Mediterranean Sea, is due to a) the use of more accurate satellite and terrestrial data, b) the higher resolution that can be achieved by combining different data sources, and c) the combination of heterogeneous data in the spectral domain with I/O algorithms.

2 Geoid Determination Methodology

Five geoid models, i.e., two purely altimetric, a gravimetric and two combined ones, have been estimated for the area under study. The development of the altimetric and gravimetric models as well as that of the two combined ones was based on the well-known remove-compute-restore method. The processing procedures for each case are well documented (see, e.g., Tziavos et al. 1998) so they will be briefly summarized here to place more emphasis on the numerical results.

2.1 Altimetric geoid modeling

The determination of a purely altimetric geoid model has been discussed in detail in Vergos and Sideris (2003a) and begins by using raw SSHs which have to be corrected for the various geophysical effects influencing the satellite measurements as well as the instrumental errors affecting the altimeters. This step results in the construction of Corrected SSHs (CSSHs) for each satellite mission. Since for some (e.g., GEOSAT), observations refer not only to oceanic but also land and shallow-water regions, a bathymetric mask has to be applied to remove the two latter. A depth limit can be set arbitrarily so measurements corresponding to depths smaller than that will be rejected. In the authors' opinion, the selection of this depth value is area-dependant and its smallness does not play a significant role, since if it is too small then there will remain erroneous observations in the data, most of which can be removed with a simple 3 rms test at a later step.

The so-derived SSHs refer to oceanic regions only and have to be reduced from the mean sea surface to the geoid by removing the contribution of the QSST, i.e., the nearly time independent part of the SST. Global SST models, which are based on altimetry and/or oceanographic data, or local solutions which employ in-situ measurements of salinity, pressure, temperature, etc, can be used to derive the QSST information. Once the SSHs are reduced to the geoid they can be regarded as geoid heights and are thus processed to give the final altimetric geoid model using the remove-compute-restore method. Thus, the contribution of a geopotential model is removed to derive reduced SSHs. However, these measurements still contain the influence of time varying oceanic effects and the radial orbit error. Such effects can be removed or reduced with crossover adjustment. Since the present study had a local character, a regional crossover adjustment scheme with one bias and one tilt parameter was applied (Rummel 1993). A local crossover adjustment can reduce not only the radial orbit error, mostly a bias in the data, but also some time-varying effects which are part of the influence of the sea state to the altimeter measurements.

To remove any remaining erroneous observations from the crossover adjusted residual SSHs, a simple 3 rms test was used, which was regarded to be sufficient for such purposes. The application of a 3 rms test assumes that the remaining errors in the data have a random nature, i.e., there are no biases left. This will be decided by examining the mean value of the reduced SSHs. If the mean value is small enough, e.g., below 0.10 m, the 3 rms test is performed and then the altimetry SSHs are RTM-reduced to derive the final residual SSHs. On the other hand, if the mean value is higher, then it means that some biases are still present in the data (even after the crossover adjustment) and should be removed prior to the 3 rms test. This can be done by first RTM-reducing the SSHs, since it is expected that the reduction will smooth the data (Forsberg 1984). This smoothness can be viewed by examining not only the mean value but the standard deviation (σ) of the reduced field too. The bathymetric effects should be accounted for with caution, since as shown by Vergos and Sideris (2003 a, b) if an inaccurate depth model is used then an error of about 2-6 cm is introduced in the final altimetric geoid. The residual SSHs available at this point were derived as

$$N_{res} = N_{obs} - N^{GM} - QSST - N^{RTM} \quad (1)$$

where N_{obs} denotes the observed SSHs, N^{GM} is the contribution of the geopotential model and N^{RTM} is the effect of the bathymetry. The so derived SSH_{res} represent the medium wavelengths of the geoid height signal and can be safely regarded as residual geoid heights N_{res}.

After that step, the final residual geoid height estimates can be derived by first gridding the data. This was performed for all solutions by LSC. The necessary variance and correlation length needed by the method were determined by computing the data empirical covariance function. The final step to determine the altimetric geoid model is to restore the contribution of the geopotential model and that of the bathymetry. The procedure described in this section is given schematically in Fig 1.

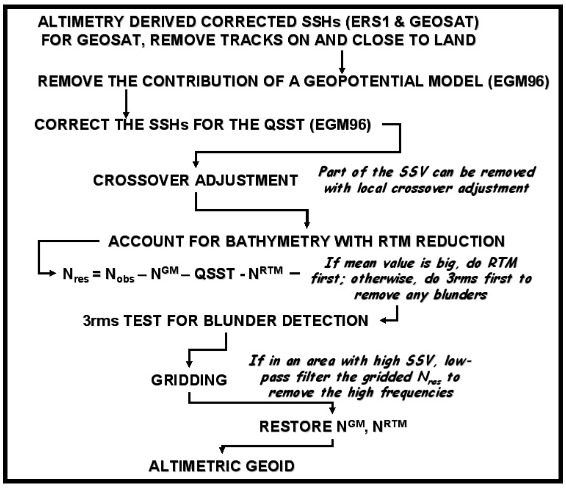

Fig. 1: Altimetric geoid modeling.

