Improving the Estimation of Bottom Ocean Topography with Altimetry Derived Gravity Data Using the Integrated Inverse Method

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Abstract. The possibility of improving the gravity response of the bottom topography of the Earth's oceans using gravity data is investigated in two extended test areas. The first area is located in the Mediterranean Sea southwest of the island of Gavdos, Greece and bounded by $33^o \leq \phi \leq 35^o$ and $15^o \leq \lambda \leq 25^o$. The other one is across the Mid Atlantic Ridge bounded by $40^o \leq \phi \leq 50^o$ and $330^o \leq \lambda \leq 340^o$. The integrated inversion method of gravity data as proposed by Knudsen, is used in an attempt to improve our knowledge of the ocean bathymetry and its gravity response. The KMS99 satellite altimetry-derived global marine gravity field is used with a-priori statistical characteristics of depths in a least squares collocation procedure to produce new depths. Two different global bathymetry models, namely JGP95E and Sandwell and Smith V9, are used to derive the depth a-priori statistical information and to test how the gravity data can improve the depth estimation. Two- and three-layer models are used to represent the Earth's structure. The improvement in the estimation of the bottom topography is investigated through an RTM reduction of shipborne gravity data and ERS1-GM satellite altimetry SSHs. Throughout this study the EGM96 geopotential model, complete to degree and order 360, is used as a reference field to model the low frequency part of the gravity field spectrum.

Keywords. Bathymetry estimation method, gravity inversion, residual terrain modeling (RTM) reduction, least-squares collocation.

1 Introduction

The effect of the ocean bathymetry on various quantities related to the gravity field (e.g. gravity anomalies, geoid heights, deflections of the vertical etc.) is of great importance for a large number of oceanographic and geodetic applications. Bathymetry influences the direction of the ocean currents, marine life, and the atmosphere in a fashion that its estimation is vital. In geodesy, Digital Depth Models (DDMs), in analogy to the Digital Terrain Models (DTMs) on land, are used during a remove-restore procedure to smooth gravity field related quantities. This is achieved through various reduction methods such as the topographic reduction, the residual terrain modeling (RTM) reduction, and the isostatic reduction, proposed and used during the last twenty years in geodesy (Forsberg 1984).

In gravity field modeling, bathymetry is combined with sea data (gravity anomalies, altimetry Sea Surface Heights (SSHs)) and a global geopotential solution in a remove-compute-restore procedure aiming at best estimating the gravity field. The primary use of the depths lies in smoothing the data so that a subsequent gravity anomaly, geoid height or deflection of the vertical prediction can be carried out. According to Forsberg (1984), when high-quality depths are available, then the smoothing of the data can reach the 50% level. The common practice in marine gravity field modeling is to use an RTM reduction to take into account the attraction of the bathymetry, and this method consists of using a model of the bathymetry equal to the actual ocean topography referenced to a mean, smooth but varying surface (Forsberg 1984).

The described use of DDMs in gravity field modeling has often given disappointing results mainly due to the use of low quality bathymetry data from global DTMs such as the 5′ ETOPO5U (Arabelos 1997). Contrary to that, the high-accuracy and high-resolution satellite altimetry data offer a great alternative for a better estimation of the bathymetry. We should mention though that this "improved" estimation can be viewed only in terms of an improved gravimetric response of the bottom ocean topography and the smoothing that the inverted from altimetry DDMs offer to gravity field quantities during a remove-restore procedure (Tscherning et al., 1994). This is so because the gravity field estimated from altimetry is band-limited to wavelengths between 10km and 120km (Smith and Sandwell 1994). The lower bound is induced by the cross-track resolution of the altimetry missions and the upper one by the singularity of the gravity to topography transfer function due to upward continuation (from the oceanic seafloor to the sea surface) and isostatic compensation.

With the advent of satellite altimetry, there have been numerous studies using altimetry data to estimate the ocean topography, at the beginning along altimetric profiles (Dixon et al. 1983; Jung and Vogt 1992) and then as a 2D plain (Arabelos and Tziavos 1998; Hwang 1999; Smith and Sandwell 1994). Some of them employ prediction in the frequency domain and others in the space domain. A good review of some of these methods is provided in Calmant and Baudry (1996).

