Evaluation of Geoid Models and Validation of Geoid and GPS/Leveling Undulations in Canada

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ABSTRACT
The purpose of this paper is to evaluate available geoid models in the region of Canada and validate the differences between geoid and GPS/Leveling undulations. The geoid heights come from six different sources; three from global geopotential solutions, namely OSU91A, EGM96, and GPM98b; and three more from national gravimetric models for the area under study, namely GSD95, GARR98, and GSD2000. The GPS-derived ellipsoidal heights and the leveling-derived orthometric ones come from the Geodetic Survey Division's *GPS on BM's* database. To minimize the differences between geoid and GPS/Leveling undulations we tested a four- and a five-parameter transformation model performing tests in both a national and a regional scale. Results show that the differences between the available geoid models range between ±65cm and ±111cm in terms of the standard deviation when the ones between geoid and GPS/Leveling undulations vary between ±10cm and ±98cm after the bias and tilt fit. The overall best agreement (±10cm) is achieved when GSD2000 geoid heights, the 1998 readjusted orthometric heights and the 5-parameter transformation model are used.

KEYWORDS
Geoid heights, GPS ellipsoidal heights, Leveling orthometric heights, transformation model.

1 Introduction

The determination of orthometric heights by traditional techniques, such as spirit leveling, is known to be a difficult task. This is especially evident in big countries like Canada where the establishment of a leveling network covering all parts of the country would be impractical from the financial point of view [Sideris et al. 1992]. Moreover leveling over areas with rough terrain, like the Rocky Mountains in western Canada, is very strenuous and time consuming. On the other hand the combined use of GPS and geoid heights presents an alternative potential to the classic geometric leveling. Differential GPS measurements are known for their high-accuracy and geoid models are becoming more and more accurate in the aim of achieving the 1cm geoid.

These facts make the study of Geoid and GPS/Leveling differences necessary for both practical surveying and scientific applications. To this extend many studies have been performed in different places all over the world and as examples we mention [Forsberg and Madsen 1990], [Fotopoulos et al. 1999b], [Kearsley et al. 1993], and [Mainville et al. 1992] among others. In this paper we want to investigate whether the combination of geoid undulations and GPS derived geometric heights, gives sufficiently accurate results in order to stand in for precise leveling.

The area under study extends from $41^o \leq \varphi \leq 73^o$ and $215^o \leq \lambda \leq 315^o$ covering the entire territory of Canada, parts of northern USA and ocean regions in the west and east coast. For the geoid undulations we used six different models, namely three global geopotential and three national gravimetric geoid solutions for Canada. The first geopotential model was OSU91A complete to degree and order 360 and developed in Ohio State University [Rapp et al. 1991]. The second was the join NASA GSFC and NIMA geopotential model EGM96 complete to degree and order 360 [Lemoine et al. 1998]. The final one used was the new ultra-high-degree, complete to degree and order 1800, global geopotential model [Wenzel 1999]. The three other geoid models represent national gravimetric geoid solutions for Canada. GSD95 is the official geoid of the country and was computed, using OSU91A as a reference field, in 1995 by the Geodetic Survey Division (GSD) of Canada [Véronneau 1997]. GARR98 is a gravimetric geoid solution for Canada and parts of the US created, using EGM96 as a reference field, in the Department of Geomatics Engineering of the University of Calgary [Fotopoulos et al.

1999a]. Finally, GSD2000 is the newly computed gravimetric geoid for Canada, which is at the final stages of its development by the GSD of Canada [Véronneau 2001]. In the next paragraph we present the comparisons between all these models in a national scale.

The GPS/Leveling data refer to 1587 benchmarks (BMs) across Canada, which are part of the first-order Canadian leveling network. On these points orthometric heights in the Canadian Vertical Geodetic Datum 1928 (CVGD28) and GPS derived ellipsoidal heights are available and will be denoted from now on as *H28* and *h* respectively [Canon 1928]. We also had available orthometric heights on 1443 benchmarks from the readjustment of the Canadian leveling network performed by GSD in 1995, which will be denoted as H98 from now on. A brief outline of the available data sets is provided in the subsequent sections.

To assess the agreement between the geoid undulations and the GPS/Leveling ones, we computed the differences for all the available geoid models and for both the 1928 and the 1998 orthometric heights. The various tests were performed for all of Canada and then for three large regions. These three regions represent; western Canada, between $215^o \le \lambda \le 250^o$; central Canada, between $250^o \le \lambda \le 265^o$; and eastern Canada, between $265^o \le \lambda \le 315^o$. In order to minimize these deviations we used a four- and a five- parameters model, which absorb most of the datum inconsistencies between the available height datasets [Heiskanen and Moritz 1967].

2 Comparisons of geoid models

Global geoid models represent the long wavelength part of the gravity field and are obtained from global geopotential solutions which are given as a set of spherical harmonic coefficients [Sideris et al. 1992]. Different datasets are used to determine these coefficients ranging from satellite observations, which give the so-called satellite-only solutions, to models which incorporate satellite altimetry and surface gravity data in the previous ones, thus usually containing more coefficients. To compute a geoid undulation value N^{GM} in spherical approximation and from such a set of spherical harmonic coefficients we use the following formula [Heiskanen and Moritz 1967]:

$$N^{GM} = R \sum_{n=2}^{n_{max}} \sum_{m=0}^{n} \left(\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda \right) \overline{P}_{nm}(\sin\phi) \qquad (1)$$

where n_{max} is the maximum degree of expansion, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the fully normalized coefficients of the disturbing potential, $\overline{P}_{nm}(\sin\phi)$ are the fully normalized associated Legendre functions, *R* is the mean radius of the earth, and φ and λ are the geodetic latitude and longitude.

Gravimetric solutions on the other hand give a better estimate of the geoid heights by taking into account local gravity measurements. In this way they represent the medium and high frequencies of the gravity field spectrum, thus giving a more detailed picture of local mass distributions and irregularities. For such a geoid computation the well-known remove-compute-restore method is used with the trend nowadays pointing to the use of spectral methods. Spectral methods like the FFT provide an efficient way of handling large datasets in shorter time periods comparing to the classical Least Squares Collocation (LSC). This is significant since modern satellite methods like satellite altimetry offer a vast amount of data. Following this method, and to derive residual geoid undulations from residual, free-air in most cases, gravity anomalies, we can apply the 1-D FFT spherical Stokes convolution using the following formula [Haagmans et al. 1993]:

$$N^{gr} = \frac{R\Delta\varphi\Delta\lambda}{4\pi\gamma} F_1^{-1}\left\{ \sum_{\varphi=\varphi_1}^{\varphi_n} F_1\{S\} F_1\{\Delta g \cos\varphi\} \right\} \qquad (2)$$

where N^{gr} is the gravimetric geoid height, *R* is the mean earth radius, γ is the normal gravity, Δg denotes gravity anomaly, and $S(\psi)$ is Stokes' function.

The geoid models available for the present study incorporate one of the aforementioned methods for the determination of geoid undulations. Their statistics, for the area under study, are presented in Table 1 and in Figure 1 we depict the GPM98b geoid heights and GSD2000 geoid model. All five models do not have the same resolution and the three gravimetric solutions are computed on different points. In order to compare them, we used GSD95 points and its resolution of $5' \times 5'$ as a reference. Thus, GARR98 and GSD2000 geoid heights were interpolated on GSD95 points. GSD2000 has a resolution of $2' \times 2'$ and covers the entire Canada, and most of the USA and Greenland. A bilinear type of interpolation, available in GEOIP a program from the GRAVSOFT

software package, was used [Tscherning et al. 1994]. For the comparisons between the geopotential models and the gravimetric solutions we computed the contribution of each one of the geopotential models to geoid heights on the GARR98, GSD95, and GSD2000 points. The statistics of the differences between the available geoid models are presented in Table 2. For better visualization we depict in Figures 2 and 3 some of these differences.
From Table 2 and Figures 2 and 3 we can see that the overall best agreement is achieved between GSD2000 and EGM96 and is at the ±37cm in terms of the standard deviation of the differences. This is something expected since EGM96 was used as a reference field for the development of GSD2000, thus their difference represents the high frequency part of the gravity field signal in Canada. The greatest differences are present when comparing OSU91A with the other models and they range between ±90cm to ±111cm. The latter corresponds to the differences between OSU91A and GPM98b and can be mainly attributed to the additional, more recent and accurate gravity data used in GPM98b.

