Optimal Marine Geoid Determination in the Atlantic Coastal Region of Argentina

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Abstract. The determination of an optimal, in terms of resolution and accuracy, marine geoid model for the Atlantic coastal region of Argentina is investigated using satellite altimetry and shipborne gravity data. The altimetric data used are those of the geodetic phase of the ERS1 mission while marine gravity data have been employed as well to determine a gravimetric geoid solution. Furthermore, the effect of the Quasi-Stationary Sea Surface Topography (QSST) was taken into account in correcting the altimetric Sea Surface Heights (SSHs) to derive geoid undulations. Special emphasis was placed on reducing the effects of the Sea Surface Variability (SSV) on the densely spaced altimetric SSHs with low-pass filtering. The satellite and shipborne data were combined in the spectral domain to improve the accuracy of the altimetric solution close to the coastline and derive a more rigorous solution. The accuracy of the final geoid models is assessed through comparisons with stacked TOPEX/POSEIDON (T/P) SSHs, known for their high precision. From the results achieved it was concluded that an altimetric geoid accurate to about 5-8 cm (1σ) is feasible in some areas, while the gravimetric solution gives poorer results by about 5-6 cm. The combination of satellite and shipborne data with the proposed algorithm improves the accuracy of the gravimetric geoid model by about 2 cm.

Keywords. Marine geoid, altimetry, Sea Surface Heights, shipborne gravity, MIMOST.

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1 Introduction

The main objective of this paper is the determination of a high-accuracy and high-resolution marine geoid model in the Atlantic coastal region of Argentina. The theoretical background related to the estimation of the gravimetric and altimetric geoid models (Vergos 2002) and the combined one using the Multiple Input Multiple Output System Theory (MIMOST) (Sideris 1996, Andritsanos and Tziavos 2002), will be outlined together with the description of the data available in the area under study. Finally, some numerical studies carried out in this research will be presented.

2 Computational methodology

2.1 Gravimetric geoid modeling

The gravimetric geoid determination was based on shipborne and satellite altimetry-derived gravity anomalies. The latter have been used to augment the ship data and fill-in gaps. The shipborne gravity data referred to the Geodetic Reference System 1967 (GRS67), thus they had to be transformed to GRS80. This was done using the following basic formulae (Moritz 2000):

$$\Delta g_{GRS80} = \Delta g_{GRS67} + \gamma_{GRS67} - \gamma_{GRS80} \tag{1}$$

where, Δg_{GRS80} denotes gravity anomaly in GRS80, Δg_{GRS67} gravity anomaly in GRS67 and γ_{GRS67} and γ_{GRS80} are the magnitudes of the normal gravity in GRS67 and GRS80 respectively. Normal gravity can be computed for GRS67 and GRS80 as (*ibid.*)

$$\gamma_{GRS80} = 978032.6702 \frac{1 + 0.001931851 \sin^2 \varphi}{\sqrt{1\text{-}0.00669438002 \sin^2 \varphi}} \tag{2}$$

It is often that the ship gravity anomalies refer to the sea surface and not the geoid itself, thus their use will lead to the determination of a mean sea surface and not a geoid model. These gravity anomalies have to be free-air reduced so as to produce gravity anomalies on the surface of the geoid. The free-air gravity anomalies are computed using the well-known reduction formula

$$\Delta g_f = \Delta g - \delta g_f \tag{3}$$

where δg_f is the free-air reduction. In marine regions the height needed for the reduction is that of the QSST, which varies between 0 and -0.30 m in the study area. Thus, for practical purposes it is sufficient to use the normal gradient of gravity to compute the free-air reduction as

$$\delta g_f \approx -\frac{\delta\gamma}{\delta h} h \approx -0.3086h \tag{4}$$

where h is the QSST in meters, derived from a global model.