In this study the method used, later on called *bathymetry estimation method* and developed in the space domain, has been proposed by Knudsen (1993) and is based on the well-known Parker formula (Parker 1972) for the relationship

between gravity and bathymetry. With this method, the bottom ocean topography can be improved when high-quality marine gravity anomalies and a-priori statistical characteristics from available bathymetry models are available. The main assumption of the method is of an Earth consisting of two or more layers (one or more interfaces) with each of the interfaces contributing to the gravity anomaly field. Under this assumption, we can compute cross-covariance functions between heterogeneous quantities (SSHs/geoid, gravity/depths) and approximate more precisely the gravity field (Barzaghi 1992; Knudsen 1993). An iterative least squares collocation (LSC) procedure is used, during which corrected depths are computed until the gravimetric response of the estimated bathymetry model gives sufficiently small (e.g, below the noise level of the observations) differences when compared with the observed gravity field.

To achieve a better estimation of the bottom topography, we use multi-satellite altimetry-derived marine gravity anomalies and a-priori statistical characteristics of DDMs. When gravity data are used and the density contrasts are considered as being constant in each interface, then depths to these interfaces can be estimated (Barzaghi 1992; Knudsen 1993). In our study, we use a two-layer model, representing the sea-water and Earth's upper lithosphere, and a three-layer one with the third layer representing the crust-mantle interface (Moho depths).

2 The Bathymetry Estimation Method

The bathymetry estimation method has been proposed by Tscherning et al. (1994) and is based on the integrated inversion of gravity data developed by Knudsen (1993). It has recently been used by Knudsen and Andersen (1996) and Arabelos and Tziavos (1998) with promising results. In this study, we test the method in two areas where both bathymetry and the gravity field vary significantly. The present method is very useful to fill-in bathymetry gaps where sparse depth soundings are available and/or improve the bathymetry information in places where shipboard depths have several errors.

To describe the depths to an interface, continuous base functions in terms of the inverse Fourier transform of the depth function *h(x,y)* are used (see Schwarz et al. 1990; Tziavos 1994). The covariance (CV) function associated with the depths is derived from their spectrum and an azimuthal averaging i.e.:

$$C_{\Delta h \Delta h}(s) = 2\pi \int_0^{\infty} \Phi(q) J_0(2\pi qs) q dq \tag{1}$$

where Φ (q) is the power spectrum of Δh given by:

$$\Phi(q) = \mathscr{F}\{\Delta h(x,y)\}\mathscr{F}\{\Delta h(x,y)\}^* \tag{2}$$

where $\Delta h = h - h_o$, h_o is the mean depth, $\mathscr{F}$ is the Fourier transform symbol, and J_o is the zero-order Bessel function. According to Knudsen (1993) in order to utilize the inverse Fourier transform of the depth function as the base function we need isotropic CV and power spectral density (PSD) functions. But the PSD function derived from Eq. (2) will not be isotropic (a function of the radial frequency $q = \sqrt{u^2 + v^2}$ only). Thus, we use the following azimuthal averaging to get isotropic PSD functions:

$$\Phi(q) = \frac{1}{2\pi} \int_0^{2\pi} \Phi(u,v) d\alpha \tag{3}$$

where $u = q\cos\alpha$, $v = q\sin\alpha$, α is the azimuth, and $\Phi(u,v)$ is the non-isotropic PSD i.e.:

$$\Phi(u,v) = \mathscr{F}\{\Delta h(x,y)\}\mathscr{F}\{\Delta h(x,y)\}^* \tag{4}$$

The relationship between gravity and bathymetry is given by the well-known Parker formula (Parker 1972)

$$\mathscr{F}\{\Delta g\} = 2\pi G \rho e^{2\pi q h_o} \sum_{n=1}^{\infty} \frac{(2\pi q)^{n-1}}{n!} \mathscr{F}\{(\Delta h)^n\} \tag{5}$$

where G is the Newtonian gravitational constant, q as in Eq. 2 and ρ is the constant density of the interface. Ignoring terms of order greater than $n=1$, the linear part of Eq. (5) is obtained:

$$\Delta g \approx L_{\Delta g}(\Delta h) = 2\pi G \rho \mathscr{F}^{-1}\{e^{2\pi q h_o} \mathscr{F}\{\Delta h\}\} \tag{6}$$

The depth to gravity response model described in Eq. (6) can be regarded as a simple one since it involves only one parameter (the depths). Nevertheless, in the geophysical literature there are two- and three-layer models, which account for more parameters, like the flexural rigidity of the lithosphere, the plate stiffness and the flexural length (Calmant and Baudry 1996). A more complex approach (Calmant 1994) introduces three interfaces i.e. the ocean bottom, Moho depths and a third one representing the elastic deflection of the lithosphere under the weight of an overlaying structure. The purpose of our study is not by any chance to investigate and define the most appropriate admittance function, but rather to use a simple representation of it to validate the proposed method.