Table 1. Geoid undulations in the area under study. Unit: [m]

	max	min	mean	rms	std
N^{OSU91A}	39.440	-48.983	-16.541	24.624	±18.320
N^{EGM96}	36.349	-49.104	-16.545	24.545	±18.130
N^{GPM98b}	48.869	-49.534	-15.557	25.391	±20.068
N^{GSD95}	43.875	-48.995	-15.450	25.300	±20.034
N^{GARR98}	48.462	-47.661	-14.550	24.947	±20.297
$N^{GSD2000}$	49.005	-49.585	-15.823	25.574	±20.092

Table 2. Geoid height differences in Canada. Unit: [m]

	max	min	mean	rms	std
N^{OSU91A} - N^{GPM98b}	12.213	-6.503	0.025	1.111	±1.111
N^{EGM96} - N^{OSU91A}	6.941	-10.570	-0.041	0.904	±0.903
N^{EGM96} - N^{GPM98b}	5.532	-2.919	-0.016	0.664	±0.664
N^{GSD95} - N^{OSU91A}	10.475	-10.466	0.096	0.970	±0.965
N^{GSD95} - N^{EGM96}	6.025	-5.553	0.120	0.663	±0.653
N^{GSD95} - N^{GPM98b}	6.266	-6.006	0.107	0.895	±0.888
N^{GSD95} - N^{GARR98}	-1.237	5.306	-0.900	1.099	±0.631
N^{GARR98} - N^{OSU91A}	11.158	-10.230	0.945	1.384	±1.011
N^{GARR98} - N^{EGM96}	7.477	-1.976	1.020	1.266	±0.751
N^{GARR98} - N^{GPM98b}	7.845	-2.733	1.007	1.422	±1.004
$N^{GSD2000}$ - N^{OSU91A}	11.208	-10.769	-0.291	1.036	±0.994
$N^{GSD2000}$ – N^{EGM96}	3.238	-5.624	0.250	0.446	±0.369
$N^{GSD2000}$ – N^{GPM98}	6.228	-6.308	-0.373	0.638	±0.518
$N^{GSD2000}$ – N^{GSD95}	6.727	-3.567	-0.266	0.797	±0.751
$N^{GSD2000}$ – N^{GARR98}	7.751	-1.974	1.272	1.430	±0.653

These big differences are especially evident in Western Canada, near the Rocky Mountains, and in the Northeastern parts of our area near Greenland. In these areas the discrepancies reach the 10.5m for the comparisons of OSU91A with EGM96 and GSD95, 11.2m with GARR98 and GSD2000, and 12.2m with GPM98b. The main reasons for these great differences can be found in the strong effect of the additional gravity data used and to the roughed and difficult to model terrain of the Rockies. Some big differences between EGM96 and OSU91A are also present in Hudson Bay, the St. Lawrence Gulf, and the Labrador Sea, which are mainly due to the additional and more accurate altimetry data used in EGM96 (see Figure 2(a)). The overall best agreement for the discrepancies between all models is found in Central Canada, ranging between 50cm – 1m. The main reason for these small values can be viewed in the absence of rough topography.

The differences between GSD95 and GARR98 have a standard deviation of ±63cm with a value range of 6.5m. These deviations can be explained if we take into account that GSD95 uses OSU91A as a reference field when GARR98 uses EGM96. For the comparison between GSD95 and EGM96 the standard deviation of the differences is at the ±65cm, representing the high frequency information included in GSD95 with the local gravity anomalies used versus the long wavelength nature of EGM96.

In Figure 3(b) we depict the differences between GSD2000 and GSD95, which have a standard deviation of ±75cm. These differences are correlated with the topography across the Rockies and reach their greatest value of

6.7m in Greenland. This is so because of the use of additional gravity data in the development of EGM96 versus OSU91A. Additionally some significant values of the order of 50-70cm can be seen in the ocean areas, which are mainly attributed to the multi-satellite altimetry data used in GSD2000, where Geodetic Mission data were incorporated as well. From Figures 2(b) and 3(a), where the discrepancies between GPM98b and EGM96 and GSD95, are depicted respectively, we can see a very interesting pattern. The structure of this pattern changes right across the border of Canada and the USA and it can be seen that there is a direct correlation with the borderline. We can attribute this to the use of new gravity data for USA when the same as in EGM96 data were used for Canada. The pattern of the differences that is seen in USA agrees very well with the case in the oceans were more recent and more accurate altimetry data were used.

3 Comparisons of geoid and GPS/Leveling undulations

The basic relationship between geoid, ellipsoidal, and orthometric heights is very simple, given by the following formula:

$$H = h - N \qquad (3)$$

Because of the simplicity of Eq. 3, the combined use of geoidal undulations and GPS-derived ellipsoidal heights is so appealing in comparison to the use of orthometric heights derived from spirit leveling. For our study we used 1587 benchmarks, which are part of the first-order leveling network of Canada.

On these points orthometric heights in the CGVD28 vertical datum (*H28*) are available. They were determined in 1928 and they belong to the official height system of Canada. The heights were determined with a differential adjustment, tied to the Atlantic Mean Sean Level of 1928 as computed in the Halifax, Yarmouth, and Father Point tide gauge stations and an elevation agreed upon for Rouses Point [Canon 1928]. For the 1928 set of orthometric heights, approximate normal gravity values were used instead of true gravity measurements. The second set of orthometric heights (*H98*) comes from the adjustment performed by GSD in October 1995. This was an attempt to improve CGVD28 because of various systematic distortions occurred during the time period since 1928. Factors creating these distortions can be viewed in terms of mean sea level rise and post-glacial rebound (Fotopoulos et al. 1999a). The *H98* set corresponds to Helmert like orthometric heights, which were computed using actual surface gravity measurements. A minimal constrained type of adjustment, of the entire leveling network of Canada, was performed, by retaining fixed a single point at the tide-gauge station in Rimouski, Quebec. This set of orthometric heights is characterized by GSD as a scientific database and is not to be used in practical applications. 1443 benchmarks with *H98* heights are available and the lack of data comparing to *H28* is located in the island of Newfoundland and British Columbia (see Figure 4). The known GPS-derived ellipsoidal heights (h^{GPS}) were determined during GPS campaigns conducted by GSD between 1987 and 1996, with observation periods ranging between 4 to 48 hours. They refer to ITRF96 reference frame (reference epoch 1997.0) and the GRS80 ellipsoid.

For our comparisons we used all available geoid models and both sets of orthometric heights. We first interpolated geoid undulations from the geoid models on the GPS benchmarks where orthometric and ellipsoidal heights were available, forming five N^{GM} datasets. To do so we used a bilinear type of interpolation implemented in GEOIP, a program from GRAVSOFT. In a next step, using Eq. 3, we determined on the same points the GPS/Leveling undulations N^{GPS}. In this way three different datasets were formed. 1587 points with geoid heights using H28 set, 1443 points using H98 and 1443 points using H28 but only on the points where information for the H98 heights was available. The statistics of the interpolated geoid model undulations, the H28 and H98 orthometric heights and the geometric heights are presented in Table 3. Then for each one of the GPS/Leveling geoid height datasets we determined the differences with the previously interpolated N^{GM} on BMs.

The computed differences reflect datum inconsistencies between the available height data, long-wavelength geoid errors, and GPS and leveling errors included in the ellipsoidal and orthometric heights. In order to minimize these deviations we used a four- and a five-parameter transformation models. The four-parameter model is the most commonly used in such adjustments and is given by the following formula [Heiskanen and Moritz 1967, Sideris 1992]:

$$N_i^{GPS} - N_i^{GM} = h_i^{GPS} - H_i^k - N_i^{GM} = b_3 + b_o \cos\phi_i \cos\lambda_i + b_1 \cos\phi_i \sin\lambda_i + b_2 \sin\phi_i + v_i \qquad (4)$$

where the parameters b_o, b_1, b_2 and b_3 are calculated by a least squares technique, (*i*) denotes the benchmarks, (*k*) represents either *H28* or *H98*, v_i are the residuals to be minimized, N^{GPS} and N^{GM} are the GPS/Leveling and geoid undulations, and φ and λ are the geodetic latitude and longitude respectively. The five-parameter model is similar to Eq. 4 with one additional parameter added. Following Heiskanen and Moritz [1967] it is given by the following formula:

$$N_i^{GPS} - N_i^{GM} = h_i^{GPS} - H_i^k - N_i^{GM} = b_4 + b_o \cos\phi_i \cos\lambda_i + b_1 \cos\phi_i \sin\lambda_i + b_2 \sin\phi_i + b_3 \sin^2\phi_i + v_i \quad (5)$$

where now the parameters b_o, b_1, b_2, b_3 and b_4 are determined minimizing the residuals v_i using a least squares technique. The other parameters are the same as in Eq. 4. An explicit derivation and geometrical interpretation of the two models is given in Heiskanen and Moritz [1967, Section 5.9]. We use the two different models in order to assess the possible improvement that the additional parameter offers in Eq. 5. As we mentioned before the residuals v_i, which are to be minimized, contain a wide spectrum of different in nature errors resulting from different in accuracy methods. With Eqs. 4 and 5 equal weight is given in all observations during the least squares adjustment. More rigorous models can be found in Kotsakis [2001], were an extended seven-parameter similarity transformation model is used with equal or slightly better results than that of Eq. 5, and Kotsakis [1999].