2.2 Gravimetric and combined geoid modeling

The determination of the gravimetric and combined geoid models was based on the processing scheme described in Sec. 2.1 and Fig. 1. The only exception was in the way the bathymetric information was handled in the former. In the gravimetric geoid determination the bathymetric reduction is applied in the remove step to smooth the residual gravity field, then the residual gravity anomalies are gridded and the contribution of the bathymetry is restored prior to the prediction of geoid heights. This is necessary since the bathymetry refers to masses below the geoid, which have to be restored before the use of Stokes's formula for geoid prediction. In the present study the prediction of the gravimetric geoid heights was carried out using the 1D-FFT method (Haagmans et al. 1993) and employing discrete spectra to evaluate Stokes' kernel function. No integration cap or kernel modification were used.

As far as the combined geoid modeling is concerned, two methods, i.e., the FFT-based IOST and LSC have been employed. The IOST solution was based on the equations given in Sideris (1996) and Andritsanos et al. (2001a) for the optimal combination of heterogeneous data. On the other hand the LSC model was determined using the well-known collocation formula given in Moritz (1989). In both cases the input signals were the two altimetric (ERS1 and GEOSAT) geoid models and the gravimetric one. Since there was no available information about the input errors, randomly distributed noise fields (white noise) were generated. The variance of each field was based on the σ of the differences of the respective geoid model with T/P SSHs.

3 Data for Geoid Model Estimation and Validation

The area under study is located in the Southern Aegean Sea centered on the island of Crete, Greece, $33^{o} \leq \varphi \leq 37^{o}$ and $21^{o} \leq \lambda \leq 29^{o}$. 174546 GEOSAT Geodetic Mission (GM), 105105 ERS1 GM and T/P altimetry data from the latest releases of their geophysical data records (GDRs) have been extracted for this area from the databases of NOAA (1997) and AVISO (1998), respectively. The gravimetric database comprised of a) marine gravity data available from the Institut für Erdmessung (Behrend et al. 1996) and 5′×5′ mean gravity anomalies from the GEOMED project databank and b) land gravity data provided by Lagios et al. (1996). These made up for a total number of 30437 gravity observations (see Figure 2).

The local depth models used to take into account the effect of the bathymetry were those developed by Vergos (2002) using satellite altimetry and shipborne depth soundings. Finally, the EGM96 global geopotential model, complete to degree and order 360, and the EGM96 DOT, complete to degree and order 20, were used to provide the long wavelength geoid information and the QSST respectively (Lemoine et al. 1998).

For the validation of the estimated geoid models, stacked T/P SSHs, known for their high accuracy, and the GEOMED geoid for the Mediterranean were used. The complete dataset of T/P SSHs from the 3^{rd} year of its mission was extracted so as to have more reliable results. In all cases the differences between T/P or GEOMED and the estimated geoid were computed and minimized using a four-parameter transformation model:

$$N^v = N^i - b_o \cos\phi\cos\lambda - b_1 \cos\phi\sin\lambda - b_2 \sin\phi - b_3 \quad (2)$$

where the parameters b_o, b_1, b_2 and b_3 were calculated by a least squares technique, N^v denotes T/P SSHs or GEOMED and N^i denotes the altimetric *(i=a)*, gravimetric *(i=g)* or combined *(i=c)* geoid height depending on the solution under consideration.

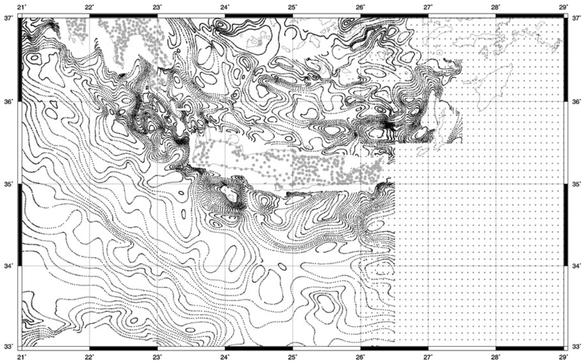

Fig. 2: Gravity data distribution (IfE: black, GEOMED: gray, Lagios: light gray circles).