The observed gravity anomalies are combined with a-priori statistical characteristics from a global or local DDM in an iterative LSC procedure. These a-priori characteristics of the depths are needed since we do not know the power spectrum $\Phi(q)$ of the depths and the iterations are necessary to overcome the non-linearity between gravity and depths. We obtain the auto- and cross-covariance functions of the gravity and depths from Eqs. (1) and (6) as

$$C_{\Delta g \Delta g}(s) = (2\pi)^3 (G\rho)^2 \int_0^{\infty} e^{4\pi q h_o} \Phi(q) J_0(2\pi qs) q dq \tag{7}$$

and

$$C_{\Delta h \Delta g}(s) = (2\pi)^2 G\rho \int_0^{\infty} e^{2\pi q h_o} \Phi(q) J_0(2\pi qs) q dq \tag{8}$$

In this way, and employing some a-priori information (mean, standard deviation *std*, correlation length ξ) of the depths we can compute the model covariance function of

the gravimetric response of the bathymetry with these statistical characteristics. Additionally, we can estimate the empirical CV function of the observed gravity data, which will be used as input to the LSC. Thus, this a-priori spectrum is tuned up so that Eqs. 1, 7, and 8 will agree with the empirical CV function of the observed data. Other model CV functions can be used and tested as well (see Barzaghi et al. 1992) but, in our case, the described one gave a very good agreement with the empirical CV function estimated from the observed data, thus we decided to use this approximation. If the model and empirical CV functions agree well, then we iteratively estimate "corrected" depths and compare their gravimetric response with the observed gravity anomaly field. In the kth iteration, the "corrected" depths are estimated by the residual gravity observations Δg_k obtained from the $(k-1)^{th}$ iteration using the collocation formula (Moritz 1980):

$$\mathbf{\Delta h}_k = \mathbf{C}_{\Delta h \Delta g}\left(\mathbf{C}_{\Delta g \Delta g} + \mathbf{D}\right)^{-1} \mathbf{\Delta g}_k \tag{9}$$

where $\boldsymbol{C}_{\Delta h \Delta g}$ is the cross-covariance matrix between the depths and observed gravity, $\boldsymbol{C}_{\Delta g \Delta g}$ is the auto-covariance matrix of the observed gravity data, $\boldsymbol{D}$ is the diagonal matrix of the noise of the observations, and Δg_k is the vector of the observed gravity anomalies. The observations in this study are gravity anomalies only, but they can also be depths or both gravity and depths. The best estimation of the bottom ocean topography is achieved when the differences between the observed gravity data and the gravimetric response of the estimated bathymetry are small. This statement about the smallness of the residuals can be interpreted in many ways and be arbitrary. We think that a safe measure is the noise level of the input gravity data, so that when the std of the differences is below that level we stop the iterations. In our case the observed gravity comes from Geodetic Mission (GM) satellite altimetry, and according to Andersen and Knudsen (1998) its accuracy, as implied by comparisons with shipborne gravity data, is at the 8mGal-10mGal.