The statistics of the computed differences between the available datasets can be found in Tables 4 to 6. The differences between geoid and GPS/Leveling undulations, when using H28 orthometric heights on all 1587 benchmarks, are presented in Table 4, when using H98 on the 1443 points they refer to in Table 5, and when using H28 data but on the same points as H98, in Table 6. The differences after the bias and tilt fit using the four- and the five-parameter models are presented in parenthesis with bold letters and with italics respectively. From Table 4 and for the 1928-orthometric heights, we can see that the best agreement, at the ±22cm level, before the bias and tilt fit is offered by GSD2000 geoid when for GARR98 and GSD95 it is at the ±28cm and ±32cm respectively. This can be mainly attributed to the more accurate long-wavelength information contained in EGM96 in comparison to OSU91A, which is the reference field used for the development of GSD95. The ±105cm of difference when using geoid heights from GPM98b is due to the absence of new data for Canada in the development of this model. On the other hand new data were used for USA and the surrounding oceans influencing in this way the harmonics over Canada as well. In a study conducted by Vergos et al. [2001] in the Atlantic part of Canada, GPM98b gives better agreement with TOPEX/POSEIDON Sea Surface Heights (SSHs), comparing to EGM96, when used for marine geoid modeling.

Table 3. Geoid, orthometric, and ellipsoidal heights on benchmarks. Unit: [m]

	max	min	mean	rms	std
N^{OSU91A}	12.910	-46.617	-20.492	23.740	±11.442
N^{EGM96}	11.940	-47.854	-20.555	23.566	±11.527
N^{GPM98b}	13.014	-47.873	-20.511	23.488	±11.446
N^{GSD95}	12.215	-47.680	-20.538	23.535	±11.493
N^{GARR98}	12.786	-46.652	-19.843	22.916	±11.462
$N^{GSD2000}$	11.554	-48.341	-20.855	23.820	±11.509
H28	2010.086	1.101	436.141	562.179	±354.720
H98	2011.220	2.104	472.072	588.913	±352.089
h^{GPS}	1999.086	-42.465	414.982	548.242	±358.273

This is further evidence that the lack of new data in the continental part of Canada is the main factor causing this high disagreement between GPM98b and GPS/Leveling geoid heights. For OSU91A and EGM96 the differences are at the ±70cm and ±38cm before the fit indicating the more accurate long-wavelength information included in the latter. After the bias and tilt fit, using the 4-parameter model, the improvement is at the 4cm for OSU91A, the 5cm for EMG96, the 18cm for GPM98b, the 9cm for GSD95 and GARR98, and the 2cm for GSD2000. The overall best agreement after the fit is achieved by GARR98 being at the ±19cm. When using the five-parameter model for the minimization of the residuals the improvement, comparing to the four-parameter one, is not great. The differences improve by 5mm for OSU91A, by 2cm for GPM98b and by 2cm for the two GSD models. The improvement for both EGM96 and GARR98 is negligible. These results indicate that the four-parameter model is adequate enough for the minimization of the differences in this kind of studies.

Table 4. Comparisons of various geoid models with 1928 orthometric heights before and after the bias and tilt fit. Unit: [m]

	max	min	mean	rms	std
N^{GPS} - N^{OSU91A}	5.099	-4.953	-0.668	0.963	±0.694
	(5.203) (*5.290*)	**(-4.712)** (*-4.756*)	**(0.000)** (*0.000*)	**(0.658)** (*0.653*)	**(±0.658)** (*±0.653*)
N^{GPS} - N^{OSU91A}	1.787	-1.905	-0.700	0.861	±0.501
(after 2rms)	**(2.337)** (*2.427*)	**(-1.385)** (*-1.459*)	**(0.000)** (*0.000*)	**(0.474)** (*0.459*)	**(±0.474)** (*±0.459*)
N^{GPS} - N^{EGM96}	1.262	-1.468	-0.605	0.717	±0.384
	(1.531) (*1.477*)	**(-1.055)** (*-1.113*)	**(0.000)** (*0.000*)	**(0.332)** (*0.323*)	**(±0.332)** (*±0.323*)
N^{GPS} - N^{GPM98b}	4.389	-2.652	-0.649	1.241	±1.058
	(4.280) (*4.179*)	**(-2.088)** (*-2.263*)	**(0.000)** (*0.000*)	**(0.889)** (*0.868*)	**(±0.889)** (*±0.868*)
N^{GPS} - N^{GPM98b}	2.365	-2.373	-0.694	1.188	±0.965
(after 2rms)	**(2.662)** (*2.515*)	**(-1.813)** (*-1.965*)	**(0.000)** (*0.000*)	**(0.811)** (*0.789*)	**(±0.811)** (*±0.789*)
N^{GPS} - N^{GSD95}	0.921	-1.338	-0.623	0.701	±0.322
	(1.680) (*1.601*)	**(-0.613)** (*-0.618*)	**(0.000)** (*0.000*)	**(0.231)** (*0.209*)	**(±0.231)** (*±0.209*)
N^{GPS} - N^{GARR98}	-0.361	-2.470	-1.317	1.346	±0.280
	(1.278) (*1.278*)	**(-0.843)** (*-0.843*)	**(0.000)** (*0.000*)	**(0.190)** (*0.189*)	**(±0.190)** (*±0.189*)
N^{GPS} - $N^{GSD2000}$	1.204	-1.012	-0.350	0.376	±0.220
	(1.630) (*1.552*)	**(-0.620)** (*-0.375*)	**(0.000)** (*0.000*)	**(0.200)** (*0.176*)	**(±0.200)** (*±0.176*)

Table 5. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit. Unit: [m]

	max	min	mean	rms	std
N^{GPS} - N^{OSU91A}	4.087	-5.923	-1.112	1.305	±0.682
	(5.191) (*5.210*)	**(-4.719)** (*-4.759*)	**(0.000)** (*0.000*)	**(0.668)** (*0.656*)	**(±0.668)** (*±0.656*)
N^{GPS} - N^{EGM96}	0.359	-2.574	-1.053	1.113	±0.360
	(1.582) (*1.574*)	**(-1.306)** (*-1.425*)	**(0.000)** (*0.000*)	**(0.302)** (*0.302*)	**(±0.302)** (*±0.302*)
N^{GPS} - N^{GPM98b}	3.419	-3.497	-1.093	1.454	±0.960
	(4.075) (*3.909*)	**(-2.089)** (*-1.904*)	**(0.000)** (*0.000*)	**(0.838)** (*0.815*)	**(±0.838)** (*±0.815*)
N^{GPS} - N^{GSD95}	0.273	-2.093	-1.090	1.176	±0.442
	(0.616) (*0.515*)	**(-0.669)** (*-0.760*)	**(0.000)** (*0.000*)	**(0.136)** (*0.129*)	**(±0.136)** (*±0.129*)
N^{GPS} - N^{GARR98}	-1.198	-2.702	-1.742	1.752	±0.205
	(0.461) (*0.431*)	**(-1.057)** (*-0.999*)	**(0.000)** (*0.000*)	**(0.143)** (*0.137*)	**(±0.143)** (*±0.137*)
N^{GPS} - $N^{GSD2000}$	0.076	-1.402	-0.744	0.786	±0.254
	(0.470) (*0.403*)	**(-0.530)** (*-0.448*)	**(0.000)** (*0.000*)	**(0.106)** (*0.101*)	**(±0.106)** (*±0.101*)