These two pre-processing steps are necessary for the data homogenization so that they can be used for the determination of the gravimetric geoid. The marine gravimetric geoid will be computed using the remove-compute-restore technique employing Stokes' formula for the prediction of residual geoid heights. Before the prediction of the geoid the gravity anomalies have to be reduced to a geopotential model during the remove step. Furthermore the effect of the topography, actually that of the bathymetry in marine areas, has to be taken into account through a topographic reduction. In this study, a residual terrain model (RTM) reduction was used to account for the bathymetry. The RTM effect on gravity is given by the approximate expression (Forsberg 1984)

$$\Delta g_{RTM} \approx 2\pi G\rho(h - h_{ref}) - c \tag{5}$$

where h is the bathymetric depth given by a global bathymetry model, h_{ref} is the depth of a smooth mean reference surface and ρ is the density contrast between Earth's crust and seawater. The reference bathymetric surface was obtained by simple averaging the fine bathymetry grid and then low-pass filtering it using a moving-average window, with a resolution around 100 km. The residual gravity anomalies are then gridded and the contribution of the bathymetry is restored prior to the calculation of the geoid height. The bathymetry refers to masses below the geoid so its effect has to be restored before the use Stokes formula for the estimation of geoid heights (Dahl and Forsberg 1998).

Different approximations to Stokes' kernel function were investigated to compute residual geoid undulations all in the spectral domain. Finally, it was decided to employ the 2D-FFT spherical Stokes convolution to evaluate the kernel function (Strang Van Hess 1990)

$$N^{gr} = \frac{R\Delta\varphi\Delta\lambda}{4\pi\gamma}\mathscr{F}_1^{-1}\{\{\mathscr{F}\{\Delta g\cos\varphi\}\mathscr{F}(\Delta\varphi,\Delta\lambda,\varphi_m)\}\} \tag{6}$$

where N^{gr} are the estimated residual gravimetric geoid heights and $\mathscr{F}$, $\mathscr{F}^{-1}$ denote the direct and inverse Fourier transforms. The final gravimetric geoid is obtained by restoring both the contribution of the reference model and the RTM effects to geoid heights.

2.2 Altimetric geoid modeling

An altimetric satellite measures the time taken by a radar pulse to travel from the satellite to the sea surface and then back to the satellite receiver. Combined with precise satellite location data, altimetry measurements yield Sea Surface Heights (SSHs). The so-derived SSHs have to be corrected for several geophysical effects (tides, tidal loading, ionosphere, wet and dry troposphere, inverse barometer and electromagnetic bias) and instrumental errors (ultra-stable oscillator, centre of gravity, corrections for instrument and algorithm effects that can not be modeled and waveforms). After applying the above corrections, Corrected Sea Surface Heights (CORSSHs) are available for one or more satellites (e.g. GEOSAT, ERS1).

Sea Surface Heights contain information about both the geoid and the sea surface topography (SST), while the latter consists of a time-dependent and a nearly time-independent component (quasi-stationary part). Stacking the repeat tracks can eliminate the effect of the time-dependent component and part of the sea surface variability effects that influence the data.

These altimetric measurements refer to the sea surface so they have to be reduced to the geoid. This is performed by estimating the QSST at each sub-satellite point and removing the contribution of the QSST from the SSH value. The quasi-stationary component of the SST is modeled by a spherical harmonic series of the Dynamic Ocean Topography (DOT) as follows:

$$\varsigma_c(\phi,\lambda) = \left[\sum_{n=1}^{n_{max}}\sum_{m=0}^{n}\left(\overline{C}_{nm}^{*SST}\cos m\lambda + \overline{S}_{nm}^{SST}\sin m\lambda\right)\overline{P}_{nm}(\sin\phi) \right. \tag{7}$$

where $\varsigma_c(\phi,\lambda)$ is the contribution of the model coefficients, n_{max} denotes the maximum degree and order of expansion of the DOT model, $\overline{P}_{nm}(\sin\phi)$ are the fully normalized associated Legendre functions, and $\overline{C}_{nm}^{*SST}$, $\overline{S}_{nm}^{SST}$ are the fully normalized DOT spherical harmonic coefficients.

After the removal of the effect of the QSST the SSHs refer to the geoid and can be used to derive an altimetric geoid model. As in the gravimetric geoid computation, the contribution of a geopotential model was removed to derive reduced SSHs

($SSHs_{red}$). The so-reduced SSHs ($SSHs_{red}$) may still contain some blunders, so a 3 rms test is used to identify and remove gross-errors. If the mean value of the reduced SSHs is small enough (e.g., bellow 10 cm), then the 3 rms test can be applied. That is so because by using a 3 rms test we assume that all systematic errors have been removed from the data and only random errors remain. If the mean value of the SSH_{red} is larger, an RTM reduction is applied first to obtain smoother residual SSHs. The computation of the RTM effects on residual geoid heights has been based on the same concept as in the gravimetric geoid. In both cases the GRAVSOFT software (Tscherning et al. 1992) has been used to create the reference bathymetric grid and estimate the RTM reduction on gravity anomalies and geoid heights.