3 Data Description

The 2′×2′ KMS99 (Andersen and Knudsen 1997) multi-satellite altimetry-derived gravity field is used in both regions as observed gravity to the bathymetry estimation method. The depth a-priori statistical characteristics come from two global models, namely the 5′×5′ JGP95E (Lemoine et al. 1998) and the 1.5′×2′ Sandwell and Smith Ver.9 (S&SV9) model (Smith and Sandwell 1997). The former is regarded as the regular bathymetric grid of this study and was the DTM used to account for the attraction of the topographic masses in the development of the EGM96 geopotential model. The latter represents another estimation of the bottom ocean topography from the combination of a multi-satellite (Seasat, GEOSAT, and ERS1) altimetry-derived gravity field with available depth soundings. Our intention in using two global DDMs is to test the new bathymetry estimates against a regular field and a DDM derived from altimetry using a different method than the one proposed in this study. From the global models, the one that gives the greatest smoothing when used in an RTM reduction of shipborne gravity data is selected to provide the a-priori statistical characteristics needed by the estimation method. For the area of Gavdos, we have available 21699 shipborne free-air gravity anomalies referred to GRS80 and coming from the Bureau Gravimétrique International (BGI) database (personal communication 2001) and a digitization of Morelli's maps (Behrent et al. 1996) performed at the Institüt für Erdmessung (IfE). In the MAR, we have 39248 free-air gravity anomalies available, coming entirely from the BGI database. These gravity data are used to validate the new estimated bathymetry models, as well. Especially for the area near Gavdos, we validate the new models further more by testing the effect of an RTM reduction to ERS1-GM SSHs (AVISO 1998), processed during an earlier study by the authors (Vergos et al. 2001). Throughout this study the EGM96 global geopotential model (Lemoine et al. 1998) compete to degree and order 360 was used as a reference field.

4 Evaluation of Global DDMs

For the area near Gavdos, the statistics of the global DDMs and their differences are shown in Table 1. The statistics of the gravity data before and after their reduction to EGM96 are presented in Table 2 together with the ones for the RTM-reduced gravity anomalies.

Table 1. Statistics of available DDMs and their differences in Gavdos. Unit: [m].

Model	max	min	mean	std
JGP95E	245.00	-3841.00	-2210.18	±823.26
S&SV9	437.00	-4061.00	-2297.67	±863.66
S&SV9-JGP95E	1619.99	-1999.25	-94.34	±198.34

Table 2. Observed and EGM96 reduced gravity data. Unit: [mGal].

	max	min	mean	std
Δg	146.90	-200.30	-26.60	±58.77
Δg_{red} (EGM96)	121.01	-115.68	1.66	±26.88
JGP95E	96.21	-124.42	3.17	±28.16
S&SV9	123.41	-91.84	1.99	±24.60

From Table 2 we can see that the S&SV9 model offers bestter smoothing to the gravity data in terms of reducing the std by almost 8.5%, while JGP95E not only does not manage to smooth the data but increases the std by almost 4.8%. Thus, for the area in Gavdos, the depth a-priori statistical characteristics will be those of the S&SV9 model. For the second test area in the Mid-Atlantic Ridge (MAR) the statistics of the DDMs as well as their differences are shown in Table 3.

Table 3. Statistics of available DDMs and their differences in MAR. Unit: [m].

Model	max	min	mean	std
JGP95E	-999.00	-5440.00	-3262.12	±719.21
S&SV9	-451.00	-6124.00	-3313.29	±722.19
S&SV9-JGP95E	2110.33	-1525.19	-35.411	±235.794

In Table 4 we present the statistics of the 39248 shipborne gravity data in MAR before and after the reduction to

EGM96 as well as after the RTM reduction using the two global models.

Table 4. Observed and EGM96 reduced gravity data. Unit: [mGal].

	max	min	mean	std
Δg	156.00	-77.18	29.60	±26.82
Δg_{red} (EGM96)	119.19	-86.90	-2.82	±23.97
JGP95E	102.77	-78.338	-4.16	±20.88
S&SV9	84.786	-65.79	-2.40	±17.69

From Table 4 it is again evident that the S&SV9 model smoothes more the gravity data in terms of the std, reducing it by almost 26.2% against 12.9% for JGP95E. Thus, for the MAR the depth a-priori statistical characteristics will be those of the S&SV9 model.

5 Estimation of the Bathymetry Models

The new bathymetry models estimated by the proposed method are developed to provide good results when used in a remove-restore procedure in gravity field modeling. For both areas, we performed two tests. The first of them was based on a two-layer (one interface) model with the two layers representing the seawater and the Earth's upper lithosphere. In this case, the interface corresponds to the ocean bottom topography and a constant density contrast of 1.67gr/cm^3 was assumed. In the second test, we used a three-layer model (two interfaces), with the third layer representing the upper mantle. In this case, the second interface corresponds to the crust-mantle boundary, with a constant density contrast of 0.6gr/cm^3, and the depths to the isostatic response of the bottom ocean topography. For both areas and all tests, the a-priori statistical characteristics of the first interface were those of the S&SV9 model (see results in section 4) and the ones for the Moho depths were arbitrary values due to unavailable external information.