Table 6. Comparisons of various geoid models with 1928 orthometric heights (on the H98 points) before and after the bias and tilt fit. Unit: [m]

	max	min	mean	rms	std
N^{GPS} - N^{OSU91A}	5.099	-4.953	-0.681	0.978	±0.702
	(5.146) (*5.192*)	**(-4.716)** (*-4.816*)	**(0.000)** (*0.000*)	**(0.672)** (*0.662*)	**(±0.672)** (*±0.662*)
N^{GPS} - N^{EGM96}	1.262	-1.468	-0.622	0.719	±0.360
	(1.582) (*1.534*)	**(-1.004)** (*-1.075*)	**(0.000)** (*0.000*)	**(0.326)** (*0.314*)	**(±0.326)** (*±0.314*)
N^{GPS} - N^{GPM98b}	4.389	-0.661	-0.661	1.252	±1.063
	(4.078) (*3.852*)	**(-2.123)** (*-1.900*)	**(0.000)** (*0.000*)	**(0.879)** (*0.838*)	**(±0.879)** (*±0.838*)
N^{GPS} - N^{GSD95}	0.584	-1.338	-0.658	0.721	±0.294
	(0.951) (*0.694*)	**(-0.637)** (*-0.730*)	**(0.000)** (*0.000*)	**(0.223)** (*0.193*)	**(±0.223)** (*±0.193*)
N^{GPS} - N^{GARR98}	-0.690	-2.470	-1.310	1.341	±0.285
	(0.622) (*0.552*)	**(-0.812)** (*-0.856*)	**(0.000)** (*0.000*)	**(0.176)** (*0.174*)	**(±0.176)** (*±0.174*)
N^{GPS} - $N^{GSD2000}$	0.404	-1.012	-0.313	0.382	±0.221
	(0.621) (*0.587*)	**(-0.814)** (*-0.417*)	**(0.000)** (*0.000*)	**(0.198)** (*0.169*)	**(±0.198)** (*±0.169*)

From Table 5, and for the readjusted 1998-orthometric heights, GARR98 offers again the best agreement, at the ±21cm, before the fit, comparing to ±25cm and ±44cm for GSD2000 and GSD95. But, after the fit GARR98 and GSD95 reduce both to almost ±14cm and ±13cm, when GSD2000 gives the overall best agreement at the ±10.6cm and ±10cm level, for the four- and the five-parameter models respectively. This would suggest that even when using the superior EGM96 to model the long-wavelength part of the geoid, the available resolution and accuracy of the gravity data is not sufficient enough to bring the agreement with GPS/Leveling at the sub-decimeter level. Nevertheless, the achieved ±10cm for GSD2000 geoid heights provide promising results for the usual, low accuracy, surveying applications. When using the four-parameter model the fit improvement is almost at the 3cm level for OSUA91A, the 6cm for EGM96 and GARR98, the 14cm for GSD2000 and GPM98b, and the 31cm for GSD95. Again the use of the five-parameter model fails to give significant improvement comparing to its four-parameter counterpart, since their differences range from a few mm to 3cm. Nevertheless the overall best fit is seen when using GSD2000, the H98 orthometric heights, and the five-parameter model.

Comparing Tables 5 and 6, where the statistics of the differences when using the H98 and the H28 on the same points are presented respectively, we can see that in all comparisons the 1998-radjusted heights give better agreement with almost all geoid models. The improved fit ranges between 1cm and 4cm indicating the superiority of the new readjusted orthometric heights. As expected the H98 heights fail to give better results comparing to the H28 ones when computing their differences with the oldest geoid model, OSU91A. This offers an additional proof that OSU91A fails to model the low-frequency part of the geoid, at least as accurately as EGM96 does. Of course this outcome holds for the specific area under study and the specific data sets and methods used. In remote areas, where no new gravity data were added during the last 15 years, this conclusion may not hold.

Comparing the results of this study with the ones from Kotsakis et al. [2001], where an extended seven-parameter similarity transformation model was used, we can conclude that the differences are not significant. When using EGM96 and the five-parameter model we achieved an agreement of ±31.4cm, for the reduced 1928 orthometric heights, and ±30.6cm for the 1998 ones respectively. For the same datasets the extended model resulted in ±29.4cm and ±30.6cm fit between the height data. Substituting EGM96 with GSD95 the five-parameter model gave ±19.3cm fit for the reduced H28 dataset and ±12.9cm for H98, when the extended one resulted in an agreement of ±18.4cm and ±12.7cm respectively. From these results we may conclude that a five-parameter transformation model is capable of minimizing the residuals enough so that an extended version is not necessary. Of course we must not neglect the fact that a few mm improvement will be important if we are after the cm-level fit between our height data. But since the present level of agreement is at the ±10cm, for the best case, such a constraint is not considered to be necessary.

As a final remark in this nationwide investigation, we present in Figure 5 West – East profiles of the residuals after the five-parameter model fit when using H98 orthometric heights and GSD95, GSD2000, OSU91A, EGM96, and GPM98b geoid heights. From (a), (b), (c), and (d) we can see that the greatest differences are located in the Western part of Canada where the Rocky Mountains are situated. This is something expected, since both geoid modeling and spirit leveling are less accurate in areas with rough topography comparing to flat areas like Central Canada. Additionally some big values appear in Eastern Canada and this, for GSD95 and GARR98, can be mainly attributed to the low-quality shipborne gravity data used for geoid modeling near the island of Newfoundland and the St. Lawrence gulf. From Figure 5(a) the improved residuals for the case of GSD2000, comparing to GSD95, are evident, since almost all large deviations are eliminated and the differences look constantly small across Canada. For OSU91A the lack of new and more accurate altimetry data adds to residuals at the ±60cm for the same area. The improvement in modeling the low frequency part of the geoid that EGM96 offers, comparing to OSU91A, is evident from Figure 5(c). We can see that the very big differences in British Columbia, at the ±350cm for OSU91A, are reduced to ±80cm for EGM96. The same improvement can be seen in the Rocky Mountains where once again EGM96 outperforms OSU91A. The absence of new gravity data for Canada in the development of GPM98b is evident in Figure 5 (d), so that the differences are great across Canada and not in the Western part only. The new data used in USA and the oceans and the lack of new information over Canada clearly trigger the main part of the differences between the model and GPS/Leveling undulations. As a concluding remark we mention the obvious superiority of gravimetric geoid solutions in contrast to global geopotential models. It is important to mention at this point that even with gravimetric geoid models the best level of agreement is at the ±10cm level, which is far from the cm-level goal. New, more accurate, and higher resolution gravity data are needed if cm-level geoid and GPS/Leveling residuals are to be obtained.

We will proceed to the regional comparisons where three sub-areas were selected. These three regions represent: western Canada, between $215^o \leq \lambda \leq 250^o$; central Canada, between $250^o \leq \lambda \leq 265^o$; and eastern Canada, between $265^o \leq \lambda \leq 315^o$. The total number of GPS benchmarks is 737, 172, and 534 for western, central, and eastern Canada respectively. For each one of these cases we readjusted the geoid and GPS/Leveling undulations using all available geoid models and orthometric height datasets.

The results obtained from these tests are far too many too be depicted individually, thus we present only the comparisons when using *H98* orthometric heights, since these give the overall best agreement. These results are presented in Tables 7, 8, and 9 for western, central, and eastern Canada respectively where we can see that the largest residuals are found in western Canada. This is expected, since the topography in this area is rough thus making both geoid modeling and traditional leveling less accurate than in plain regions.

For all regions GSD2000 gives the overall best agreement after the bias and tilt fit for both models used. It is interesting to see that the agreement between GSD2000 and GPS/Leveling undulations in western and eastern Canada is approximately at the ±9.5cm when for the central region reaches the ±6cm. The latter level of agreement provides substantial evidence that the current method is appropriate for orthometric height determination over large regions with smooth terrain, like central Canada. Of course even for these cases it cannot reach the millimeter level requirements of a first-order leveling network. Comparing the statistics of the three geopotential solutions for the three regional cases, we can see that they provide a relatively good agreement for central Canada at the ±12cm for OSU91A, the ±11cm for EGM96, and the ±24cm for GPM98b. But the statistics degrade for the eastern part and get worst in the western one, reaching the ±85cm, ±36cm, and ±91cm for the aforementioned models respectively. This is an indication of the fact that global geopotential solutions fail to model the high-frequency part of the gravity field spectrum, in areas with varying topography like the Rocky Mountains, caused by the distribution of the masses. Thus, they are to be avoided when GPS and geoid heights are to be used for orthometric height determination.