The residual Sea Surface Heights represent the medium wavelengths of the geoid heights and can be considered as residual geoid heights (N_{res}). After all these processing steps the N_{res} are ready to be gridded. The gridding was based on a weighted means method using the inverse of the square of the distance as the weight for each irregular observation. If the area under study is located in a closed sea, then the such-derived N_{res} constitute the final estimated residual altimetric geoid height. But, if in an open-ocean area, then the effect of the Sea Surface Variability (SSV) influences the data and appears as noise in the N_{res}. Such effects need to be removed or at least reduced so as to derive reliable predictions. Since the effect of the SSV appears as high-frequency noise in the altimetric data, it can be reduced by low-pass filtering the grid of N_{res}. This was performed using a Wiener filtering, which is equivalent to least squares collocation. Then, the final step to compute the altimetric geoid N^{alt} is to restore the contribution of the geopotential model and the contribution of the bathymetry.

2.3 Combined geoid modeling

Using the Multiple Input Multiple Output System Theory (MIMOST) for the optimal combination of heterogeneous noisy data a combined solution has been determined. That was done in order to investigate whether the combined use of shipborne gravity and satellite altimetry data improves the geoid compared to the purely gravimetric case. The algorithm and the related formulae are given in Sideris (1996) and Andritsanos and Tziavos (2002).

The input signals in the combined solution were the residual gravimetric and altimetric geoid heights before the restore of the contribution of the geopotential model. That was done to avoid introducing long-wavelength errors. Since no information was available about the errors of the input data, simulated noise was used as input errors in the prediction. White noise was generated in MATLAB® using as standard deviation (σ) that of the differences of each solution with T/P SSHs.

3 Area under study and data availability

The area under study was located in the Atlantic Coastal region of Argentina, bounded by 34°S to 55°S in latitude and 56°W (304°E) to 70°W (290° E) in longitude. The marine gravity data available were 17352 gravity anomalies provided by BGI (Bureau Gravimétrique International). Since there were some gaps between the ship tracks, the KMS01 $2' \times 2'$ altimetry derived free-air gravity anomaly field (Andersen and Knudsen 1998) has been used as fill-in information. The distribution of the marine gravity data is depicted in Figure 1. KMS01 is the newest compilation of a global altimetry-derived marine free-air gravity field by the KMS group at the Danish Surveying and Cadastre.

To derive the long-wavelength information of the gravity field we used the EGM96 global geopotential model (Lemoine et al. 1998) complete to degree and order 360. The topographic/bathymetric data for the RTM reduction were those of the Smith and Sandwell model (Smith and Sandwell 1997), which resulted as a combination of depths derived from altimetry.

The QSST was computed from the EGM96 DOT model, which is a spherical harmonic expansion of the SST, complete to degree and order 20 (Lemoine et al. 1998). That model was derived during the simultaneous adjustment for the development of the EGM96 geopotential model and its representation of the QSST for the area under study is given in Figure 2. Finally, the altimetric data were 70510 SSHs measurements from the geodetic mission (GM) of the ERS1 satellite and 80864 repeated T/P SSHs from the entire 3rd Year of the satellite's mission (Aviso 1998).

4 Geoid model development

4.1 Altimetric geoid model with ERS1-GM data

The ERS1 satellite altimetry SSHs were provided in the usual Geophysical Data Records (GDRs) format and were corrected for all geophysical and instrumental errors in a pre-processing step according to the models and methods described in the AVISO handbook (AVISO 1998).

As the ERS1-GM CORSSHs refer to sea surface, they were reduced to the geoid by removing the effect of the QSST. That was performed using Eq. 8 to predict QSST values on the irregular ERS1 points.