5.1 Estimation of the bathymetry model in Gavdos

The main bathymetric features of the area near Gavdos are: the Prolemy Trench and the Hellenic Trench close to the isle of Gavdos; the Mediterranean Ridge, the Cyrene Seamount, and the Cyrene basin in the central part of the area; and finally the Medina (Malta) Ridge on the far west end. These features are depicted in Figure 1.

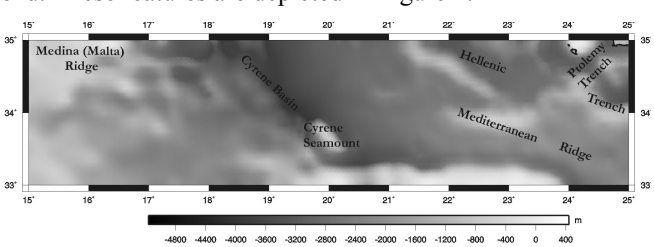

Fig. 1: Bottom topography in Gavdos (S&SV9).

As input to the estimation method, we used the KMS99 gravity anomaly field, which is depicted in Figure 2. Inspection of this plot shows strong negative gravity anomalies in the eastern part of the area, associated with the Hellenic and Prolemy Trenches; positive anomalies are found over the Mediterranean Ridge, the Cyrene seamount and the Malta Ridge. One would expect large negative anomalies over the Cyrene Basin, but these are not depicted in the gravity field. This can be mainly attributed to isostatic compensation of the Cyrene Basin bathymetry, so that its signature is not present in the gravimetric response.

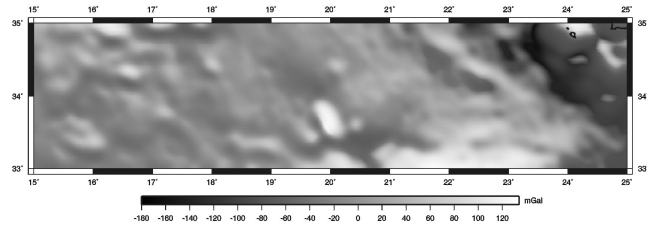

Fig. 2: The observed KMS99 free-air gravity anomalies.

The a priori statistical characteristics of the interface needed by the estimation method, are those from Table 1 (mean and std value) with a constant density contrast between the upper lithosphere and the sea-water of Δρ=1.67 gr/cm^3 and a correlation length of the depth CV function of ξ=25km. The gravity response of the interface having these statistical characteristics produced a CV function with larger variance and considerably smaller correlation length than that of the observed gravity data. This can be attributed to unknown geophysical features in the area, errors in the S&SV9 DDM, the value of the density contrast used, and to the absence of small wavelength characteristics of the bathymetry in the gravity field. Since we had no information about the real value of $\Delta\rho$, we decided to modify the other characteristics of the interface. After several trials, a decreased std to 750m and an increased correlation length of 45km gave a very good fit between the empirical and the model CV functions (see Figure 3). The two-layered statistical model was finally based on a varying topography with –2.297km mean depth, depth std of 0.75km, correlation length equal to 45km and a density contrast of 1.67gr/cm^3.

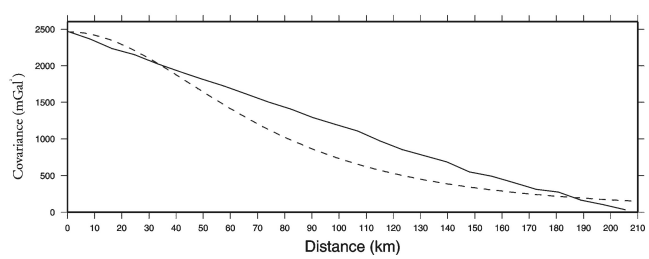

Fig. 3: Empirical CV function of gravity anomalies (solid line) and model CV function (dashed line) using modified parameters (bottom).

Based on these statistical characteristics, the estimation of the new bathymetry model was carried out according to the iterative LSC procedure described in Sect. 2. A very good agreement between the observed gravity anomaly field and the gravimetric response of the new bathymetry model was achieved after three iterations. The results of the iterations are presented in Table 5. This agreement is at the ±0.75mGal level and is considered as satisfactory compared to the noise level of the input gravity data (10 mGal). The statistical characteristics of the new bathymetry model are presented in Table 6 and the new model is depicted in Figure 4.