Table 7. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit in Western Canada. Unit: [m]

	max	**min**	**mean**	**rms**	**std**
$N^{GPS} - N^{OSU91A}$	4.087 **(4.636)** (*3.800*)	-5.923 **(-5.369)** (*-4.759*)	-1.297 **(0.000)** (*0.000*)	1.324 **(0.851)** (*0.793*)	±0.878 **(±0.851)** (*±0.793*)
$N^{GPS} - N^{EGM96}$	0.359 **(1.557)** (*1.525*)	-2.574 **(-1.286)** (*-1.273*)	-1.236 **(0.000)** (*0.000*)	1.263 **(0.359)** (*0.358*)	±0.369 **(±0.359)** (*±0.358*)
$N^{GPS} - N^{GPM98b}$	3.419 **(3.642)** (*4.223*)	-3.497 **(-1.755)** (*-1.857*)	-1.144 **(0.000)** (*0.000*)	1.459 **(0.915)** (*0.830*)	±1.230 **(±0.915)** (*±0.830*)
$N^{GPS} - N^{GSD95}$	-0.888 **(0.425)** (*0.395*)	-2.093 **(-0.614)** (*-0.602*)	-1.405 **(0.000)** (*0.000*)	1.416 **(0.120)** (*0.119*)	±0.148 **(±0.120)** (*±0.119*)
$N^{GPS} - N^{GARR98}$	-1.441 **(0.402)** (*0.354*)	-2.434 **(-0.543)** (*-0.524*)	-1.833 **(0.000)** (*0.000*)	1.845 **(0.115)** (*0.111*)	±0.119 **(±0.115)** (*±0.111*)
$N^{GPS} - N^{GSD2000}$	-0.458 **(0.388)** (*0.391*)	-1.402 **(-0.295)** (*-0.308*)	-0.941 **(0.000)** (*0.000*)	0.957 **(0.099)** (*0.097*)	±0.139 **(±0.099)** (*±0.097*)

Table 8. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit in Central Canada. Unit: [m]

	max	**min**	**mean**	**rms**	**std**
$N^{GPS} - N^{OSU91A}$	-0.611 **(0.377)** (*0.371*)	-1.428 **(-0.447)** (*-0.417*)	-1.043 **(0.000)** (*0.000*)	1.051 **(0.125)** (*0.125*)	±0.152 **(±0.125)** (*±0.125*)
$N^{GPS} - N^{EGM96}$	-0.551 **(0.423)** (*0.422*)	-1.582 **(-0.531)** (*-0.524*)	-0.980 **(0.000)** (*0.000*)	1.011 **(0.116)** (*0.116*)	±0.142 **(±0.116)** (*±0.116*)
$N^{GPS} - N^{GPM98b}$	-0.120 **(0.596)** (*0.621*)	-2.643 **(-0.788)** (*-0.777*)	-1.270 **(0.000)** (*0.000*)	1.372 **(0.243)** (*0.242*)	±1.230 **(±0.243)** (*±0.242*)
$N^{GPS} - N^{GSD95}$	-1.071 **(0.217)** (*0.241*)	-1.701 **(-0.255)** (*-0.206*)	-1.302 **(0.000)** (*0.000*)	1.304 **(0.070)** (*0.065*)	±0.095 **(±0.070)** (*±0.065*)
$N^{GPS} - N^{GARR98}$	-1.479 **(0.337)** (*0.336*)	-2.272 **(-0.233)** (*-0.168*)	-1.841 **(0.000)** (*0.000*)	1.845 **(0.096)** (*0.087*)	±0.116 **(±0.096)** (*±0.087*)
$N^{GPS} - N^{GSD2000}$	-0.527 **(0.222)** (*0.243*)	-0.994 **(-0.235)** (*-0.170*)	-0.726 **(0.000)** (*0.000*)	0.731 **(0.065)** (*0.061*)	±0.074 **(±0.065)** (*±0.061*)

Table 9. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit in Eastern Canada. Unit: [m]

	max	min	mean	rms	std
$N^{GPS} - N^{OSU91A}$	-0.070 **(0.791)** (*0.808*)	-1.589 **(-0.646)** (*-0.704*)	-0.880 **(0.000)** (*0.000*)	1.051 **(0.271)** (*0.262*)	±0.291 **(±0.271)** (*±0.262*)
$N^{GPS} - N^{EGM96}$	0.132 **(0.898)** (*0.838*)	-1.700 **(-0.822)** (*-0.741*)	-0.826 **(0.000)** (*0.000*)	0.851 **(0.234)** (*0.227*)	±0.240 **(±0.234)** (*±0.227*)
$N^{GPS} - N^{GPM98b}$	0.955 **(1.819)** (*1.686*)	-2.266 **(-1.469)** (*-1.411*)	-0.965 **(0.000)** (*0.000*)	1.251 **(0.509)** (*0.486*)	±0.534 **(±0.509)** (*±0.486*)
$N^{GPS} - N^{GSD95}$	0.273 **(0.456)** (*0.452*)	-1.341 **(-0.832)** (*-0.796*)	-0.586 **(0.000)** (*0.000*)	0.728 **(0.130)** (*0.128*)	±0.302 **(±0.130)** (*±0.128*)
$N^{GPS} - N^{GARR98}$	-1.198 **(0.466)** (*0.435*)	-2.702 **(-0.870)** (*-0.804*)	-1.583 **(0.000)** (*0.000*)	1.737 **(0.141)** (*0.137*)	±0.225 **(±0.141)** (*±0.137*)
$N^{GPS} - N^{GSD2000}$	0.076 **(0.327)** (*0.329*)	-1.030 **(-0.422)** (*-0.466*)	-0.479 **(0.000)** (*0.000*)	0.491 **(0.096)** (*0.094*)	±0.147 **(±0.096)** (*±0.094*)

4 Conclusions

A detailed study on the possibility to substitute classic spirit leveling with GPS/geoid derived orthometric heights was presented. We used; three gravimetric geoid solutions and three global geopotential models to derive geoid undulations; two different sets of orthometric heights, one from the original 1928 adjustment of the entire Canadian leveling network and another from the 1998 readjustment when real gravity measurements were used; and finally GPS derived ellipsoidal heights on BMs. For the minimization of the residuals a four- and a five-parameter transformation models were used, successfully managing to absorb the datum inconsistencies between our height data and GPS, leveling, and long-wavelength geoid, errors.

The comparisons between the available geoid models indicate that the main problems exist in Western Canada mainly due to the roughed topography and the lack of sufficient gravity measurements. The superiority of EGM96 over OSU91A to model the long-wavelength part of the geoid is evident, when the comparisons of GPM98b indicate that the model may not be adequate enough for use in the continental part of Canada due to the lack of new data in this region during its development. For marine regions though, the advantages of the new altimetry data incorporated in GPM98b make it to give better results than EGM96 when ocean geoid modeling in concerned [Vergos et al. 2001].

The overall best agreement, ±10cm, between geoid and GPS/Leveling undulations was achieved when we used height information from GSD95, the 1998-readjustment of the Canadian leveling network, and the five-parameter similarity transformation model. The use of the five-parameter model slightly improves the residuals comparing to the four-parameter one, with their differences ranging between 1mm to 3cm. Orthometric heights from the 1998-readjustment provide better results than the 1928 ones at the 1cm to 4cm level. Furthermore the results for Central Canada reached the ±6cm level for GSD2000 indicating the advantages of this latest gravimetric geoid solution and the better agreement achieved in areas with smooth topography in contrast to those with more varying one.

When low-order leveling surveys are conducted the combined use of geoid and GPS ellipsoidal heights offers a great alternative to traditional methods, providing accuracies at the ±10cm and ±6cm level for national and regional scales respectively. On the other hand, when first-order networks are concerned, meaning that the accuracies wanted are at the sub-cm-level, the presented method is not a feasible alternative to traditional leveling. To achieve a sub-cm-level accuracy with the use of geoid and GPS orthometric heights, more accurate and higher resolution gravity data should be obtained. Overall, more sophisticated methods, which will take into account the a-priori accuracies of the different height data, need to be employed and further investigated.