At this point, the ERS1-GM Sea Surface Heights referred to the geoid were ready to be used for the estimation of a purely altimetric geoid. The contribution of the EGM96 geopotential model was then removed and the resulting reduced SSHs were checked for their mean value. After examining the mean value of the reduced field it was not found to be small enough for a 3rms test to be performed. Thus, the bathymetry was first taken into account with an RTM reduction and after that a 3 rms test for blunder detection has been applied. After the 3 rms test 678 points were removed and the resulting point data were gridded using the aforementioned algorithm on a 3′×3′ grid.

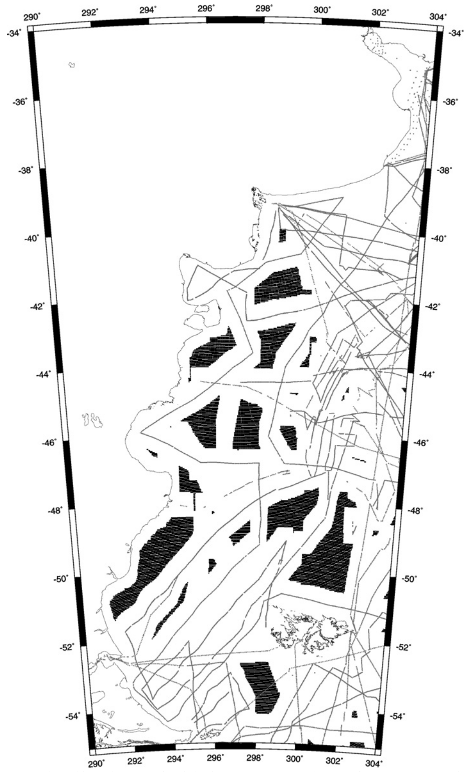

Fig. 1: Distribution of shipborne (gray) and KMS01 (black) data in the area under study.

To reduce the high-frequency SSV-like effects the data were low-pass filtered using Wiener filtering. The cut-off frequency was determined empirically based on a criterion of maximum noise reduction with minimum signal loss. A number of cut-off frequencies were tested and finally a cut-off frequency corresponding to a wavelength of 20 km was chosen. That selection gave the best results as far as both the noise reduction and the minimization of the differences with T/P SSHs are concerned.

The final altimetric geoid solution was obtained by restoring the contribution of the EGM96 geopotential model and that of the RTM effects of the bathymetry. Table 1 presents the statistics of the final ERS1-GM geoid for the area under study which is also depicted in Figure 3 for visualization purposes.

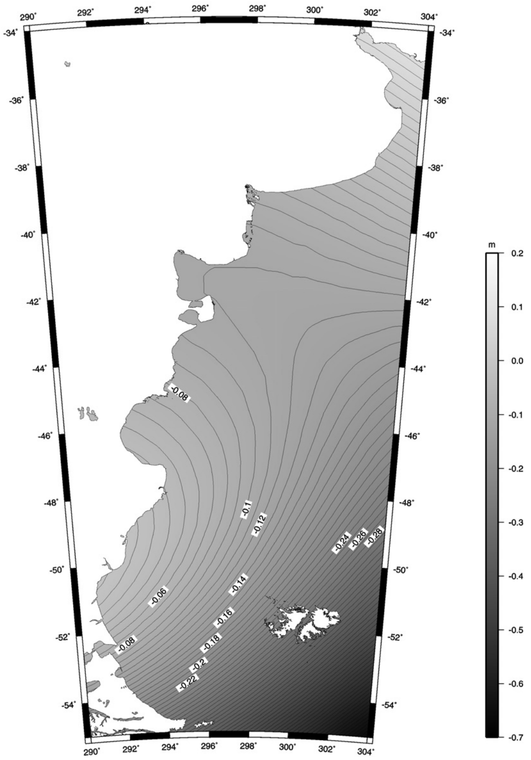

Fig. 2 The EGM96 QSST in the area under study.

Table 1. Statistics of the ERS1-GM geoid. Unit: [m].

	max	min	mean	σ
N^{alt}	19.665	0.505	11.460	±2.996

4.2 Gravimetric geoid model

The gravimetric geoid model was determined using the computational procedure described in paragraph 2.1. The main difference with the altimetric geoid modeling is that the RTM reduction is restored before the prediction of the gravimetric residual geoid heights. Table 2 presents the statistics of the final gravimetric geoid for the area under study which is also depicted in Figure 4.