Table 5. Statistics of differences between observed and calculated gravity. Unit: [m]

Iteration	max	min	mean	std
1	32.37	-30.05	0.27	±4.07
2	12.63	-12.69	0.02	±1.03
3	12.43	-12.51	0.00	±0.75

Table 6. Statistics of the new bathymetry model. Unit: [m]

No	max	min	mean	std
2575	-380.00	-5160.00	-2730.97	±756.45

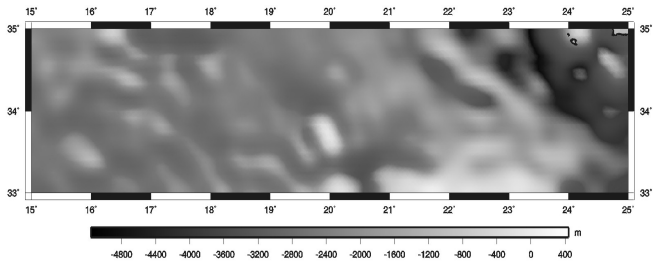

Fig. 4: The new bathymetry model in Gavdos.

Comparing the patterns between Figures 1 and 4 we can see that the main bathymetric features of the area (Ptolemy Trench, Hellenic Trench, Cyrene Seamount, Cyrene Basin and Malta Ridge) depicted by the S&SV9 model are preserved. Only the Cyrene Basin is not depicted very well and this can be attributed to the fact that its response is not present in the input gravity field. This can be due to the isostatic compensation and/or sedimentation burial of the depth features of the basin.

In the second test for the same area, we assumed a three-layer model with the a-priori statistical characteristics shown in Table 7. The gravity response of the interface having these statistical characteristics produced a CV function with considerably smaller variance and correlation length than that of the observed gravity data. This can be attributed to an overestimation of the isostatic compensation in the specific area. After several trials the modified parameters, presented with boldfaced numbers in Table 7, provided a very good agreement between the empirical and the model CV functions. The last column of Table 7 represents the correlation of the layer below the interface to the one above it.

Table 7. A-priori and modified statistical characteristics for the three-layer model. Unit: [km]

Interf.	mean	std	ξ	Δρ (gr/cm^3)	corr.
1	-2.29	0.86 **0.89**	65 **70**	1.67	1
2	-25	2 **0.55**	50 **55**	0.6	-1 **–0.97**

The estimation of the new bathymetry model was carried out using these modified parameters and the input gravity data. After three iterations, a very good agreement between the observed and the calculated gravity field was achieved. The statistical characteristics of the bathymetry model having this gravimetric response are shown in Table 8.

Table 8. Statistics of the new bathymetry model.

Interf.	max	min	mean	std
1 (in m)	-180.00	-5740.00	-2777.24	±916.49
2 (in km)	-20.82	-14.70	-17.86	±0.62

5.2 Estimation of the bathymetry model in MAR

We performed the same tests (two- and three-layer models) in the MAR, as well. The main bathymetric features in this area are: the Min Atlantic Ridge; the Kurchatov and Maxwell fracture zones; the Kings Trough; the Azores Biscay Rise; the Procupine Abyssal Plain; and the Peek and Freen Deeps as shown in Figure 5.

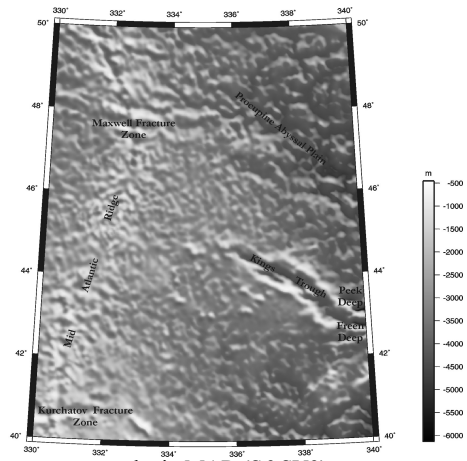

Fig. 5: Bottom topography in MAR (S&SV9).