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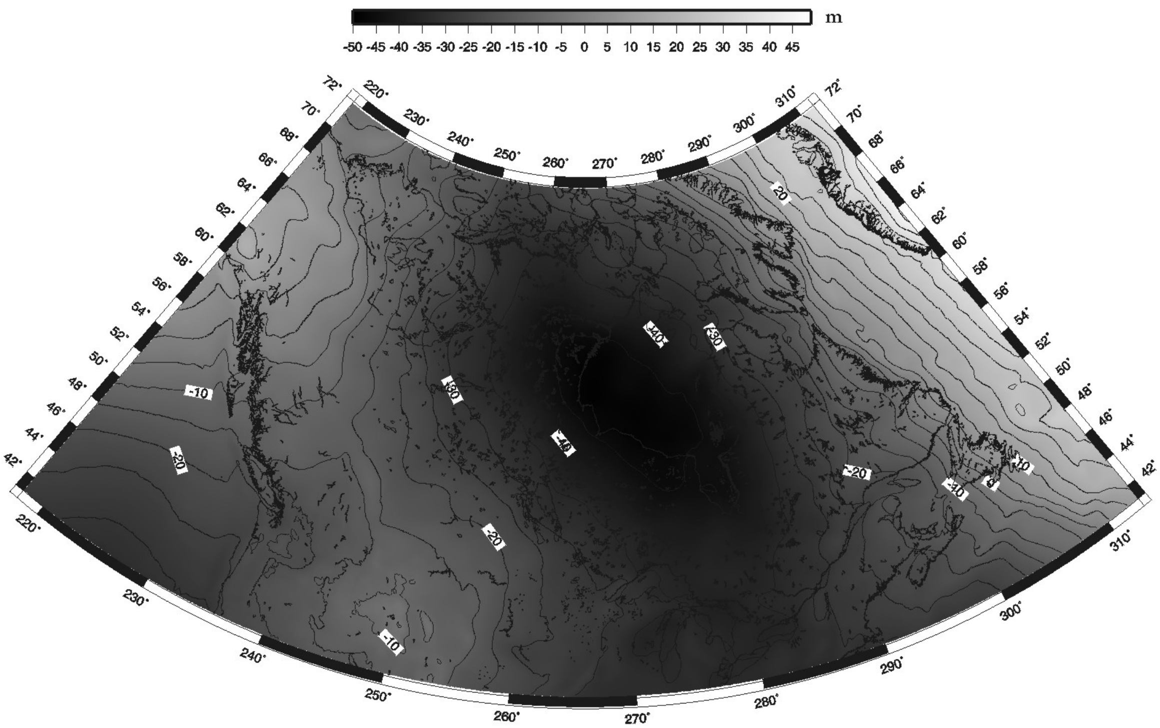
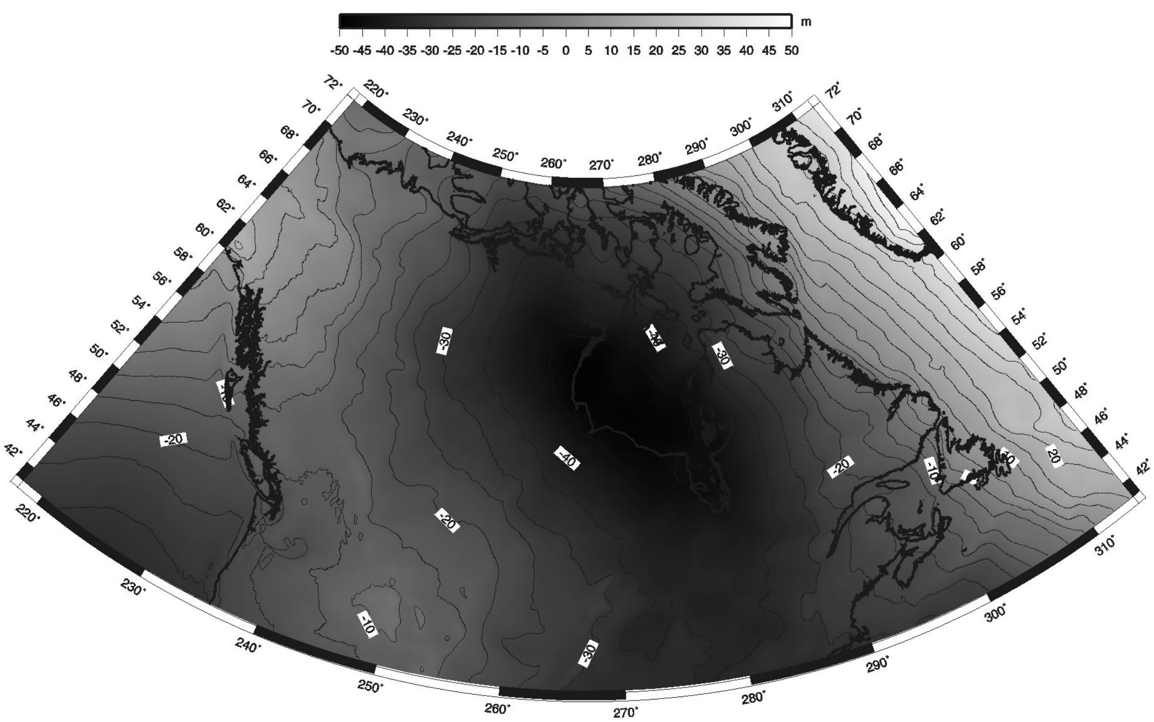
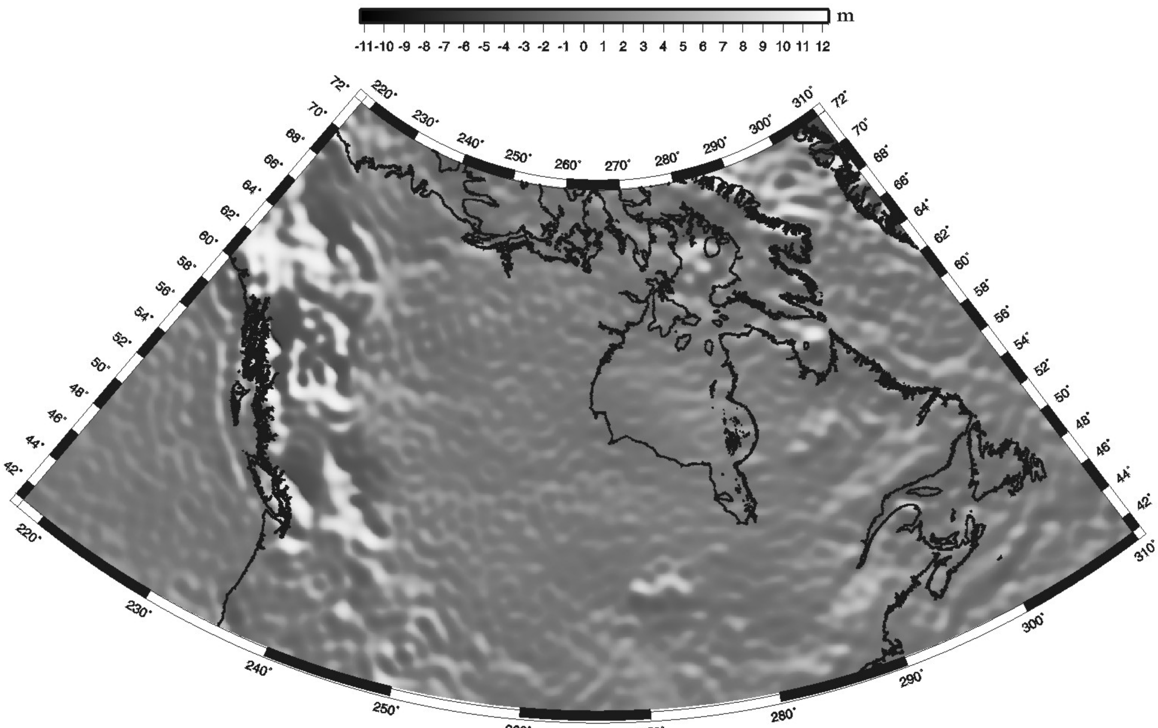


(a)



(b)

Figure 1: Geoid undulations from GPM98b geopotential model (a) and GSD2000 (b) (contour interval 5m).