4 Geoid Model Estimation

The development of the altimetric geoid models was based on the aforementioned ERS1 GM and GEOSAT GM data. Special emphasis was placed on crossover adjusting the data to reduce not only the orbital errors but sea variability effects as well. From that test it was concluded that even with the improvements in the satellite orbit determination, crossover adjustment is necessary since for both datasets it resulted in a reduction of both the mean and σ values by about 3-6 cm. Furthermore, it was found that by RTM-reducing the SSHs to account for the effect of the bathymetry the reduced field was much smoother (mean and σ reduction by about 4 cm) compared to the one prior to the reduction. This is a good indication that if a good DDM is available then it should be used to reduced altimetric SSHs prior to gridding or prediction (e.g., if the prediction of gravity anomalies is intended).

Following the processing scheme described in Sec. 2 the determination of all geoid models was performed on a 5′×5′ grid. It should be mentioned that to speed up the computations of the LSC-based combined solution, we decided to restrict it to a smaller inner area bounded between $33^o \leq \varphi \leq 37^o$ and $23^o \leq \lambda \leq 26^o$. This inner area was selected because it is centered on the isle of Gavdos which was our main focus. Table 1 summarizes the statistics of all five solutions. Note that the min and mean values in the LSC solution differ, because they refer to a smaller inner area.

Table 1 Statistics of the final altimetric, gravimetric and combined geoid solutions. Unit: [m].

	max	min	mean	σ
N^{GEOSAT}	38.10	0.67	16.59	±9.68
N^{ERS1}	37.98	0.73	16.68	±9.69
N^{gr}	38.04	2.09	16.99	±9.33
N^{IOST}	37.99	1.41	16.83	±9.51
N^{LSC}	33.20	5.83	20.17	±9.83

The differences between the gravimetric geoid solution and the altimetric ones is at the 15 cm level (1σ) and the highest values are concentrated in the eastern part of the area where the GEOMED data have been used. On the other hand the differences between the combined and altimetric models are smaller by about 8 cm something expected since the altimetry data have been used in the determination of the combined solutions. Figure 3 depicts the combined LSC solution.

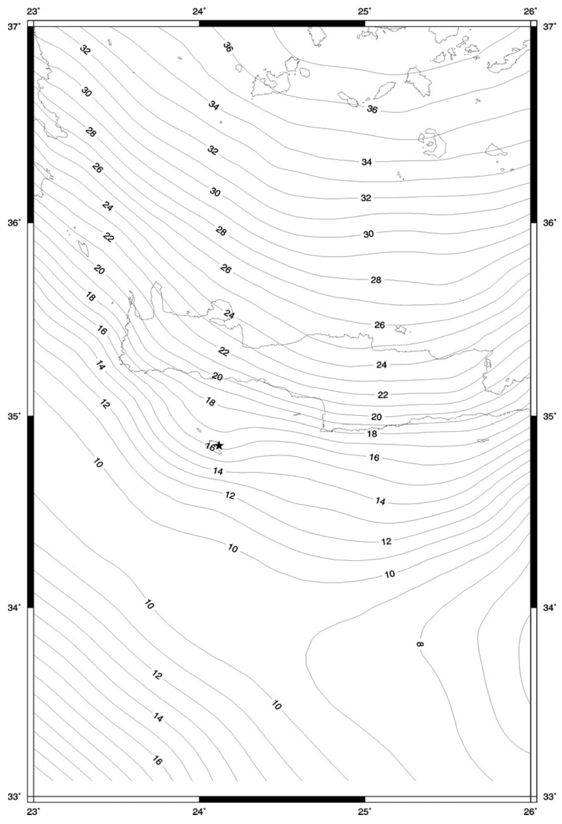

Fig. 3: The combined geoid model estimated with LSC (the black star shows the location of the Gavdos TG station).

5 Validation of the Estimated Geoid Models

The accuracy and precision of the models estimated were assessed by a) estimating the differences to T/P SSHs, b) estimating the differences to GEOMED geoid heights and c) comparing the geoid height estimate that each model gives for the Gavdos TG station. Table 2 presents the comparisons between T/P and the geoid models developed in this study. In all cases the differences were determined as $N^{T/P}$-N^{i} where *i* represents the geoid solution. The altimetric solutions agree with T/P at the ±10 cm level, which is better by about ±26 cm compared to the gravimetric geoid. The IOST and LSC solutions show an improved agreement with T/P by about 17 and 19 cm (1σ), compared to the gravimetric one, respectively. This leads to the conclusion that the combination of altimetry and gravity improves the accuracy of the gravimetric geoid. Additionally, it improves the altimetric geoid close to the coastline, where altimetry suffers from errors.