For the two-layer test, the a-priori statistical characteristics of the interface are those from Table 3 (mean and std value) with a constant density contrast between the upper lithosphere and the seawater of $\Delta\rho$=1.67 gr/cm^3 and a correlation length of the depth CV function of ξ=40km. The gravity response of the interface having these statistical characteristics produced a CV function with considerably larger variance and correlation length than that of the observed gravity data. This can be attributed to the isostatic compensation of the bottom topography in the area, i.e., long-wavelength bathymetry features are not depicted in the gravity field. After several trials, a std to 310m and a correlation length of 20km gave a very good fit between the empirical and the model CV functions. The two-layer statistical model was finally based on a varying topography with –3.313km mean depth, depth std of 0.31km, correlation length equal to 20km and a density contrast of 1.67gr/cm^3. Using the bathymetry estimation method, we achieved after three iterations a very good agreement between observed gravity and the gravimetric response of the new estimated bathymetry model. The statistical characteristics of the new bathymetry model are presented in Table 9 and the new model is depicted in Figure 6.

Table 9. Statistics of the new bathymetry model. Unit: [m]

No	max	min	mean	std
2240	-750.00	-5560.00	-2839.73	±390.67

Comparing Figures 5 and 6 we can see that the main bathymetric features of the area depicted by the S&SV9 model are preserved in our estimate. Only the Procupine Abyssal Plane is not depicted very well, probably due to isostatic compensation. Also, some short wavelength features in the middle of the area are attenuated, obviously due to the upward continuation of the gravimetric response from the ocean bottom to the sea surface.

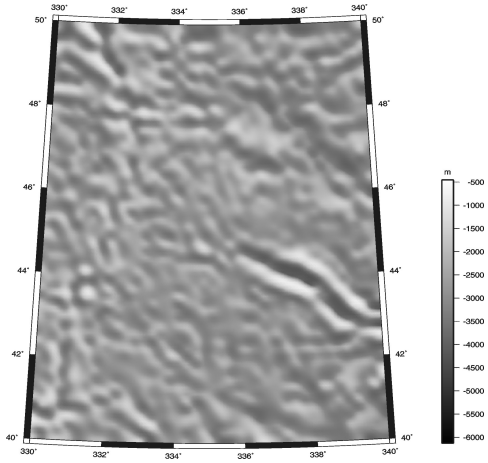

Fig. 6: The new bathymetry model in MAR.

In the second test, we assumed a three-layer model with the a-priori statistical characteristics shown in Table 10. The gravity response of the interface having these statistical characteristics produced a CV function with greater variance and correlation length than that of the observed gravity data. After several trials, the modified parameters, presented with boldfaced numbers in Table10, provided a very good agreement between the empirical and the model CV functions.

Table 10. A-priori and modified statistical characteristics for the three-layer model. Unit: [km]

Interf.	mean	std	ξ	Δρ (gr/cm^3)	corr.
1	-3.31	0.72 **0.65**	40 **35**	1.67	1
2	-25 **-18**	2 **2**	50 **35**	0.6	-1 **-0.97**

Using these modified parameters and the input gravity data the estimation of the new bathymetry model was carried out. After three iterations, a very good agreement between the observed and the calculated gravity field was achieved. The statistical characteristics of the bathymetry model having this gravimetric response are shown in Table 11.

Table 11. Statistics of the new bathymetry model.

Interf. No	max	min	mean	std
1 (in m)	-330.00	-8540.00	-3400.59	±723.76
2 (in km)	-25.67	-6.28	-16.46	±2.08

6 Validation of the Estimated Bathymetry Models

In order to assess the improvement of the new bottom ocean topography models, two tests were carried out. In the first test, the smoothing of shipborne gravity anomalies was studied (1) without using bathymetry information, (2) considering the bathymetry of the JGP95E and S&SV9 models, and (3) considering the bathymetry of the new models. Then for the area in Gavdos we performed steps (1) to (3) but using this time ERS1-GM altimetry SSHs. The effect of the attraction of the masses was taken into account with an RTM reduction applied to the gravity and SSH values using the fine grid of each model for the computation in the entire area. The reference grids were formed in each case by taking moving averages over the adjacent blocks.