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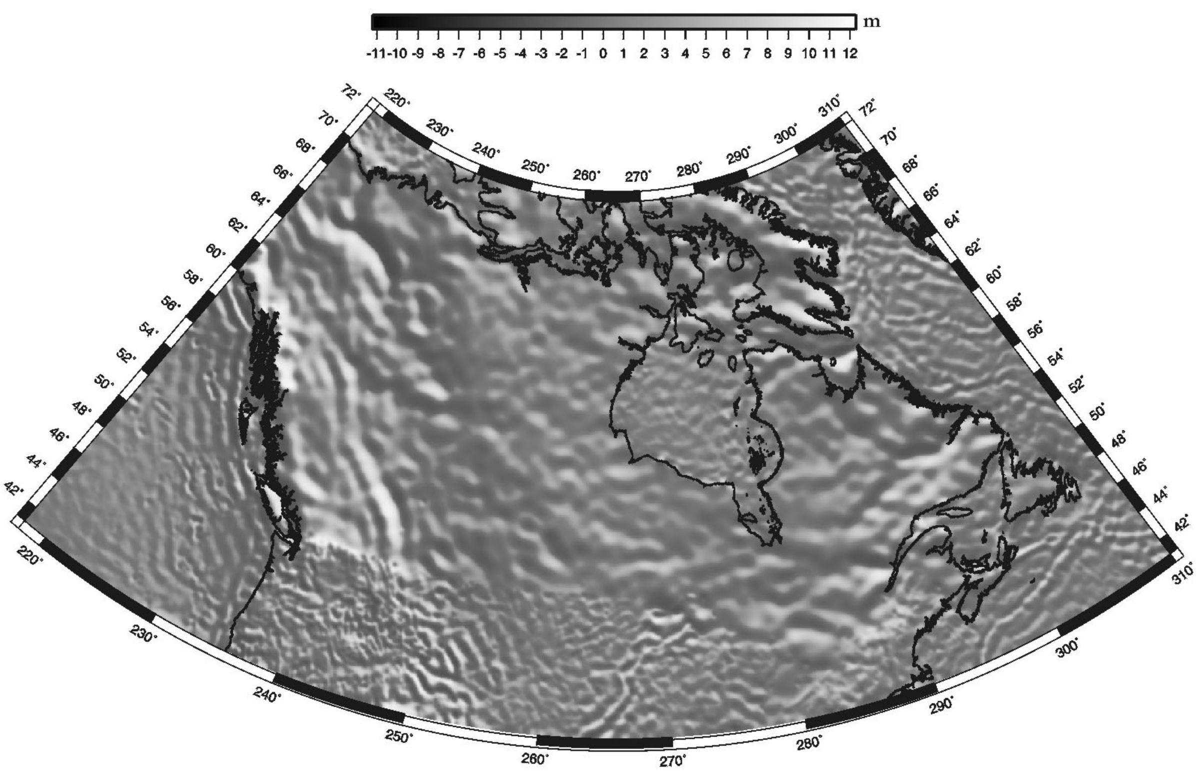
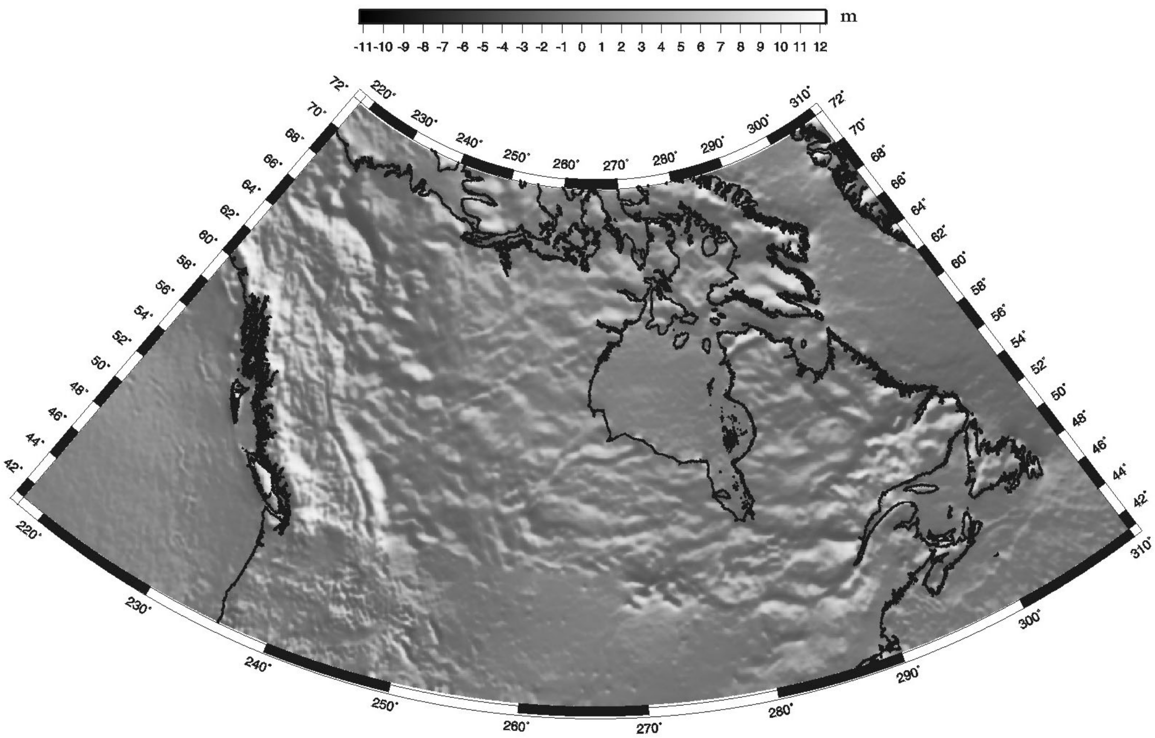
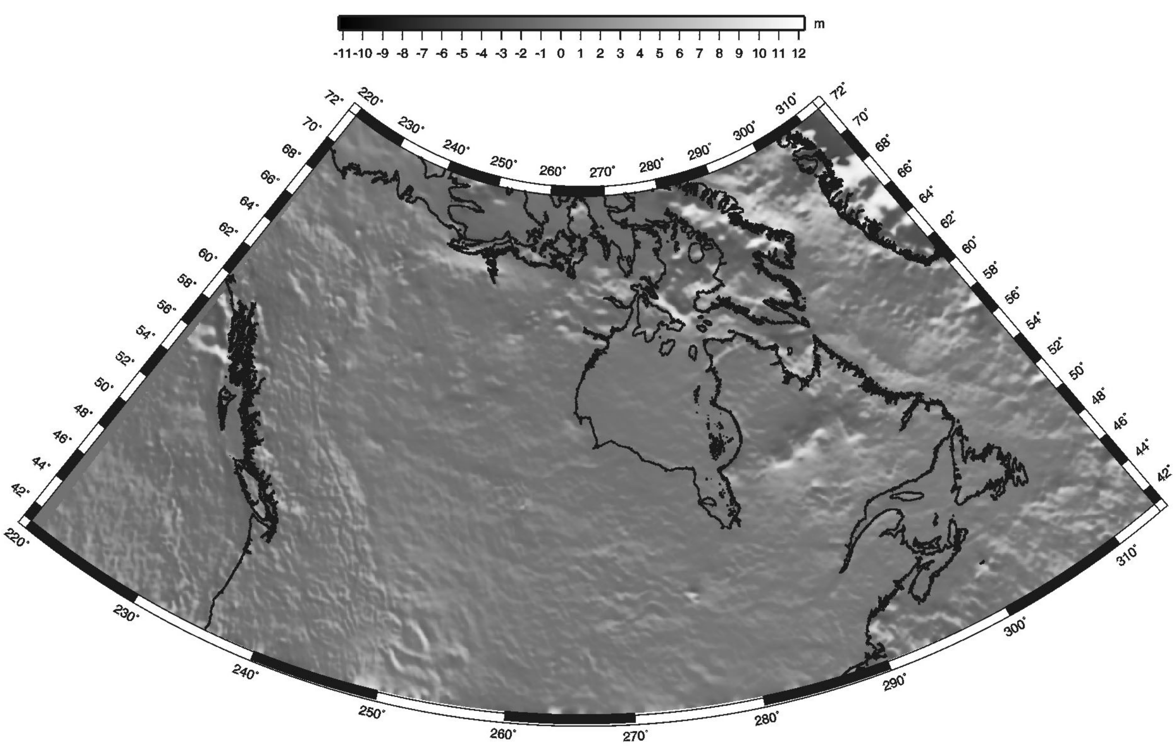


(b)

Figure 2: Geoid height differences between EGM96 and OSU91A (a) and EGM96 and GPM98b (b).