Table 2. Statistics of the gravimetric geoid. Unit: [m].

	max	min	mean	σ
N^{grav}	19.333	0.642	11.362	±2.957

4.3 Combined solution

The estimation of the combined solution was performed in a smaller area between 40°S to 50°S in latitude and 56°W (304°E) to 66°W (294° E) in longitude. Table 3 shows the statistics of the gravimetric geoid, the ERS1-GM geoid and the combined solution in the reduced area and Figure 5 shows the final solution from MIMOST. The input noise for each dataset was generated using the σ of the differences T/P SSHs and the gravimetric (±25 cm) and altimetric geoid (±20 cm) geoid models.

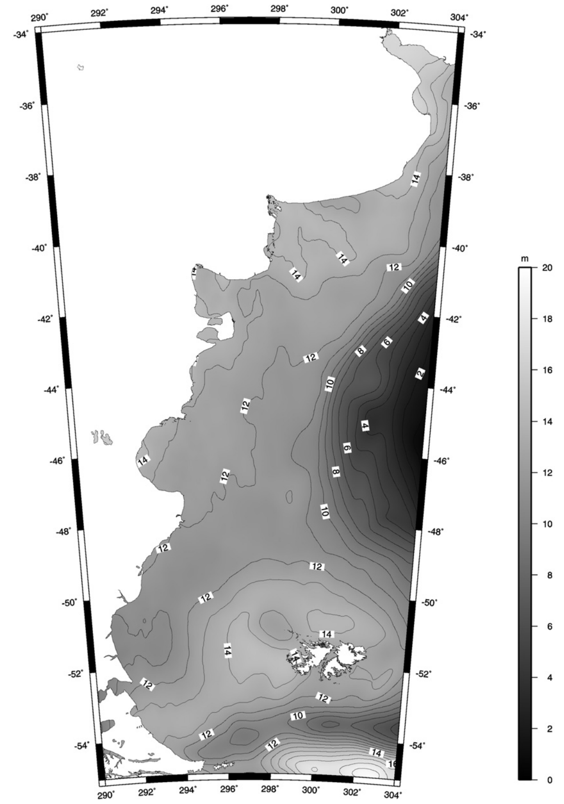

Fig 3: The final ERS1-GM altimetric geoid model for Argentina.

Table 3. Statistics of the geoid models in the inner area. Unit: [m].

	max	min	mean	σ
N^{alt}	17.225	0.642	10.253	±3.273
N^{grsv}	16.542	0.545	10.293	±3.290
N^{comb}	17.103	0.574	10.266	±3.279

5 Validation of the estimated geoid models

The accuracy of the models was assessed through comparisons with stacked T/P SSHs. The computed differences between T/P and each geoid solution were minimized using a four-parameter transformation model, as

$$N^{y} = N^{i} - b_0 \cos\varphi\cos\lambda - b_1 \cos\varphi\sin\lambda - b_2 \sin\varphi - b_3 \quad (8)$$

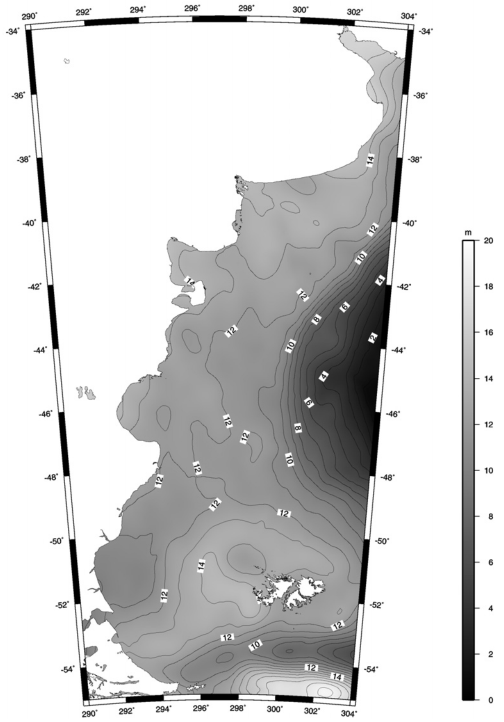

Fig 4: The final gravimetric geoid model for Argentina.

where N^{y} is the stacked T/P SSHs, N^{i} is the altimetric, gravimetric or combined geoid solution and the parameters b_0 b_1 b_2 and b_3 were calculated using a least squares technique. The statistics of the differences after the bias and tilt fit between the T/P SSHs and the estimated geoid solutions are given in Table 4. From that Table it can be seen that the overall best agreement is achieved for the altimetric geoid solution while the data combination improves the gravimetric geoid by about 2 cm.