For the area in Gavdos, the statistics of the raw shipborne gravity data referenced to EGM96 ($\Delta g_{f\ red}$) and the RTM-reduced ones (raw – EGM96 - RTM) are shown in Table 12. From this table we can see that, for both tests, the new models smoothed the gravity data by about 50% more than the best of the global ones (S&SV9) did. For our models the std decreased from 26.88mGal to 22.78mGal for the first test and 22.76mGal for the second one (reduction at the 15.3% level). The improvement is also seen in terms of the mean value, which dropped almost to 0 mGal when the estimated models were used. On the other hand, when the global DDMs were employed, the mean value increased. Performing the same test but this time on ERS1-GM SSHs, we arrived at the same conclusions. Table 13 shows the statistics of the referenced to EGM96 SSHs as well as the RTM-reduced values using the new models and the two global ones. Again, the new models perform better than the global ones and reduce the std by 3cm compared to 2cm for S&SV9 and an increase of 2cm for JGP95E.

Table 12. Statistics of the EGM96-reduced and RTM-reduced sea-gravity data (21699 values). Unit: [mGal]

Model	$\Delta g_{f\,red}$	TEST1	TEST2	JGP95E	S&SV9
mean	1.66	0.07	-0.04	3.17	1.99
std	26.88	22.78	22.76	28.16	24.60
std impr.	-	15.3%	15.4%	-4.8%	8.5%

Table 13. Statistics of the EGM96-reduced and RTM-reduced ERS1-GM SSHs (8712 values). Unit: [m]

Model	SSH_{red}	TEST1	TEST2	JGP95E	S&SV9
mean	0.02	0.00	0.00	0.04	0.00
std	0.24	0.21	0.21	0.26	0.22
std impr.	-	12.5%	12.5%	-8.3%	8.3%

For the MAR area we used 39248 shipborne gravity anomalies to assess the improvement in bathymetry estimation. In Table 14, we present the statistics of the gravity anomalies referenced to EGM96 as well as the RTM-reduced ones. The estimated bathymetry model from test (1) provides the overall best smoothing by decreasing the std to 17.07mGal from 23.97mGal (28.8% improvement) and the mean to -1.71mGal from -2.82mGal (39.6% improvement). The model from test (2) does not perform so well (std improvement 12.7% and mean improvement 8.5%) and this can be mainly attributed to the arbitrary characteristics used for the second interface. The best of the global models (S&SV9) performs slightly worse than ours from test (1), in terms of the std improvement, but about 3 times worse in terms of the mean.

Table 14. Statistics of the EGM96-reduced and RTM-reduced sea-gravity data (39248 values). Unit: [mGal]

Model	$\Delta g_{f\,red}$	TEST1	TEST2	JGP95E	S&SV9
mean	-2.82	-1.71	-2.58	-4.16	-2.40
std	23.97	17.07	20.93	20.88	17.69
std impr.	-	28.8%	12.7%	12.9%	26.2%

7 Conclusions – Future Plans

A method of improving the estimation of bottom ocean topography and its gravimetric response by inverting gravity field related quantities has been presented. The smoothing of the gravity field was considerably better when

the new bathymetry models were used instead of the global ones. For the area near Gavdos the std of the shipborne gravity anomalies decreased 15.3% when the two- and three-layer models were used. For both models the mean value of the RTM-reduced gravity anomalies dropped to almost 0mGal. The best of the global models decreased the std by only 8.3% and didn't manage to decrease the mean value. In the case of the altimetry data the estimated models performed equally well by decreasing the std 12.5% comparing to 8.3% for the S&SV9 model. The fact that the data used to validate the new models were not the same as the input to the inversion provides an external validation of the improvement in bathymetry estimation.

For the area in MAR the two-layer model decreased the std of the observed shipborne gravity data by 29% while the three-layer one by 13%. The S&SV9 model provided a std decrease of 26% performing slightly worse than our model from TEST1. The inferior smoothing of the more complicated two-interface model can be attributed to the arbitrary characteristics used for the Moho interface.

For both areas and all tests the new models and S&SV9 outperformed JGP95E. This provides substantial proof to the fact that altimetry data can be used to improve the gravity response of the bottom ocean topography. From the results of this study it can be concluded that the *bathymetry estimation method* successfully manages to provide depth models which, when used in an RTM reduction during a remove-restore procedure, give a better smoothing of quantities related to the gravity field. It should be mentioned though that the estimated bottom ocean topography models might not necessarily represent an improved sea bathymetry, due to the insufficient knowledge of the true density contrasts and the characteristics of the Moho interface. Another major factor is the band-limited depth information that altimetry-derived gravity offers. To overcome this limitation, and in the frame of our future investigations, available depth soundings will be implemented in the LSC procedure.

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