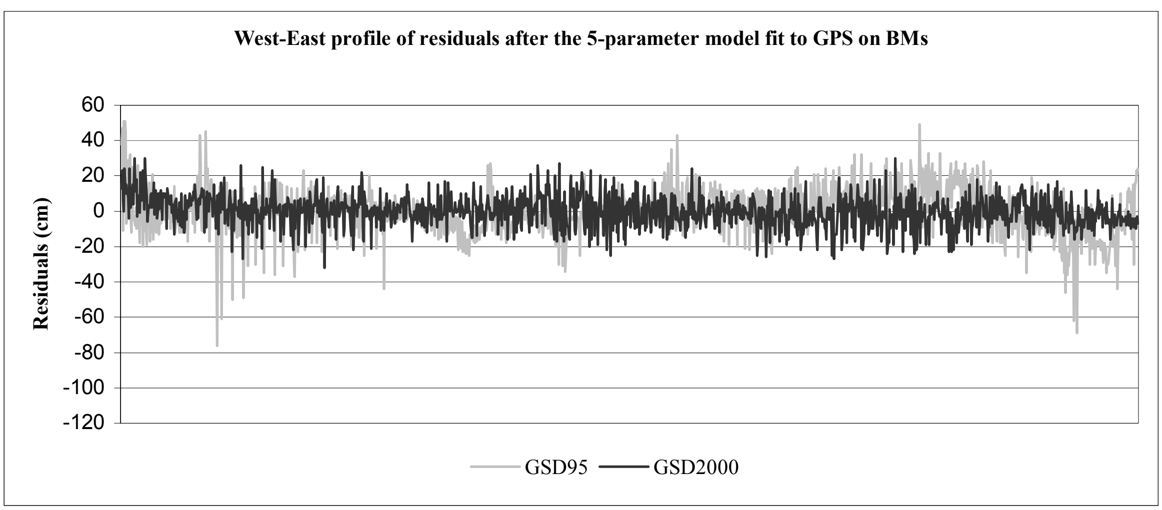
(a)



(b)

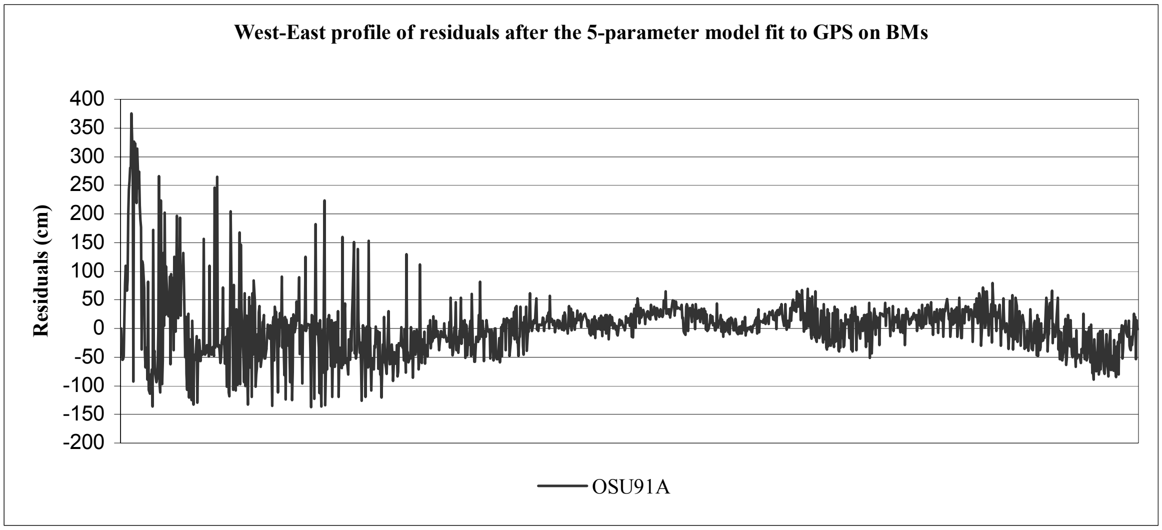
Figure 3: Geoid height differences between GSD95 and GPM98b (a) and GSD2000 and GSD95 (b).

Fig. 4: Distribution of GPS benchmarks in Canada.



(a)

Figure 5: West – East profiles of residuals after bias and tilt fit with the five-parameter model for GSD95 and GSD2000 (a), OSU91A (b), EGM96 (c), and GPM98b (d).



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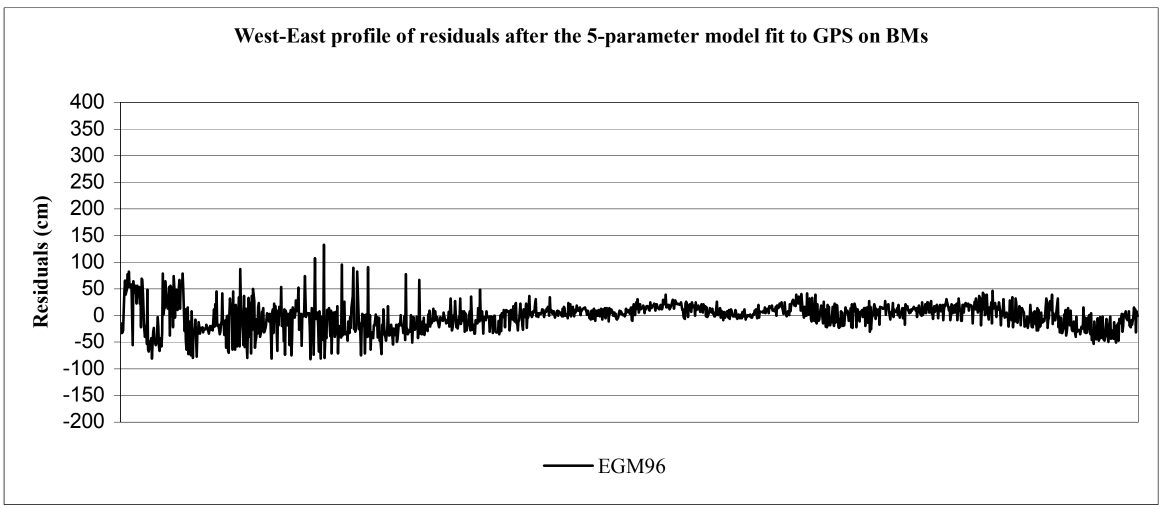
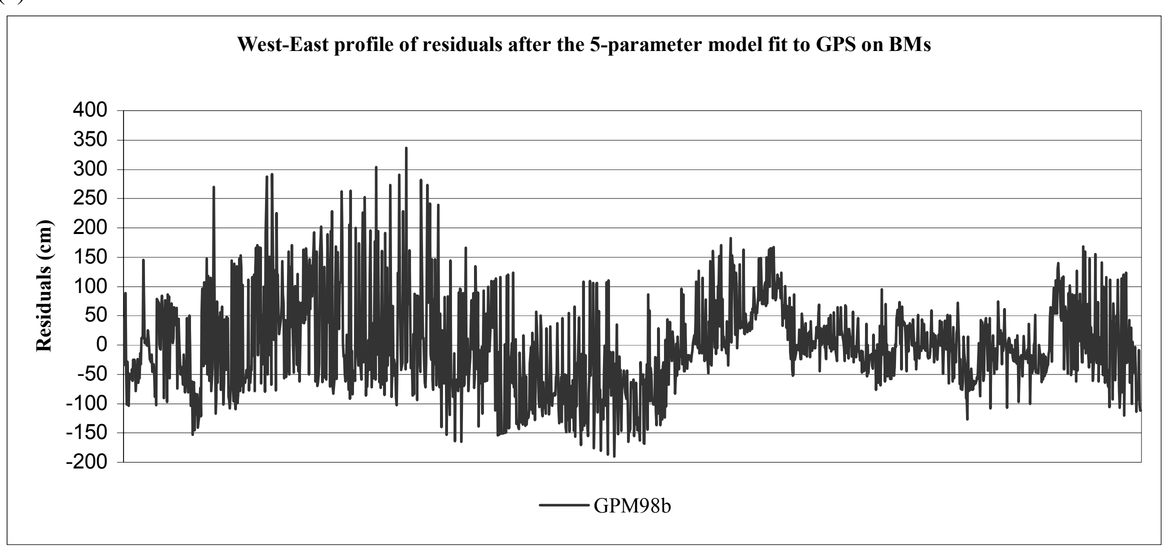


(c)



(d)

Figure 5: (continued)