Worth mentioning though, is that the σ of the differences for the comparisons with the altimetric models is quite high, at the ±20 cm level, while a value close to ±9 cm would be expected based on previous studies (Li and Sideris 1997; Vergos 2002). Plotting the differences it was noticed that their largest and smallest values are located close to the coastline and more specifically between $-45^{o} \leq \varphi \leq -44^{o}$ and $294^{o} \leq \lambda \leq 296^{o}$ and close to Falkland Islands where the effects of SSV and other oceanic phenomena are very strong. In the rest of the region, the differences are within their expected values ranging between -60 and 60 cm. In our opinion, this is an indication that the accuracy of the altimetric geoid models is much better than the comparisons with T/P imply. Neglecting a few T/P points that refer to the aforementioned regions the σ of the differences reduces to about ±5 to ±8 cm for the altimetric geoid models. The same improvement of more than ±9 cm holds for the gravimetric and combined models too.

So, it can be concluded that by only stacking the T/P data part of the oceanic effects, which clearly influence the SSHs used for the comparisons, cannot be removed. Probably, the T/P data had to be low-pass filtered as well in their along-track direction, to further reduce the effect of the SSV and make the comparisons more representative.

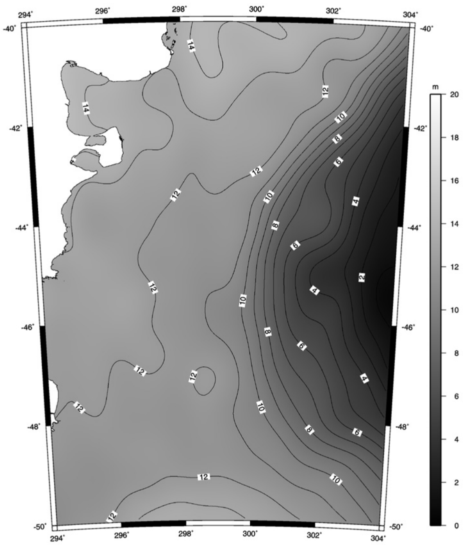

Fig 5: The combined geoid model.

Table 4. Geoid height difference between T/P and the estimated models. Unit: [m].

	max	min	mean	σ
$N^{T/P} - N^{ERS1}$	1.00	-1.20	0.00	±0.20
$N^{T/P} - N^{gr}$	1.66	-0.93	0.00	±0.25
$N^{T/P} - N^{comb}$	1.49	-0.93	0.00	±0.23

6 Conclusions

Altimetric, gravimetric and a combined final geoid height solution have been determined in the Atlantic coastal region of Argentina. The MIMOST theory for the optimal combination of heterogeneous data has been applied to improve the gravimetric geoid solution close to the coastline.

From the results and validation procedures carried out, it is evident that when altimetry and shipborne gravity data are handled properly, i.e. corrected for all error sources, blunders removed, accurate geopotential and DOT models used, the data are corrected for the QSST signal, the bathymetry is taken into account using an accurate model, the altimetry data are low-pass filtered, then, altimetric geoid modeling accurate to about ±7 cm is feasible, while the combined solution improves the gravimetric one, by about 2 cm, in terms of the σ of the differences with T/P SSHs. These differences refer to purely oceanic areas (not close to the coastline) and regions where the effect of the variability of the oceans is not very strong. Some new results must be acquired using GEOSAT-GM data.

The effect of oceanic phenomena in the densely spaced GM datasets, especially in areas with high ocean dynamics, is profound and should be reduced by low-pass filtering the altimetric datasets. If this step is neglected, then the resulting geoid solutions are less accurate by about 2-5 cm. A point that needs further research is on the use of crossover adjustment for the reduction of such noisy signals. Finally, the altimetry data should be corrected for the QSST signal to refer to the geoid and not the sea surface, while the question that arises is not on the necessity of such a correction, but on the selection and the development of accurate DOT models.

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