Table 2. Geoid height difference between T/P and the estimated models. Unit: [m].

	max	min	mean	σ
$N^{T/P} - N^{GEOSAT}$	0.34	-0.19	0.00	±0.11
$N^{T/P} - N^{ERS1}$	0.37	-0.23	0.00	±0.10
$N^{T/P} - N^{gr}$	1.27	-0.88	0.00	±0.36
$N^{T/P} - N^{IOST}$	0.59	-0.49	0.00	±0.19
$N^{T/P} - N^{LSC}$	0.40	-0.32	0.00	±0.17

Table 3 presents the geoid height differences between GEOMED and the estimated geoid models after the fit of a four-parameter transformation model. It is noticing that the range of the differences is in all cases close to about 2.7 m, which is quite high. Furthermore the σ ranges for all geoid models between ±42 and ±45 cm signaling that the GEOMED geoid model deviates from our solutions consistently. By plotting the differences with all geoid models it was found that they show some significant values throughout the area under study. This can be attributed to the different geopotential model, i.e., OSU91A, which was used in the development of the GEOMED geoid, in contrast to EGM96 that was employed in the present study.

Table 3. Geoid height difference between GEOMED and the estimated models. Unit: [m].

	max	min	mean	σ
$N^{GEOMED} - N^{GEOSAT}$	1.00	-1.74	0.00	±0.44
$N^{GEOMED} - N^{ERS1}$	0.99	-1.74	0.00	±0.43
$N^{GEOMED} - N^{gr}$	0.96	-1.82	0.00	±0.42
$N^{GEOMED} - N^{IOST}$	0.93	-1.72	0.00	±0.45
$N^{GEOMED} - N^{LSC}$	0.93	-1.75	0.00	±0.43

The final validation test was performed by estimating the geoid height value that each of the new models gives at the Gavdos TG station. For the combined LSC solution the geoid height was predicted by regarding the TG station as an additional estimation point, while in the other cases it was interpolated from the final solutions. The interpolation was performed in all cases using LSC and estimating the empirical covariance function for each individual solution to derive the necessary variance and correlation length. Table 4 presents the estimated geoid heights at the TG from the different solutions together with the precision in the prediction of each height. All estimates are very close and give a geoid height value at the Gavdos TG station of about 16.67 m. The differences between the predictions range from a minimum of 0.5 cm to a maximum of only 6 cm, while neglecting the gravimetric geoid height results in a maximum difference of only 2.8 cm. This signals that all solutions are consistent to each other and the methodology followed leads to precise results. The precision of the estimated height is ±3.33 cm in the worst case while it reaches the sub-cm level for the height from LSC. This, combined with the fact that the accuracy of the models is at the ±5-10 cm level, gives evidence of the rigorousness of the geoid models both in terms of the processing methodology and the accuracy and precision achieved.

Table 4. Estimated geoid height at the Gavdos TG station from the models developed. Unit: [m].

Model	N(m)	σ_N (cm)
N^{GEOSAT}	16.690	±2.47
N^{ERS1}	16.682	±2.05
N^{gr}	16.633	±2.82
N^{IOST}	16.662	±3.33
N^{LSC}	16.695	±0.92

6 Conclusions

With the aim of estimating an accurate and precise geoid model in support of the GAVDOS project, heterogeneous data and various methods have been used. This study aimed at providing the necessary methodological background so as to be ready to employ the additional gravity data that will become available to the project. The new data include marine, land and airborne gravity observations which will complement the existing gravity database. Thus, it was apparent that before implementing the entire database, different processing schemes had to be validated to choose the most rigorous one both in terms of accuracy and precision.

From the results of this study and the validation carried out, it became evident that when altimetry and shipborne gravity data are handled properly, i.e. corrected for all error sources, blunders removed, accurate geopotential and DOT models used, the data are corrected for the QSST signal, the bathymetry is taken into account using an accurate model, and the altimetry data are crossover adjusted, then, altimetric geoid modeling accurate to about ±5 cm is feasible. The combined solution is more accurate, compared to the gravimetric one, by about 17-19 cm, in terms of the σ of the differences with T/P, depending on whether LSC or IOST was used. It seems that LSC gives slightly better results (only by about 2 cm), but the IOST method is much faster. This last point becomes especially important taking into account that the complete gravimetric database by itself consists of approximately 100000 observations.

In terms of the accuracy of the models, the altimetric solutions performed much better compared to the gravimetric one, but that is probably due to the GEOMED data only. As shown by Vergos (2002) for the same area, when good quality marine gravity observations are used, then the gravimetric geoid is inferior to the altimetric ones by only 2-4 cm. Both combined geoid models (based on LSC and IOST) improved the gravimetric one in terms of the accuracy and the altimetric ones close to the coastline. The precision of the estimated geoid models ranged between ±0.9-3.3 cm, while they proved to be very consistent to each other, since the differences between the geoid heights that each one provided at the Gavdos TG were of the order of a few cm in the worst case.

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